

STIFFENING OF STRUCTURAL COMPONENTS IN WAFFLE SLABS FOR VIBRATION CONTROL UNDER HUMAN WALKING

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Abstract

The stiffening of structural components in waffle slabs as remedial measure to reduce acceleration under human walking is assessed. The stiffened structural components were: concrete slab, beams and joist. The stiffening of the concrete slab was achieved with the increment of concrete modulus of elasticity and with the increment of the depth of the structural components, using normal or lightweight concrete. The increment of the stiffness of the beams and joist was achieved with the increment of their depths. The assessment of the stiffening of these structural components to control vibration was carried out in detailed finite element models of a waffle panel. The concrete slab and solid heads were meshed with shell finite elements, beams, joists and columns with frame finite elements. The numerical models were calibrated and validated with the natural frequency obtained with experimental ambient vibration tests. As a result of the assessment of the stiffening of structural components in the numerical models, the remedial measures to reduce the acceleration induced by human activities are: the increment of concrete modulus of elasticity, depth of structural components of waffle slabs, increment of slab thickness, increment of joists depth, but the weight is adversely unfavourable. The use of lightweight concrete is also a remedial measure to reduce acceleration, but in some cases the acceleration increases as the mass is reduced. On the contrary, the acceleration is increased with the increment of the principal beam depths.

Keywords: Two-way joist floor, human-activities, vibration, natural frequency, stiffening.

1 INTRODUCTION

A two-way joist concrete slab floor or waffle slab is a slab made of reinforced concrete with concrete ribs running in two directions on its underside. [1] developed a mathematical model for the analysis of vibration at the centre of a bay and the transmission of vibration along a waffle floor. In this mathematical model, the waffle slab was idealized as an equivalent plate section with a constant thickness, equalling the cross-sectional moments of inertia of the waffle slab section and a plate section of unit width. Additionally, the columns were idealized as linear springs. The mathematical model was evaluated with finite element models in the software ANSYS and experimental results from several manufacturing buildings with waffle slabs, resulting in good agreement. The displacement at the centre of the span and the frequency of the floor were evaluated with the increment of the slab thickness, size of the bays and the stiffness of the columns. [2] evaluated the behaviour of reinforced concrete waffle slabs under human rhythmic activities. The dynamic load model to represent the effect of rhythmic activities was the function suggested by [3], which includes periodic mathematical functions decomposed in Fourier series. In the three-dimensional numerical models, the computed acceleration and speed exceed tolerable limit values, which induces discomfort on users. [4] reported experimental tests on twenty-six waffle concrete floor slabs using the heel-drop as impact source, using accelerometer and mobile phone to register the natural frequencies and modal damping ratios. The natural frequencies were also obtained with analytical equations for isotropic and orthotropic plates, available in the literature, which were in good agreement with the experimental ones. The results for fundamental frequencies show that mobile phones are effective tools for recording acceleration in floor slabs.

In the steel design guide 11 of the [5] and [6] and [7], remedial measures to resolve floor vibration problems that can occur in existing building are given. These remedial measures are: reduction of effects, relocation, reducing mass, stiffening, retrofit, damping increase with non-structural elements, passive control in the form of tuned mass dampers, floating floor from rhythmic activities and reduction of vibration transmission. However, not all remedial solutions are appropriated for different types of floor system although some of them may be universal [7]. The response of the vibration of floor systems is mainly related with the stiffness, mass and damping of the structural elements which make it up. In this paper, practical solution of stiffening the structural components in waffle slabs is evaluated, including: modulus of elasticity, increment of the depth of slab thickness, beams and joist. Additionally, reducing mass is evaluated in the concrete slab with the use of lightweight concrete.

The outline of this paper is as follows. In section 2, the properties of the waffle slab are described in detail. In section 3, the finite element model of the waffle slab is calibrated with the experimental frequencies and accelerations. In section 4, the stiffening of structural components to mitigate acceleration under human activities is assessed. Finally, in Section 5, conclusions derived from this work are given.

2 PROPERTIES OF THE WAFFLE SLAB STUDIED

This two-way joist concrete slab was experimentally tested by [8]. This test was carried out on a multi-storey office building, located in Mexico, which consists of five storeys of 3.96 m in height and three basements for parking of 3.30 m in height, as shown in Figure 1a. The basements floors were designed and constructed with RC waffle slabs supported on a RC framed structure and the upper stories with steel-deck composite slabs supported on a steel framed structure. The tests were carried on the first basement floor, at a height of -2.66 m below the ground, which has no overloads than the dead load during the test.

The waffle slab chosen to study is the panel P_1 , localized between axes 7-8 and LL-C, which is 10.34 m in length and 10.00 m in width, shown in Figure 1b. The concrete slab lies in two-way joist with continuous span, which are also supported on three beams and on a wall at the edges. The beams are supported at the corners of the panel on solid heads over square section columns. The concrete slab is 60 mm thick with embedded sheets of 6x6 in (152.4 x 152.4 mm) welded wire reinforcement, 6/6 gage, as illustrated in Figure 2, where it is seen that the total deep of the of the slab is 410 mm. The joists are 350 mm deep with a variable thickness, 100 mm at the bottom and 190 mm at the top, as shown in Figure 2, spacing at 735 mm and built with domes of 63.5 x 63.5 cm. Principal beams are: T-4, T-5, T-H, which are 80 cm deep and 60 cm wide, and the wall is 60 cm thickness. The types of the columns are: C-7A, C-8A and C-16A, with a depth and width of 80x60, 74x75, 80x80 cm, respectively. The same column sections are at the lower and upper levels of the tested waffle slab. Columns, solid heads, beams, joist and slab were built of concrete with an ultimate compressive strength of 25 MPa and a modulus of elasticity, $E_c=21707.945$ MPa. The wall was built of concrete with an ultimate compressive strength of 30 MPa and a modulus of elasticity, $E_c=23779.862$ MPa. The dynamic concrete modulus of elasticity, E_d , was computed as $E_d=1.35E_c$ [6]. The steel reinforcement has a yield strength $\sigma_y=345$ MPa and a modulus of elasticity, $E_s=205.94$ GPa.

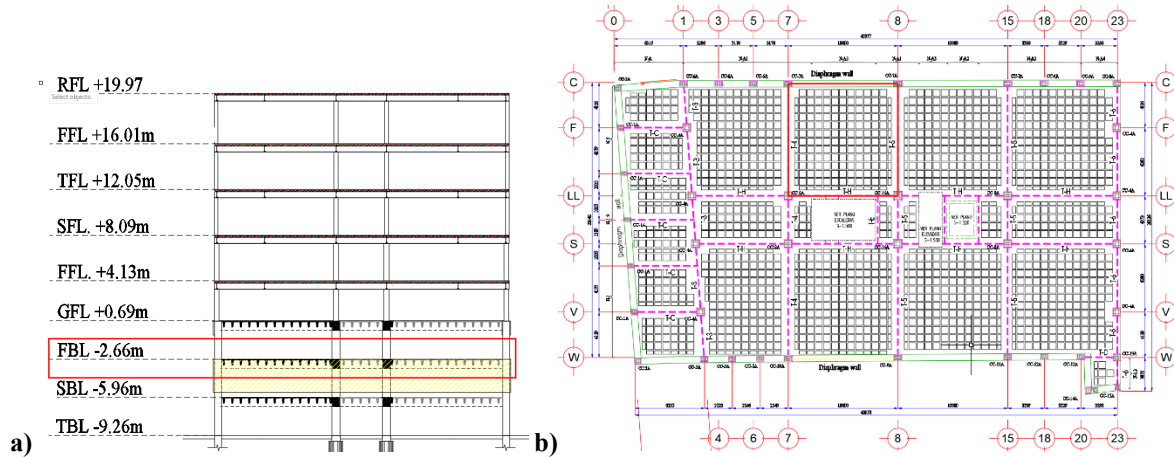


Figure 1: Building: a) stories and b) location of the interior waffle slab panel at first basement floor (taken from).

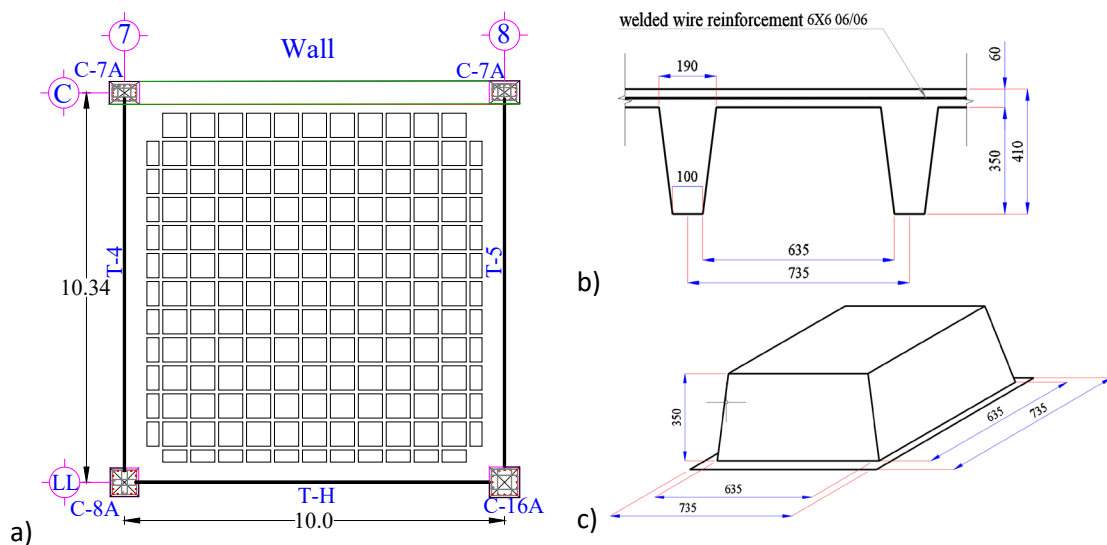


Figure 2: Details of: a) geometry of the interior waffle slab panel (m) and b) joist (mm) and c) dome dimensions (mm).

3 VALIDATION OF THE NUMERICAL MODEL

3.1 Calibration of the frequencies

The model of the isolated panel was analysed. The isolated model corresponds to the panel localized between axes 7-8 and LL-C, shown in Figure 1b. The geometry of the models was built in the Software Autodesk AutoCAD 2023 [9], which was imported to the software SAP2000 [10], where materials were defined, and sections were assigned to every structural component. Joists, beams and columns were meshed with frame finite elements, slabs and walls were meshed with shell finite elements. Half height of columns and walls were modelled above and below the waffle slab as illustrated in Figure 3, where the tree directions of the displacements were restrained at the ends of columns and walls. The isolated panel was meshed with 363 frame finite elements and 221 shell finite elements, in which three modes were calculated.

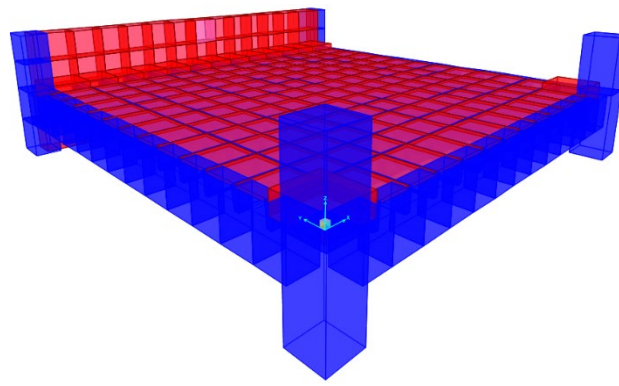
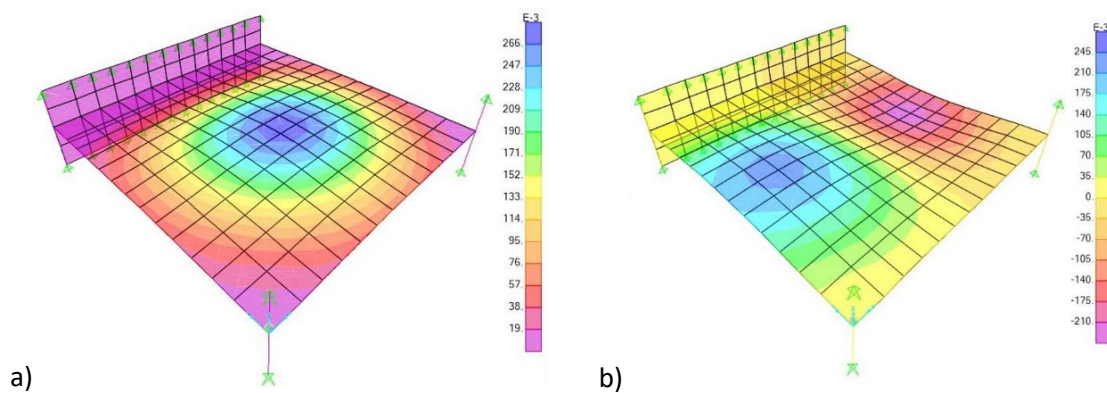


Figure 3: Two way joist concrete isolated panel model.

In the isolated panel model, the first three mode shapes were computed, which are shown in Figure 4. In the first mode, the maximum modal shape amplitude occurs at the centre of the span; in the second mode, it occurs at the quarter span of the short length; and in the third mode, it occurs at free edge. The experimental and numerically computed frequencies in the isolated model are given in Table 1, where the difference is up to 9.39% for mode 3. Consequently, it is concluded that is valid to study an isolated panel model instead of a complete floor model to save computational cost.



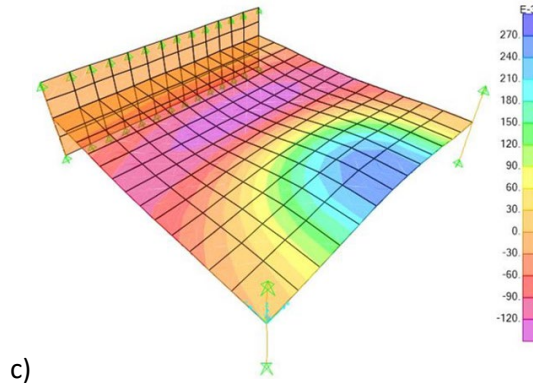


Figure 4: Modal shapes of the isolated panel model. Mode: a) 1, b) 2 and c) 3.

Table 1: Experimental and numerically computed frequencies.

Mode	Frequency (Hz)		Difference (%)
	Experimental	Isolated model	
1	13.000	12.745	-1.96
2	19.156	20.629	7.69
3	21.813	23.861	9.39

3.2 Calibration of the dynamic load model

A load models were evaluated to incorporate human walking dynamic effects in the two-way joist concrete slab panel. This load model is for a high frequency floor system, where it is assumed that it has a fundamental frequency higher than 10 Hz [11]. The walking load model, labelled as high frequency load model (HFLM), is taken as a series of consecutive steps whereby each step load is described by a polynomial (HIVOSS [12]). The normalized step load is given by:

$$\frac{F(t)}{P} = K_1 t + K_2 t^2 + K_3 t^3 + K_4 t^4 + K_5 t^5 + K_6 t^6 + K_7 t^7 + K_8 t^8 \quad (1)$$

where P is the bodyweight, the coefficients K_1 to K_8 depend on f_s and are given in Table 2. The load duration, t_s , is given by:

$$t_s = 2.6606 - 1.757 f_s + 0.3844 f_s^2 \quad (2)$$

For $t > t_s$, $F(t) = 0$.

Table 2: Coefficients K_1 to K_8 for given f_s (taken from HIVOSS 2008).

K_n	$f_s \leq 1.75$	$1.75 \leq f_s < 2$	$f_s \geq 2$
K_1	$-8 \times f_s + 38$	$24 \times f_s - 18$	$75 \times f_s - 120.4$
K_2	$376 \times f_s - 844$	$-404 \times f_s + 521$	$-1720 \times f_s + 3153$
K_3	$-2804 \times f_s + 6025$	$4224 \times f_s - 6274$	$17055 \times f_s - 31936$
K_4	$6308 \times f_s - 16573$	$-29144 \times f_s + 45468$	$-94265 \times f_s + 175710$
K_5	$1732 \times f_s + 13619$	$109976 \times f_s - 175808$	$298940 \times f_s - 553736$
K_6	$-24648 \times f_s + 16045$	$-217424 \times f_s + 353403$	$-529390 \times f_s + 977335$
K_7	$31836 \times f_s - 33614$	$212776 \times f_s - 350259$	$481665 \times f_s - 888037$
K_8	$-12948 \times f_s + 15532$	$-81572 \times f_s + 135624$	$-174265 \times f_s + 321008$

The curve of the load model to incorporate human walking dynamic effects, given in eq. (1), is illustrated in Figure 5. In this model, $P=951.245\text{N}$ and $f_s=1.87\text{ Hz}$. In the numerical model of the isolated panel waffle slab, the dynamic load model given in eq. (1) was applied at the centre of the span. It was assumed that the walker is acting only at the centre of the slab, he is not moving, because the maximum modal amplitude of mode 1 occurred at this place, where the peak acceleration is expected. Two types of analyses were performed: modal dynamic and time history dynamic. Constant modal damping was used in the former and Rayleigh damping model was used in the latter. In the modal dynamic analyses, three and twelve modes were included in the calculation of the dynamic responses. The damping was assumed 2% for concrete floor systems with no furniture neither finishes [13].

In Figure 6, the accelerations computed with a HFLM are transient and periodic, reaching the same peak acceleration after 0.5 s; they fit better with the transient behaviour of the experimental accelerations. It is important to point out that the part of the record, 4 s, which is compared with the numerical response, includes the time where the peak acceleration occurs. This peak acceleration most likely occurred when the pedestrian crossed the centre of the span. The numerical model was calibrated not only with the experimental frequency of the first three mode shapes, but also with the accelerations induced with the load model of human walking in eq. (1) and solved with time history analysis.

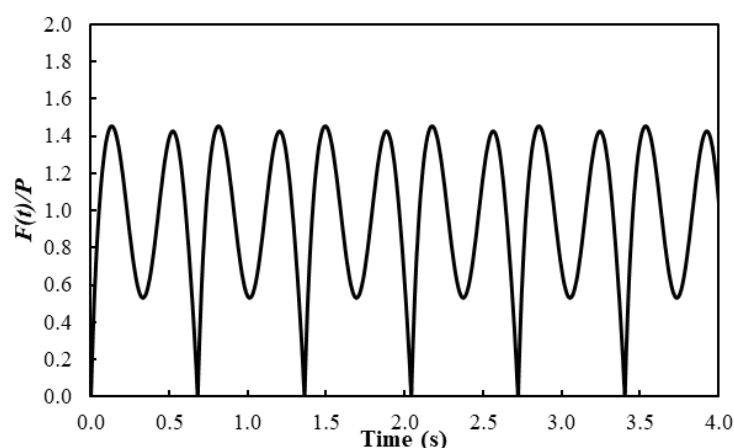


Figure 5: Dynamic load models of a person walking [12] eq. (1)

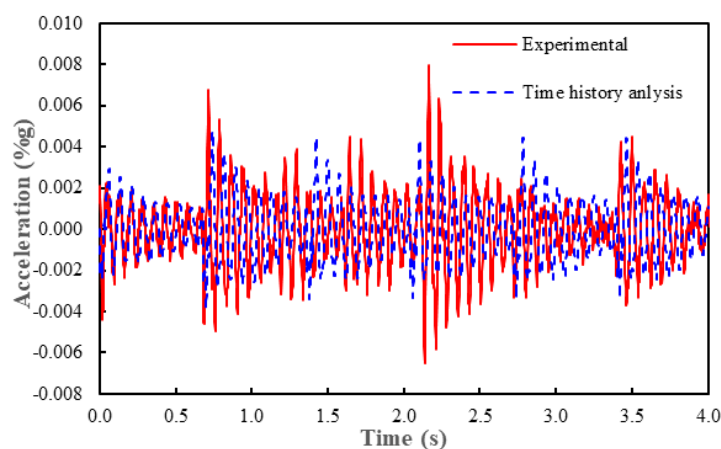


Figure 6: Experimental and numerical accelerations in the panel under human walking with damping of 2% with a HFLM eq. (1).

4 ASSESSMENT OF THE STIFFENING OF STRUCTURAL COMPONENTS TO MITIGATE ACCELERATION

Stiffening of structural components is evaluated as remedial measure to mitigate accelerations. The stiffness of the structural elements of the isolate waffle slab panel, shown in Figure 1b, is incremented. The increasing of the structural stiffness is achieved by increment of the modulus of elasticity of concrete, increment of slab thickness, increment of beam depths and joist depths and filling up the domes located along the perimeter edges and rows of domes at the centre of the span. Furthermore, reducing mass is achieved in the concrete slab with the use of light weight concrete (LWC) instead of normal weight concrete (NWC). The original dimension of the structural components of the two-way joist concrete slab floor slab are given in section 2 and illustrated in Figure 2.

4.1 Stiffening with the increment of the concrete modulus of elasticity

All the structural components of the two-way joist concrete slab panel were stiffened with the increment of the concrete modulus of elasticity, keeping the original sizes of the structural components, given in Figure 2. The concrete modulus of elasticity is related with the ultimate compressive strength, f_c , the original model has an ultimate compressive strength of 25 MPa and an $E_d=29.3$ GPa. f_c was increased to 30, 40 and 50 MPa and, consequently, the E_d is modified as tabulated in Table 3. Increasing E_d , the decrement Δa_p was up to -30.78%, for $E_d=41.445$ GPa, as tabulated in Table 3, which is a significant reduction of the acceleration. Therefore, the stiffening of the structural components of waffle slabs with the increment of the concrete modulus of elasticity is not a remedial measure to mitigate vibration.

Table 3: Variation of acceleration for different E_d .

f_c MPa	E_d GPa	a_p (%g)	f_n (Hz)	W(kN)	Δa_p (%)	Δf_n (%)
25	29.306	0.00459	12.79	572.9		
30	32.103	0.00449	13.37	572.9	-2.16	4.59
40	37.069	0.00416	14.35	572.9	-9.30	12.22
50	41.445	0.00318	15.16	572.9	-30.78	18.57

4.2 Stiffness increment of the slab thickness

The original slab thickness, t_s , was 6 cm, as shown in Figure 2, which was increased to 8 cm, 10 cm and 12 cm, respectively, using normal weight concrete. The variation of the a_p is tabulated in Table 4, where there is a significant reduction of the acceleration, up to -33.54 %, for a slab thickness of 12 cm. Additionally, the frequency is slightly reduced, up to - 4.57 %, but the weight of the floor clearly increases, up to 25.49%, with the increment of the slab thickness.

It is clear that a reduction of acceleration is achieved with the increment of slab thickness, but it is accompanied with an inherent increase of the mass. Then, the stiffening of the slab thickness with the increment of the depth is a remedial measure to reduce acceleration; however, it is also important to keep in mind that the gravitational and seismic design may be more expensive for the weight increment.

Table 4: Acceleration variation with different thickness of the slab.

t_s (cm)	a_p (%g)	f_n (Hz)	W(kN)	Δa_p (%)	Δf_n (%)	ΔW (%)
6	0.00459	12.79	572.9			
8	0.00368	12.582	621.6	-19.84	-1.60	8.5
10	0.00327	12.381	670.3	-28.65	-3.17	16.99
12	0.00305	12.203	718.9	-33.54	-4.57	25.49

4.3 Stiffness increment of beams

The second stiffened structural elements were the principal beams, whose original dimensions were 80 cm deep and 60 cm wide as illustrated in Figure 2. The beam depth, h_b , was increased progressively in increments of 5 cm, from 80 to 100 cm to stiffen these structural components, using normal weight concrete. The variation of the a_p is tabulated in Table 5, where the acceleration also increased unexpectedly up to 10.70 % for a beam depth of 85 cm. This increment may be attributed to the fact that the stiffness is added up to the edges of the isolated panel model, where modal amplitudes of modes one and two have no contribution at the edges as illustrated in Figure 4, only the modal amplitude of mode 3 have little contribution at the short edge. The f_n increase up to 6.15% and the weight of the slap panel model increases up to 20.05%. Consequently, stiffening of principal beams is not a remedial measure to mitigate acceleration in two-way joist concrete slab floor systems.

Table 5: Acceleration variation with different beam depths.

Beam size (cm)	a (%g)	f_n (Hz)	W(kN)	Δa (%)	Δf_n (%)	ΔW (%)
60 x 80	0.00459	12.79				
60 x 85	0.00508	13.009	601.623	10.70	1.71	5.01
60 x 90	0.00492	13.213	630.347	7.21	3.31	10.03
60 x 95	0.00470	13.401	659.070	2.42	4.78	15.04
60 x 100	0.00450	13.576	687.793	-1.94	6.15	20.05

4.4 Stiffness increment of joists

Other stiffened elements were the joists, whose depth, h_j , was progressively increased in increments of 5 cm, from 41 to 66 cm, using normal weight concrete. The variation of the a_p is tabulated in Table 6, where the acceleration decreased up to -59.3% for a joist depth of 66 cm, using normal weight concrete. In this case, the f_n and the weight of the isolated panel increase up to 18.21 and 45.49%, respectively. Then, the stiffening of joist with the increment of the depth is a remedial measure to reduce acceleration; however, the gravitational and seismic design may be more expensive for the weight increment.

Table 6: Acceleration variation with different joist depths.

Joist depth (cm)	a_p (%g)	f_n (Hz)	W(kN)	Δa_p (%)	Δf_n (%)	ΔW (%)
41	0.00459	12.787	572.9			
46	0.00423	13.37	624.6	-7.9	4.56	9.02
51	0.00290	13.878	677.4	-36.8	8.53	18.24
56	0.00282	14.33	729.1	-38.6	12.07	27.26
61	0.00273	14.741	780.8	-40.5	15.28	36.29
66	0.00187	15.116	833.5	-59.3	18.21	45.49

the panel are kept. Consequently, the weights of the panel models of combination C-1 to C-4 are lower than the original weight of the panel with normal weight concrete, only the model with the dimension of combination C-5 keeps the original weight. Surprising, there is an increment of the acceleration with the reduction of the mass for combinations C-1 to C-2 as tabulated in Table 9, where the maximum acceleration increment is up to 34.92% for combination C-1. In fact, the reduction of the acceleration occurs for combinations C-3 to C-5, where the weights are lower or equal to the original panel, but the structural components have larger size. Therefore, the stiffening of the structural components of waffle slabs with LWC is a remedial measure to mitigate vibration, but particularly keeping the original weight of the two-way joist concrete slab floor system.

Table 8: Combinations of the sizes of the structural components with LWC.

Combination	Slab thickness (cm)	Beams (cm)	Joist depth (cm)
C-1	6	60 x 80	41
C-2	7	60 x 92.5	47
C-3	8	60 x 105	53
C-4	9	60 x 117.5	59
C-5	10	60 x 130	65

Table 9. Variation of acceleration for different sizes of the structural components LWC

Combination	a_p (g)	f_n (Hz)	W(kN)	Δa_p (%)	Δf_n (%)	ΔW (%)
Reference	0.00459	12.79	572.9			
C-1	0.00689	16.17	357.9	50.11	26.49	-37.53
C-2	0.00508	17.71	412.7	10.61	38.47	-27.96
C-3	0.00392	19.10	466.4	-14.68	49.37	-18.59
C-4	0.00280	20.39	521.2	-38.88	59.44	-9.02
C-5	0.00202	21.59	574.9	-55.91	68.86	0.35

5 CONCLUSIONS

Increment of the beam depth is not a remedial measure to control accelerations as a surprising increment of acceleration occurs. Conversely, remedial measures to mitigate vibration are:

- Increment of the concrete modulus of elasticity. There is a considerable reduction of acceleration and the original weight is kept.
- Increment of the slab thickness. The acceleration is reduced but it is accompanied with a moderate weight increment.
- Increment of the joist depth. The acceleration decreased; however, the gravitational and seismic design may be more expensive for the considerable weight increment.
- Increment of only pairs of central joists depth. The stiffness increment of central joists is a remedial measure to reduce acceleration and there is a minor weight increment.
- Depth increment of the structural components, substituting normal with lightweight concrete. There is a reduction of the acceleration, but it should be revised that an increase of acceleration does not occur with the reduction of mass.

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