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AN IDEA FOR FURTHER SIMPLICITY IN SEISMIC NONLINEAR RESPONSE HISTORY ANALYSIS WHEN ELIMINATING THE STOPS BECAUSE OF FAILURE OF NONLINEAR SOLUTIONS

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Abstract

In nonlinear response history analysis, it may be acceptable to continue the analysis, even when the iterative nonlinear solutions fail. This approach has recently been applied to nonlinear response history analysis according to the seismic code of New Zealand, NZS 1170.5:2004. The numerical tests have displayed that not only the above-mentioned claim is valid, but also can considerably accelerate the analysis. The resulting analysis is hence simpler and faster than the conventional analysis according to NZS 1170.5:2004. In this paper, attention is paid to that the analysis according to NZS 1170.5:2004 or its revised version addressed above is based on sequential time integration computations, and after the first computation, each new computation is performed with a time step half the step in the previous computation. The idea here is to simplify the relation leading to the integration step in the first time integration computation, and perform the first computation with a larger step. As a result, the analysis will become simpler. Moreover, effectiveness of the approach addressed in the start of this abstract is more likely, when the integration step is larger. Accordingly, by applying the suggestion stated in this paper, the analysis may become faster, because of two reasons. An example is studied, considering variations of nonlinear solutions' parameters and the severity of the nonlinear behavior. The numerical results show that, when performing the first time integration computation with steps equal to the step of the earthquake record, the analysis is always simpler and the run-time may decrease considerably. For all seismic nonlinear response history analyses, the suggestion in this paper is applicable, and simplifies the calculations. The analysis run-time however reduces in some of the analyses, and for these cases, the change in the accuracies can be negligible. Further tests are essential.

Keywords: Seismic Nonlinear Response History Analysis, NZS 1170.5:2004, Integration Step, Response Accuracy, Analysis Simplicity, Analysis efficiency.

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1 INTRODUCTION

True behavior of structural systems is nonlinear and dynamic, especially when the structures are subjected to severe earthquakes [1-3]. Nonlinear response history analysis using an integration scheme for time-stepping and an iterative method for nonlinear solution is the most versatile approach for the analysis [3, 4], irreplaceable in many cases [5]. Complexities however exist, including:

(a) The number of the analysis input data is considerable.
(b) The possibility of failure of nonlinear solutions exists.
(c) The analysis run-times are generally high.
(d) The analysis and the resulting responses are in general inexact.

Because of these features, application of nonlinear response history analysis is not well-accepted, in the structural/earthquake engineering society. The preference is mostly to use other dynamic analysis methods; see [1, 2, 6-10]; the other analysis methods consider different assumptions and approximations in modeling structural nonlinearities, while nonlinear response history analysis can model the nonlinearities in most detail. Considering these, and the fact that nonlinearities are of the major contributors in seismic behavior at severe earthquakes, it is, for important cases, essential to simplify and accelerate seismic nonlinear response history analyses. This can encourage the engineers in charge of seismic structural analysis to use nonlinear response history analysis in their projects. With this perspective, a step towards faster and simpler analysis is recently taken [11-13]; and based on that, the only legalized procedure for nonlinear seismic response history analysis, i.e. the procedure in the seismic code of New Zealand, NZS 1170.5.2004 [6, 7], is considerably improved; see [13]. In more detail, by replacing the analysis procedure in [6, 7] (see Fig. 1(a)) with the procedure in Fig. 1(b) [13], the possibility of failure of nonlinear solution, though persists, will not complicate the analysis. In this paper, the objective is to improve the procedure in Fig. 1(b), by simplifying the second step of the procedure, inherited from the seismic code of New Zealand, NZS 1170.5:2004 [6, 7]. An initial study is presented very recently in [14]; however, in that study [14], three changes are considered simultaneously, and besides, the presented example is not sensitive to the change that will be proposed in this paper. In continuation, first the Step 2 in the procedures in Figs. 1(a) and 1(b) will be simplified. The resulting procedure, will then be tested through an example, considering variations of three parameters. The observations, and the importance of the achievements, will later be discussed, and finally, the paper will be closed with a brief set of the achievements and an overlook of the future.

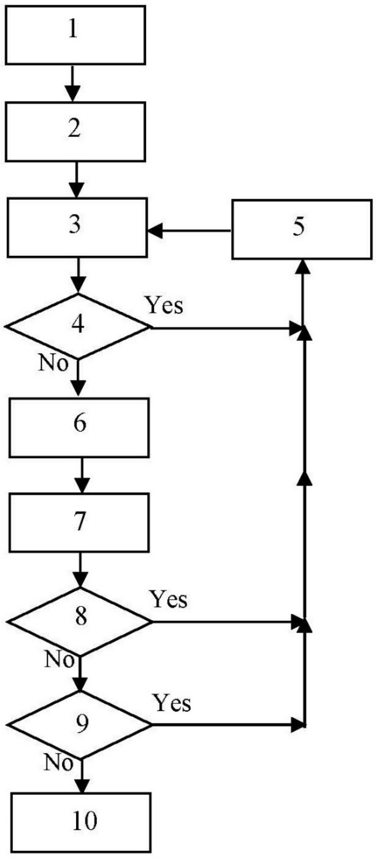
2 AN IDEA AND SUGGESTION TO CHANGE THE ANALYSIS PROCEDURE

The second step of either the analysis procedures in the seismic code of New Zealand, NZS 1170.5:2004 [6, 7] or the procedure proposed in [13] talks about the integration step in the first time integration computation of the analysis; see Fig. 1. The consequence is obtaining the integration step of the first time integration computation from:

$$\Delta t \cong\leq \mathrm{Min}\left(\frac{T_1}{\chi}, T_\mathrm{n}, \Delta t_\mathrm{cr}, \Delta t_\mathrm{CFL}, {}_f\Delta t,\ 0.01\right) \tag{1}$$

In Eq. (1), T_1 is the period associated with the first translational natural mode of vibration, χ equals 100, unless when the behavior is involved in impact and $\chi = 1000$, T_n is the period of the highest mode, in the same direction as T_1, required to achieve the 90 % mass as described in the modal response spectrum method [1, 6], Δt_cr is the upper-bound because of the linear

1. Selection of the integration scheme and the method and details of iterative nonlinear solution; $i = 0$
2. Selection of the integration step with attention to the equation in the code, considering the characteristics of the structure, earthquake record, and the integration scheme
3. Carry out a direct time integration analysis
4. Has the analysis stopped before the end of the time interval because of problems in convergence of nonlinear solution?
5. Halve the integration step
6. $i = i + 1$
7. Determine the maximum absolute response (peak) of the carried out analysis
8. Is $i = 1$ or one of the last two direct time integration has not ended the analysis time interval?
9. Is the relative difference of the last two peaks more than 5%?
10. The last response is final; stop



(a)

1. Selection of the integration scheme and the method and details of iterative nonlinear solution; $i = 0$
2. Selection of the integration step with attention to the equation in the code, considering the characteristics of the structure, earthquake record, and the integration scheme
3. Carry out a direct time integration analysis
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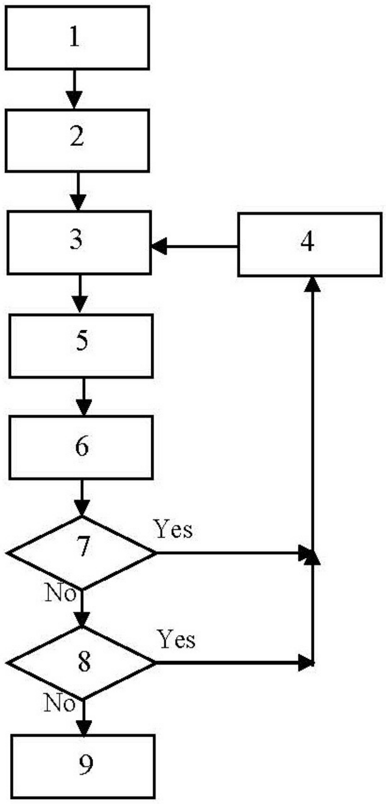

(b)

Figure 1: Two procedures for nonlinear response history analysis: (a) Stated in the seismic code of New Zealand, NZS 1170.5:2004 [6, 7], (b) Proposed in [13].

theory of numerical stability [15], Δt_{CFL} is the upper-bound because of the CFL condition [16], and ${}_f\Delta t$ is the excitation's digitization step [1-3]. (Because of the term "0.01" in Eq. (1), it is essential to note that all the terms in Eq. (1), including the "0.01", are in seconds.)

Mathematically, the integration step obtained from Eq. (1) is equal to or smaller than each of the terms in the parentheses in the right hand side of this equation. This implies that by omitting five of the six terms in the parentheses, the resulting integration step may enlarge. The run-time of the response history analysis may hence reduce; see [3], unless when the only remaining term in the parentheses is the actual minimum of the six terms. From the point of view of simplicity, such an approach will always simplify the analyses and in particular the pre-processing computation will be simplified, because of the no need to compute most of the terms in Eq. (1). And about the accuracy, it is worth paying attention to the following facts:

1. The main purpose of the terms in the right hand side of Eq. (1) is the accuracy of the target response, including numerical stability.
2. The accuracy of the target response is similarly checked in the ending four steps of the two analysis procedures; see Fig. 1.

Therefore, even without the accuracy considerations in Step 2 of the analysis procedure, the final target response will satisfy the accuracy requirements, in the ending steps of the two procedures; see Fig. 1. Accordingly, eliminating five of the six terms in the parentheses at the right hand side of Eq. (1) will not necessarily weaken the accuracy of the final response.

Consequently, the suggestion in this paper is to obtain the integration step, for the first time integration computation, simply from:

$$\Delta t = {}_f\Delta t \tag{2}$$

A main result is simplification of the computation. Especially the computations essential for setting the values of T_1, T_n, Δt_{cr}, Δt_{CFL} are eliminated. Besides, in view of the sequential nature of the procedures in Fig. 1 and the ending steps in these procedures, using Eq. (2) instead of Eq. (1) may reduce the analysis run-time with negligible effects on the accuracy.

3 NUMERICAL INVESTIGATION

3.1 Preliminary notes

For the first example, presented in [13], the results of Eqs. (1) and (2) are both equal to 0.01 seconds, and hence, though the change proposed in this paper can be applied and simplify the pre-processing computations, there will be no change in the target response and analysis run-time. The case is different for the second example.

The second example, in [13], is studied here, using Eq. (2) instead of Eq. (1), in the procedure in Fig. 1(b). The results are compared with the results obtained from the two procedures in Fig. 1. The main purpose is to check the analysis run-times. The accuracy is studied from the point of view of the ending lines of the procedures, and the more simplicity of the analysis is obvious. Besides, the results of the following variations are also tested:

$$\bar{k} = 4,5,6 \tag{3}$$

$$\bar{\delta} = 10^{-2},10^{-4},10^{-6} \tag{4}$$

$$I_n = 1,3 \tag{5}$$

In Eqs. (3-5), $\bar{k}$ stands for the maximum number of iterations [13, 17, 18], $\bar{\delta}$ is the nonlinear tolerance [13, 17, 18], and I_n is a multiplier of the earthquake record, implying the severity of nonlinear behavior [13, 19, 20].

3.2 The problem and some further details of the analyses

The problem under study is defined in Fig. 2 and Table 1 (${}_f\Delta t$ is the earthquake's digitization step; $\ddot{u}_g$ denotes the ground acceleration; g implies the acceleration of gravity, and m_i and k_i stand for the mass and stiffness of the i th floor, respectively) and reasons for the nonlinear behavior are implied in Fig. 3. The lateral acceleration of the fourth floor (important for human comfort [21]) is considered as the target response. The C-H ($\rho_\infty = 0.8$) scheme [22] is set for time integration, and fractional time stepping [23, 24] is selected as the iterative nonlinear solution method. Finally, in view of Eqs. (1) and (2), the integration steps in the beginning of the two procedures in Fig. 1 are equal to 0.0025 seconds (see [13]), while the integration step obtained from the suggestion in this paper equals 0.02 seconds.

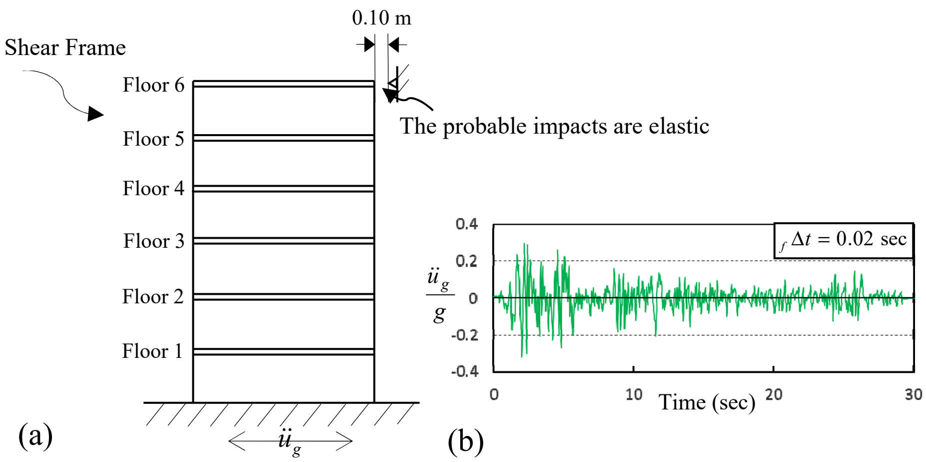

Figure 2: A pictorial introduction to the example: (a) Structural model, (b) Ground acceleration [13].

Property	Floor (i)					
	1	2	3	4	5	6
$10^{-9} \times m_i$ (Kg)	1.8	1.8	1.8	1.8	0.6	0.6
$10^{-11} \times k_i$ (N/m)	1.2	1.2	1.2	1.2	0.2	0.2
Damping (N sec/m)	Classical [1], considering 2% damping for the 1st and 3rd natural modes					

Table 1: Main properties of the structural model [13].

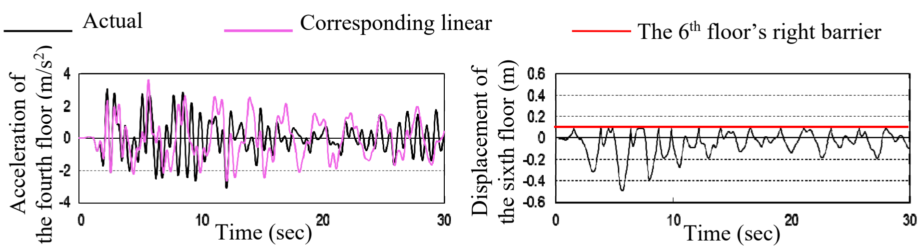

Figure 3: Two exact responses: (a) Acceleration of the fourth floor, (b) Displacement of the sixth floor [13].

3.3 The numerical results

In view of the presented discussions, the eighteen cases in Eqs. (3)-(5), Fig. 2, and Table 1, are undergone three time history analyses each, according to the two procedures in Fig. 1 and that implied in Section 2. The results are reported in Tables 2-7, where in each cell, the first number introduces the peak of the final target response, the second number stands for the total number of time integration computations, the third number denotes the total number of the integration steps (representing the analysis run-time), and the last number addresses the percentage of the run-time reduction compared to when using the procedure in Fig. 1(a).

3.4 A review on the observation

Even without any test, it is obvious that the proposed suggestion is applicable to all seismic nonlinear response history analyses, simplifies the analyses, and may affect the analysis run-time and accuracy. Besides, in the view of the numerical investigation, reported in Sections 3.2 and 3.3:

(a) The simplification because of the suggestion in this paper, has negligibly affected the accuracy of the target response. (The same is true for the simplification because of the change proposed in Fig. 1(b) compared to the procedure in Fig. 1(a).)
(b) The change proposed in this paper has in all cases reduced the analysis run-time of the analyses according to the procedures in Fig. 1.
(c) The reductions in the analysis run-time, addressed in the previous observation, are considerable, up to about 60 fold; see the case $\bar{k} = 4$ and $\bar{\delta} = 10^{-6}$ in Table 7.

To explain the first observation, it is sufficient to pay attention to the fact that the error-control in the last steps of the procedure in Fig. 1(a) does not change in Fig. 1(b) or by the suggestion in this paper. Therefore, the simplification is achieved, without meaningfully sacrificing the accuracy of the target response. Explanation of the second observation is simple, considering the role of the time integration step in the analysis run-time [3] and the fact that the result of Eq. (2) is always larger than or equal to the result of Eq. (1). Explanation of the third observation seems complex and dependent on the difference between the results of Eqs. (1) and (2), and is therefore problem-dependent. Accordingly, any further explanation is postponed to until after studying more examples.

4 DISCUSSION

In view of the small number of the presented tests, the only achievements of this paper are:

(1) A change is proposed in a recent procedure for seismic nonlinear response history analysis [13] that can simplify the analysis with negligible effects on the accuracy.
(2) When replacement of Eq. (1) with Eq. (2) changes the integration step in the first time integration computation, the new procedure may considerably (up to about 60 fold) accelerate the nonlinear time history analysis.

Considering that from the points of view of structural and earthquake engineering, the main weak points of seismic nonlinear response history analysis are the complexity of the analysis and the considerably high analysis run-time, the above-mentioned achievements are of high practical importance and may increase engineers interest in using nonlinear response analysis, and eventually better study of the structures' seismic behaviors and better seismic designs.

In view of the above-mentioned importance and that the presented numerical study is brief, the future of this study is promising at least in two directions: (1) extension of the numerical study, and (2) detailed explanation of the observations. Finally, it is worth noting that the above-mentioned simplicity is independent of the continuous decrease of the records digitization steps, which originate in the improvements in measurement instrumentation [25].

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	3.1071, 2, 40515, 0	3.0842, 2, 40892, 0	3.0854, 3, 84667, 0
5	3.1071, 2, 40515, 0	3.0465, 2, 40892, 0	3.0845, 2, 41287, 0
6	3.1071, 2, 40515, 0	3.0842, 2, 40892, 0	3.0845, 2, 41287, 0

Table 2: Results of the analysis according to the procedure in Fig. 1(a), when $I_n = 1$.

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	3.1071, 2, 40515, 0	3.0842, 2, 40892, 0	3.0845, 2, 41252, 51.27%
5	3.1071, 2, 40515, 0	3.0465, 2, 40892, 0	3.0845, 2, 41287, 0
6	3.1071, 2, 40515, 0	3.0842, 2, 40892, 0	3.0845, 2, 41287, 0

Table 3: Results of the analysis according to the procedure in Fig. 1(b), when $I_n = 1$.

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	3.0465, 2, 4626, 88.58%	3.0666, 3, 11173, 72.68%	3.0660, 3, 11603, 86.30%
5	3.0465, 2, 4626, 88.58%	3.0666, 3, 11173, 72.68%	3.0663, 3, 11698, 71.67%
6	3.0465, 2, 4626, 88.58%	3.0666, 3, 11173, 72.68%	3.0663, 3, 11709, 71.67%

Table 4: Results of the analysis according to the procedure implied in Section 2, when $I_n = 1$.

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	12.5919, 2, 40560, 0	12.5596, 2, 41066, 0	12.5524, 5, 337888, 0
5	12.5912, 2, 40560, 0	12.5596, 2, 41066, 0	12.5595, 2, 41707, 0
6	12.5919, 2, 40560, 0	12.5596, 2, 41066, 0	12.5595, 2, 41707, 0

Table 5: Results of the analysis according to the procedure in Fig. 1(a), when $I_n = 3$.

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	12.5919, 2, 40560, 0	12.5596, 2, 41066, 0	12.5595, 2, 41610, 87.69%
5	12.5912, 2, 40560, 0	12.5596, 2, 41066, 0	12.5595, 2, 41707, 0
6	12.5919, 2, 40560, 0	12.5596, 2, 41066, 0	12.5595, 2, 41707, 0

Table 6: Results of the analysis according to the procedure in Fig. 1(b), when $I_n = 3$.

$\bar{k}$	$\bar{\delta}$		
	10^{-2}	10^{-4}	10^{-6}
4	12.5962, 2, 4843, 88.06%	12.5731, 2, 5582, 86.41%	12.5731, 2, 5794, 98.29%
5	12.5962, 2, 4843, 88.06%	12.5731, 2, 5582, 86.41%	12.5731, 2, 6021, 85.56%
6	12.5962, 2, 4843, 88.06%	12.5731, 2, 5582, 86.41%	12.5731, 2, 6150, 85.25%

Table 7: Results of the analysis according to the procedure implied in Section 2, when $I_n = 3$.

5 CONCLUSION

A change in a recent procedure for seismic nonlinear response history analysis is proposed and tested through an example and its several variations. As displayed in Fig. 4, the suggestion is setting the integration step in the first time integration computation of the analysis procedure to the earthquake record's digitization step. The tests and the theory demonstrate:

(a) Simplification of all analyses by the suggested change.
(b) The fact that by the proposed change considerable reduction in the analysis run-time is probable, while the loss of the accuracy is negligible.

In view of the role of nonlinear seismic response history analysis and its simplicity and analysis run-time, in better acceptance of this analysis in structural and earthquake engineering, and besides, the observed considerable reductions in the analysis run-time, further study on the presented discussion and specifically studying further examples is recommended.

1. Selection of the integration scheme and the method and details of iterative nonlinear solution; $i = 0$
2. Set the integration step equal to the digitization step of the earthquake record
3. Carry out a direct time integration analysis
4. Halve the integration step
5. $i = i + 1$
6. Determine the maximum absolute response (peak) of the carried out analysis
7. Is $i = 1$?
8. Is the relative difference of the last two peaks more than 5%?
9. The last response is final; stop

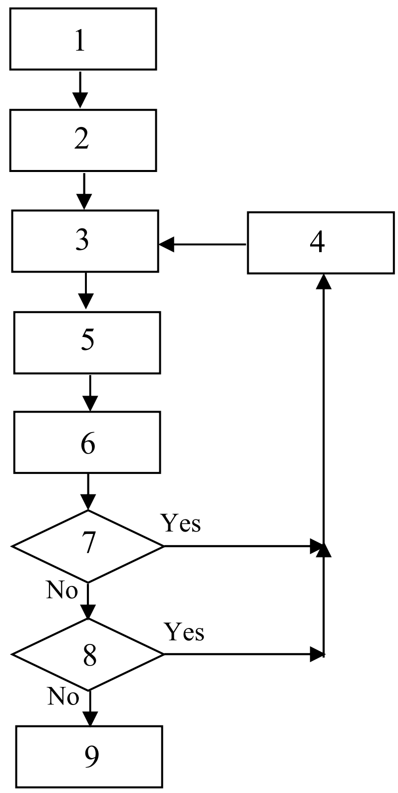

Figure 4: A new procedure for nonlinear response history analysis, based on the procedures stated in the seismic code of New Zealand, NZS 1170.5:2004 [6, 7], later revised and improved in [13].

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