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INVESTIGATIONS ON THE ENERGY-DISSIPATION CHARACTERISTIC OF THE ROCKING FOUNDATION IN LAYERED SOIL SUBJECTED TO DYNAMIC LOADINGS

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Abstract

Past studies have demonstrated that the rocking behavior of shallow foundations offers significant isolation benefits. This mechanism achieves seismic isolation by extending the natural period of the structure through soil-structure interactions. Additionally, the rocking behavior absorbs seismic energy through soil damping. This paper presents an impedance-based approach to investigate the energy-dissipation rate of rocking foundations in layered soil subjected to dynamic loading. The proposed method considers both material damping and radiation damping in the layered soil. It was found that soil depth, foundation mass ratio, and external forcing frequency significantly influence the energy dissipation rate of rocking foundations. The vibrational behavior of rocking foundations in the rotational direction may be more effective at dissipating system energy compared to the horizontal direction. Furthermore, as the foundation mass ratio gradually increases, a peak energy dissipation rate can be observed within the low-frequency range. When the forcing frequency surpasses the frequency at which the peak energy-dissipation rate occurs, the energy-dissipation rate begins to decline as frequencies increase. In summary, the research findings should aid in incorporating the energy-dissipation characteristics of soil layers into the analysis and design of rocking foundations, thereby enhancing the seismic resistance of bridge structures. Furthermore, the proposed method shows significant potential for estimating the energy-dissipation rates of rocking foundations during the preliminary design phase.

Keywords: Soil-Structure Interaction, Foundation Vibration, Earthquake Engineering, Soil Dynamics.

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1 INTRODUCTION

Seismic design of bridge systems has become increasingly critical in ensuring the resilience and functionality of transportation infrastructure in earthquake-prone regions. As our understanding of seismic behavior and structural dynamics has advanced, the methodologies for designing bridges capable of withstanding the dynamic forces generated during seismic events have also progressed. The devastating consequences of past earthquakes, such as the 1971 San Fernando event, have highlighted the necessity for resilient seismic design to avert significant failures and maintain essential transportation connections. By incorporating advanced technologies, including seismic isolation and energy dissipation devices, engineers can now design bridges that not only survive major earthquakes but also remain operational in their aftermath, significantly reducing the economic and social costs associated with post-earthquake recovery. Recently, several studies have indicated the behavior of rocking foundations may generate seismic isolation effects to reduce the internal force in bridge columns.

Housner [1] first observed the seismic isolation benefits of rocking foundation behavior during the strong 1960 Chile earthquake. He noted that structures that experienced rocking were able to survive the intense shaking with minimal or no structural damage, while some stable and rigid structures collapsed. Gajan *et al.* [2] conducted centrifuge shaking table tests on rocking soil-foundation interaction (SFI) systems to examine the mechanical behavior of rocking foundations under cyclic and seismic loading. They pointed out that during strong earthquakes, the rocking mechanism of shallow foundations may induce nonlinear soil behavior, effectively dissipating seismic energy, but potentially causing permanent displacements leading to structural damage. Anastasopoulos *et al.* [3] further proposed a novel seismic design concept that involves reducing bridge foundation dimensions. This allows the soil beneath the foundation to enter the plastic zone during strong shaking, absorbing input energy and preventing the formation of plastic hinges at the column base, thereby reducing the likelihood of column collapse. Sharma and Deng [4] conducted a series of field tests on rocking shallow foundations near Alberta, Canada, using scaled bridge foundation models subjected to low-frequency cyclic loadings. Their experiments also observed soil nonlinearity, energy dissipation effects, residual foundation displacements, and reduced foundation stiffness. However, Deng *et al.* [5] and El-Hawat *et al.* [6] indicated that if the rocking mechanism of the SFI system fails to significantly dissipate seismic energy, it may subject bridge piers to high inertial forces and cause large displacements of the bridge deck. Ultimately, this may lead to collisions between the deck and abutments, resulting in structural damage.

Past research has demonstrated the significant seismic isolation benefits of rocking foundation mechanisms. This mechanism extends the structural natural period through dynamic soil-structure interaction (SSI) behavior, achieving an isolation effect, and absorbs seismic energy through soil material damping and radiation damping. Consequently, the analysis and design of rocking foundations require careful consideration of dynamic SSI effects. While current research [7-10] has developed analytical methods for analyzing dynamic SSI behavior, there has been a lack of systematic investigation into the energy dissipation efficiency of foundation rocking mechanisms. This study aims to investigate the dynamic interaction behavior between rocking foundations and layered soil, and proposes a method to evaluate the energy dissipation rate for rocking foundations under seismic excitation. The proposed method can consider soil material damping and radiation damping. This study also conducts a dimensionless parametric analysis on the energy dissipation rate of rocking foundations. The analyzed parameters include embedment ratio, mass ratio, soil depth ratio, and shear-wave velocity variation rate in soil layers, systematically investigating the energy dissipation characteristics for a SFI system. This study has the potential to enhance the development of innovative foundation design strategies, which may ultimately lead to the creation of safer and more resilient bridge structures in seismic-prone areas.

2 METHODOLOGY

This study investigates the energy dissipation rate of a rigid square foundation embedded in nonuniform soil overlying a uniform half-space, as shown in Figure 1. The dynamic SFI behavior in both horizontal and rocking directions are considered, which explores the effects of foundation geometry and soil parameters on the energy dissipation of rocking foundations. The square foundation, measuring $2R$ by $2R$ and embedded to a depth E, is considered to be fully adhered to the adjacent soil. The nonuniform soil has a linear gradient in shear-wave velocity that increases with depth, while maintaining consistent properties such as Poisson's ratio (ν_1), mass density (ρ_1), and material damping ratio (ξ). The shear-wave velocity at the top of the soil layers is defined by V_{s1}. The uniform half-space is described by its shear-wave velocity (V_{s2}), Poisson's ratio (ν_2), mass density (ρ_2), and material damping ratio (ξ).

When the foundation top is subjected to a harmonic moment and reaches steady-state response, the energy dissipated by the target SFI system within one cycle of external force can be expressed as

$$E_{dr} = \int_0^{2\pi/\omega} M_o Cos[\omega t]\,\mathrm{Re}[\dot{\Theta}(t)]dt \tag{1}$$

In the equation above, M_o represents the moment amplitude, Θ is the foundation rotation angle, ω denotes the external force frequency, and t represents time. Furthermore, the elastic strain energy of the interaction system is defined as $E_{sr} = M_o^2 / (2K_{sr})$, where K_{sr} represents the static foundation stiffness in rotational direction. The energy dissipation of the system can be expressed as the product of the elastic strain energy and a dimensionless factor e_{dr}, as follows:

$$E_{dr} = E_{sr} \times e_{dr} = \frac{M_o^2}{2K_{sr}} \times e_{dr} \tag{2}$$

When the dissipated energy E_{dr} is divided by the force period T, it yields

$$\lambda_r = \frac{E_{dr}}{T} = \frac{M_o^2 V_s}{4\pi G R^4} \times f_r \tag{3}$$

$$f_r = \frac{a_0 e_{dr}}{k_{sr}} \tag{4}$$

where λ_r denotes the energy dissipation rate induced by a rocking moment, f_r is the relevant energy dissipation factor, V_s is the characteristic shear-wave velocity of the soil layer, G is the characteristic shear modulus of the soil layer, $a_0 = \omega R / V_{s1}$ represents a dimensionless frequency ratio, and $k_{sr} = K_{sr} / (GR^3)$ denotes the dimensionless rotational stiffness coefficient. Similarly, the energy dissipation rate generated by horizontal vibrations in response to a harmonic horizontal force can be further derived as follows.

$$\lambda_h = \frac{E_{dh}}{T} = \frac{H_o^2 V_s}{4\pi G R^2} \times f_h \tag{5}$$

$$f_h = \frac{a_0 e_{dh}}{k_{sh}} \tag{6}$$

where λ_h denotes the energy dissipation rate induced by a horizontal force, f_h is the relevant energy dissipation factor, H_o represents the force amplitude, e_{dh} denotes a ratio of the dissipated energy (E_{dh}) to the strain energy, k_{sh} is the dimensionless foundation stiffness coefficient in horizontal direction. In brief, the rocking response of a foundation can be induced by a rotational moment or a horizontal force. The relevant energy dissipation rate can be estimated by λ_r and λ_h, respectively, as defined in Eqs. (3)and (5). Note that the energy dissipation rates depend on the dimensionless factors, f_r and f_h. This feature makes the energy dissipation factors suitable for quantifying the energy dissipation characteristics of general SFI systems. In addition, the energy dissipation factors depend on the dimensionless factors e_{dr} and e_{dh} which are determined by foundation impedance functions. They can be further analyzed based on the principles of soil dynamics. More details can be found in the previous work [11-12]. Therefore, an impedance-based approach based on Eqs. (4) and (6) is presented in this section to investigate the energy dissipation characteristics for general SFI systems.

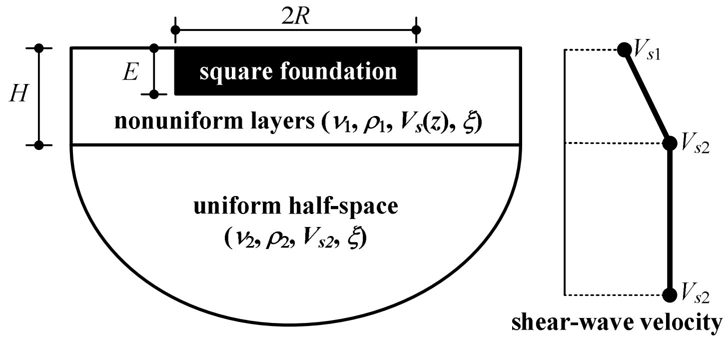

Figure 1: Simulation model of a square foundation embedded in nonuniform soil

3 RESULTS AND DISCUSSIONS

The section aims to investigate the energy dissipation characteristics of the square foundation, as shown in Figure 1, subjected to harmonic forces in horizontal and rotational directions, respectively. A finite element model has been developed to simulate the target SFI system, utilizing solid elements to represent the square foundation and horizontal layer elements to model the nonuniform soil. Relevant details of the finite element model can be found in the previous work [8, 12]. This study uses a numerical program, LSASSI [13], which is an enhanced code of SASSI, to analyze the impedance functions for the target SFI problem. SASSI is a computer code for performing finite element analyses of SSI during dynamic excitations, which was first developed by Lysmer *et al.* [14] in 1981 for mainframe systems. LSASSI was created to improve efficiency in personal computer systems. For the target SFI problem, five simulation conditions are listed in Table 1. Condition I is designed to investigate the effect of embedment ratios (T) on the energy dissipation factors. The embedment ratio, $T = E/R$, is defined as the ratio of foundation embedment depth to the half side-length. Condition II is established to evaluate the effect of foundation mass ratios. The mass ratio, $b = M_f/(\rho R^3)$, is defined as the ratio of the foundation mass to the reference soil mass. Condition III is considered to analyze the effect of shear-wave velocity ratio (V_{s1}/V_{s2}) in the nonuniform soil. Lastly, Conditions IV and V are specifically designed to investigate the effect of the depth ratio (H/R) for the nonuniform soil with a depth, H. A series of finite element models were established regarding various foundation embedment depths and soil parameters. This study uses the foundation impedance functions analyzed from LSASSI to compute the energy dissipation factors, f_r and f_h, induced by the rotational moment and horizontal force, respectively, based on Eqs. (4) and (6). The subsequent subsections will examine the distinct effects of four dimensionless parameters on the energy dissipation factors for the target SFI system.

Table 1: Simulation parameters of the target problem

Condition	Varying parameters	Fixed parameters
I	$T = 0.5, 1, 2$	$b = 1$, $V_{s1}/V_{s2} = 0.8$, $H/R = 2$
II	$b = 1, 5, 10$	$T = 0.5$, $V_{s1}/V_{s2} = 0.8$, $H/R = 2$
III	$V_{s1}/V_{s2} = 0.3, 0.6, 0.8$	$T = 0.5$, $b = 1$, $H/R = 2$
IV	$H/R= 2, 4, 8$	$T = 0.5$, $b = 1$, $V_{s1}/V_{s2} = 0.8$
V	$H/R= 2, 4, 8$	$T = 0.5$, $b = 1$, $V_{s1}/V_{s2} = 0.3$

3.1 Effect of embedment ratios

Figure 2 shows the effects of foundation embedment ratios on energy dissipation factors. As the embedment ratio increases, the energy dissipation factor significantly decreases, especially when the rotational moment acts on the foundation top. A low embedment ratio of T = 0.5 is found to have greater dissipation factor compared to higher embedment ratios. It may be attributed to the large rotations found in shallowly embedded foundations.

3.2 Effect of mass ratios

Figure 3 displays the effects of foundation mass ratios on energy dissipation factors. When the mass ratio b = 1, the foundation inertia slightly participates in dynamic interaction behavior, the energy dissipation factor is found to increase with the dimensionless frequency ratio due to the influence of radiation damping in soil. As the mass ratio gradually increases, a peak energy dissipation factor can be observed in the low-frequency range. Once the frequency ratio exceeds the frequency point that induces the peak dissipation factor, the energy dissipation rate begins to decrease with increasing frequency ratios. The analysis results demonstrate that the frequency ratio range in which the rocking foundation can efficiently dissipate energy is related to the mass ratio. Furthermore, when the foundation mass ratio is fixed, the energy dissipation factor in the rotational direction is higher than in the horizontal direction. This may suggest that rocking foundations are more capable of dissipating system energy through rotational vibrations. These findings have significant implications for the foundation design in seismic regions, as they highlight the importance of considering both mass ratio and frequency ratio in optimizing energy dissipation through SSIs.

3.3 Effect of shear-wave velocity ratios

Figure 4 illustrates the effects of shear-wave velocity ratios in nonuniform soil on energy dissipation factors. The energy dissipation factor for V_{s1}/V_{s2}=0.8 is greater than that for V_{s1}/V_{s2}=0.3 when the frequency ratio is fixed. The analyzed results imply that the nonuniform distribution of shear-wave velocity has significant influences on the energy dissipation rate for the target SFI system. A strong influence could be observed when the dimensionless frequency ratio falls within the range of 2 and 4.

3.4 Effect of layer depth ratios

Figures 5 and 6 examine the effects of layer depth ratios on energy dissipation factors when V_{s1}/V_{s2} = 0.8 and 0.3, respectively. When the shear-wave velocity ratio V_{s1}/V_{s2} = 0.8, the layer depth ratio is observed to have minimal effects on the energy dissipation factor. However, the layer depth ratio could significantly affect the energy dissipation factor when V_{s1}/V_{s2} = 0.3. In such case, the deeper nonuniform soil exhibits greater energy dissipation than the shallower soil. Hence, the layer depth ratio and the shear-wave velocity ratio jointly affect the energy dissipation rate for the target SFI problems.

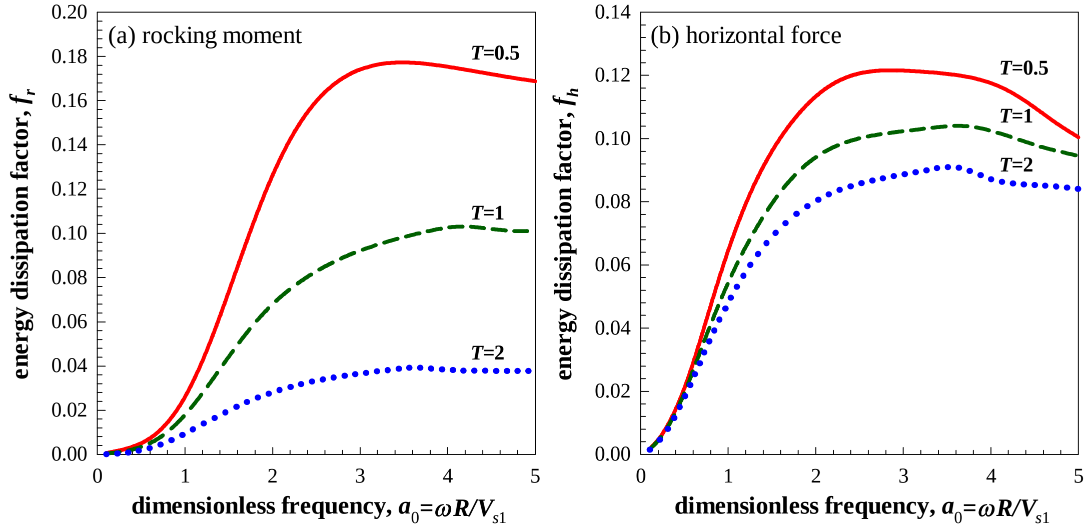

Figure 2: Energy dissipation factors for various embedment ratios (Condition I)

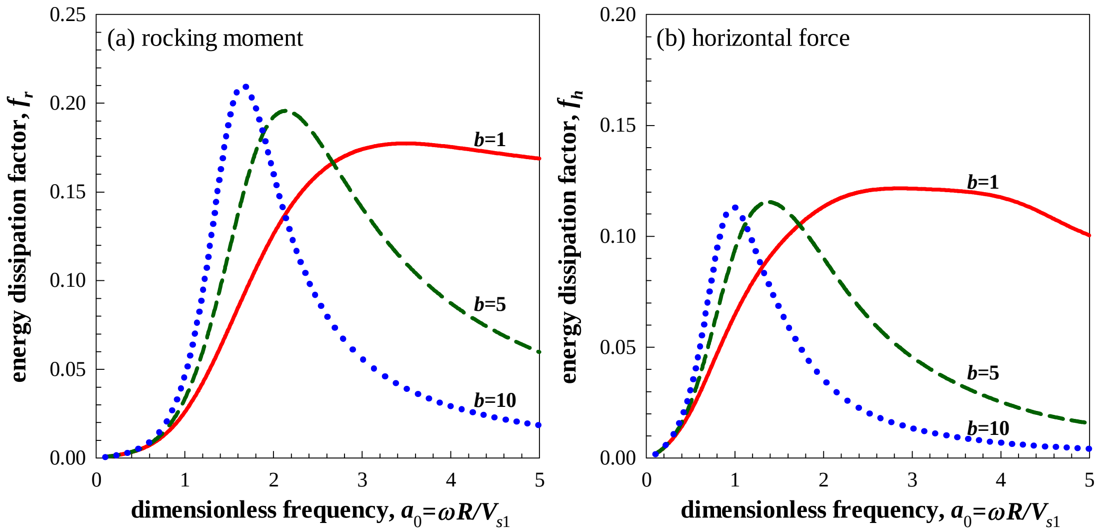

Figure 3: Energy dissipation factors for various mass ratios (Condition II)

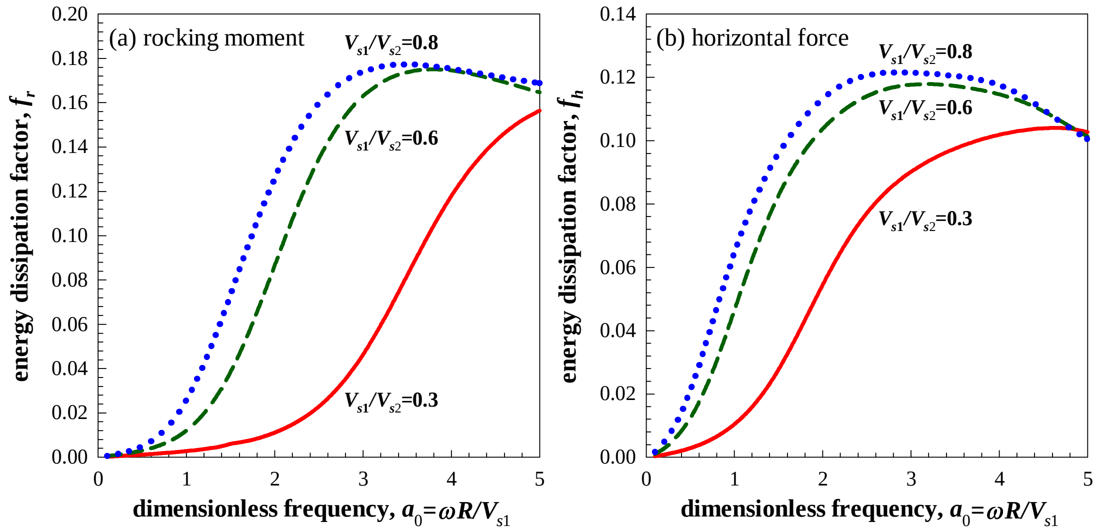

Figure 4: Energy dissipation factors for various shear-wave velocity ratios (Condition III)

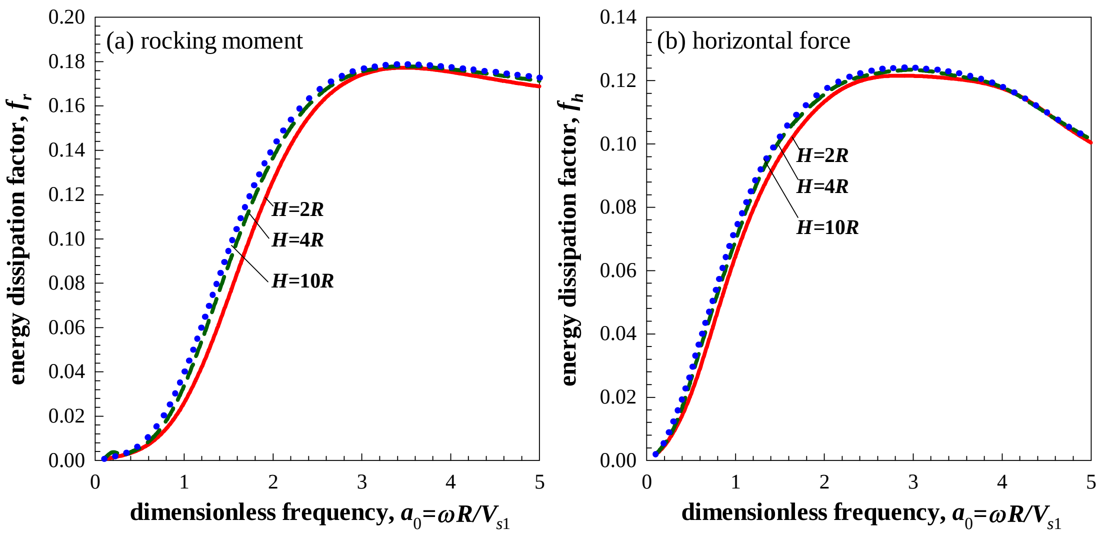

Figure 5: Energy dissipation factors for various layer depth ratios (Condition IV)

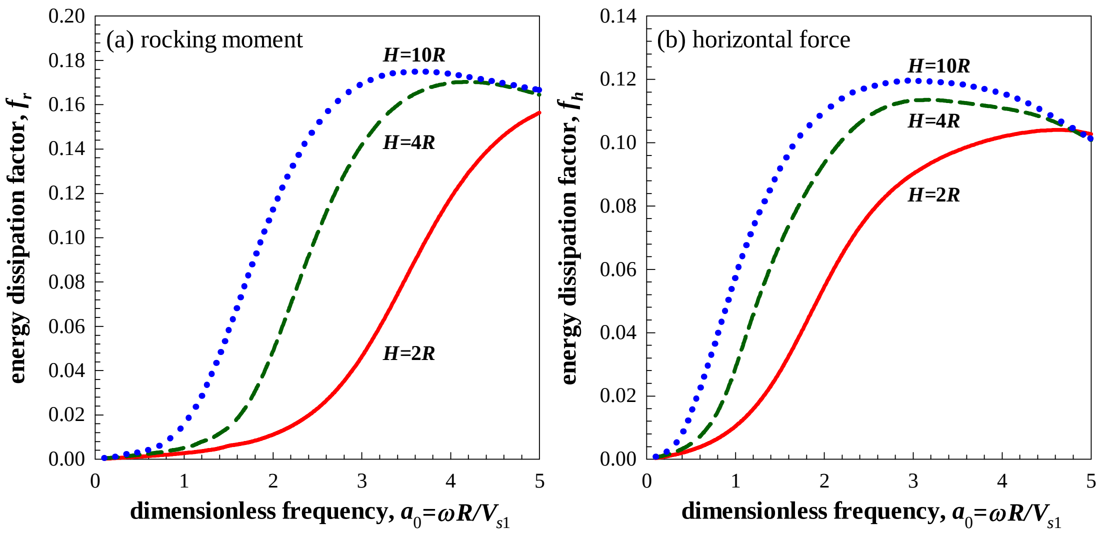

Figure 6: Energy dissipation factors for various layer depth ratios (Condition V)

4 CONCLUSIONS

This paper investigates the energy dissipation characteristics of rocking foundations in layered soil subjected to dynamic loadings, using an impedance-based approach that considers both material and radiation damping. The study revealed the significant influence of foundation embedment ratio, foundation mass ratio, shear-wave velocity ratio, layer depth ratio, and forcing frequency ratio on the energy dissipation rate for square foundations embedded in nonuniform soil. Key findings include:

- In certain instances, rocking foundations may be more effective at dissipating system energy through rotational vibrations compared to horizontal vibrations.
- As the foundation mass ratio increases, a peak energy dissipation rate can be observed within the low-frequency range. Beyond this peak, the energy-dissipation rate declines as frequency ratios increase.

- The embedment ratio significantly affects energy dissipation, with lower embedment ratios exhibiting greater energy dissipation due to higher rotations in shallowly embedded foundations.
- The shear-wave velocity ratio in nonuniform soil significantly influences energy dissipation, particularly within a dimensionless frequency ratio range of 2 to 4.
- The layer depth ratio and the shear-wave velocity ratio collectively influence the energy dissipation rate. Deeper nonuniform soil exhibits greater energy dissipation when the shear-wave velocity ratio is low.

In summary, the findings presented above can assist in integrating the energy-dissipation characteristics of soil layers into the analysis and design of rocking foundations, ultimately improving the seismic resistance of bridge structures. The proposed method shows considerable potential for estimating the energy-dissipation rates of rocking foundations during the preliminary design phase. By carefully considering parameters such as mass ratio, frequency ratio, foundation embedment, and soil properties, engineers can optimize the energy dissipation capabilities of SSIs, leading to safer and more resilient bridge designs in seismic-prone areas.

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