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FATIGUE TOPOLOGY OPTIMIZATION UNDER CYCLIC LOADING FOR INNOVATIVE STRUCTURAL DESIGN

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Abstract

This study proposes a fatigue topology optimization approach for structural systems subjected to cyclic loading. Recognizing the impact of repeated loading on material endurance, the method incorporates fatigue considerations into the optimization process to enhance the durability and sustainability of load-bearing structures. By factoring in cyclic loading effects, this approach aims to reduce excessive material use while improving structural resilience, aligning with objectives for efficient and sustainable design. The proposed method is particularly useful in optimizing structural components in steel frameworks, where fatigue resistance and material efficiency are critical under dynamic loads. Emphasizing a balance between strength and durability, this study offers a practical tool for engineers aiming to design resilient structures that can endure cyclic stress without unnecessary material consumption. Furthermore, this framework is envisioned as a valuable solution for cyclic load environments, supporting more sustainable and resource-efficient engineering practices.

Keywords: Fatigue topology optimization, cyclic loading, material efficiency, sustainable design.

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1 INTRODUCTION

Structural design has traditionally focused on achieving an optimal balance between material efficiency and mechanical performance. However, in applications where structures are subjected to cyclic loading, fatigue becomes a critical consideration in ensuring durability and serviceability. Fatigue failure, which results from progressive damage accumulation under repeated stress cycles, significantly affects the lifespan and safety of structural components[1–3]. As a result, there has been growing interest in integrating fatigue constraints into topology optimization (TO) to design lightweight and durable structures capable of withstanding cyclic loads[4,5].

Topology optimization has been widely used in structural engineering to derive efficient load-bearing configurations by strategically redistributing material within a given design domain. The development of the Bi-directional Evolutionary Structural Optimization (BESO) method [6] and its extensions has enabled the incorporation of various design constraints, including stress, displacement, and manufacturability considerations[7,8]. More recently, researchers have sought to extend TO frameworks to address fatigue-related challenges in engineering applications[9,10].

Recent research has focused on integrating fatigue constraints into topology optimization (TO) to design lightweight structures capable of withstanding cyclic loads. For minimal mass TO, Holmberg et al. [11] presented a technique that combines principal stress estimates with probability-based high-cycle fatigue analysis. Using S-N curves and Palmgren-Miner's rule, Oest and Lund [12] created an effective method for fatigue-constrained TO. By using differential equations, Suresh et al. [13] proposed a continuous-time fatigue TO algorithm eliminating enabling non-proportional loading. These studies have demonstrated the importance of considering fatigue in TO, with results showing distinct differences between fatigue-constrained designs and those optimized for static loads or compliance minimization.

In addition to fatigue constraints, sustainability considerations have gained prominence in structural optimization research. Given that the steel industry is responsible for a significant share of global CO_2 emissions, optimizing material usage while maintaining structural integrity under cyclic loading can contribute to more environmentally friendly engineering practices [14,15]. By suggesting an optimal design approach for composite columns in high-rise structures, Park et al. [16] showed that employing more concrete and high-strength materials may be both ecologically as well as economically sensible. An optimization algorithm for circular columns made of reinforced concrete was created by Jelušič and Žula [17], who found that design decisions included trade-offs between CO_2 emissions and material cost. In order to promote more ecologically friendly engineering methods in the structural engineering field, these studies as well as the thorough analysis by Mei and Wang [18] highlight the rising significance of incorporating sustainability goals into structural optimization research.

Despite these advancements, existing fatigue TO methodologies often lack a comprehensive approach that simultaneously addresses fatigue performance, material efficiency, and sustainability. Therefore, this study presents an innovative fatigue topology optimization approach that integrates cyclic loading effects into the optimization framework to enhance the sustainability of structural components. Unlike previous works, which primarily focus on stress-based optimization, the proposed method explicitly incorporates fatigue damage evolution in the optimization process. The approach is demonstrated on benchmark problems, including a cantilever beam, a simply supported beam, and a U-shaped plate, where topology optimization is performed by developing the BESO algorithm. The effectiveness of the proposed methodology is validated by comparing the optimized topologies with existing designs, demonstrating significant improvements in material efficiency and fatigue resistance. Furthermore, the study quantifies the environmental impact of the optimized structures by evaluating reductions in embodied carbon emissions, aligning with sustainable engineering objectives. By bridging the gap between fatigue-based TO and sustainability, this research provides a novel solution for designing resilient structures under cyclic loading conditions.

2 PROBLEM STATEMENT

The proposed methodology integrates fatigue constraints, sustainability considerations, and topology optimization (TO) to develop lightweight and durable structural designs. This approach employs an iterative process that systematically evaluates material redistribution under cyclic loading conditions while minimizing the embodied carbon footprint. The overall framework consists of three primary components: fatigue analysis, sustainability assessment, and the topology optimization formulation.

2.1 Fatigue analysis under cyclic loading

Fatigue failure is a progressive damage mechanism that results from cyclic stress fluctuations over time. To quantify fatigue damage, the stress states within the structure are analyzed at each cycle, using a reference loading case as a baseline. The number of cycles to failure at different effective stress amplitudes N_{f_i} is determined through Basquin’s equation:

$$N_{f_i}(x) = \frac{1}{2}\left(\frac{\sigma_{ar_i}}{\sigma_f'}\right)^{\frac{1}{b}} \tag{1}$$

where b is the material's fatigue strength exponent, σ_f' is the material's fatigue strength coefficient, and σ_{ar_i} is the fully reversed equivalent stress amplitude. According to Palmgren-Miner's rule, the total damage caused at a position by each reversal equals the amount of cumulative tiredness at that location.

2.2 Sustainability and carbon footprint assessment

The GWP measurement [19], which measures the carbon footprint linked to material use, is becoming more and more important in evaluating a structure's environmental effect. By multiplying the quantity of each element by its corresponding embodied carbon component, this method evaluates the environmental impact. These values are then added together to calculate the structure's or structural element's total embodied carbon, as shown by the equation below:

$$GWP = \sum_{i=1}^{n} (w_i \times ECC_i) \tag{3}$$

In this case, w_i stands for the material's weight, and ECC_i for the material's embodied carbon coefficient. Weight reduction is a crucial goal to support sustainable design as this method directly implies the proportionality between structural weight and GWP:

$$Minimize: C = u^T K u \tag{4.a}$$

$$Subject\ to: \ V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{4.b}$$

$$\frac{V^*}{V_0} - V_f \leq 0 \tag{4.c}$$

$$D(x) - \sum_{i=1}^{N_r} \frac{n_i}{N_{f_i}(x)} = 0 \tag{4.d}$$

$$g_{GWP} = \sum_{i \in \Omega} (w_i \times ECC_i) \leq GWP_{max} \tag{4.e}$$

$$x_i \in \{0,1\} \tag{4.f}$$

The proposed topology optimization (TO) framework aims to minimize mean compliance (C) while integrating constraints related to fatigue performance, global warming potential (GWP), and material volume. The optimization process considers several key variables, including the displacement vector (u) and the global stiffness matrix (K), both of which play a crucial role in defining structural compliance.

The volume constraints ensure that the final design adheres to a predefined material distribution. The total permissible structural volume is represented as V^*, while each individual element's volume is denoted as V_i, with the total number of elements given by N. The binary design variable x_i determines whether an element remains in the structure ($x_i = 1$) or is removed ($x_i = 0$). The volume fraction ratio is represented by V_f, indicating the proportion of material retained relative to the initial design domain volume V_0.

To incorporate sustainability considerations, the GWP constraint is enforced to regulate the environmental impact of the optimized structure. The constraint is defined in Equation (4.e), where the maximum allowable global warming potential GWP_{max} is a user-defined limit. The embodied carbon coefficient (ECC_i) quantifies the environmental footprint of each material, while the corresponding element weight (w_i) accounts for material distribution within the design domain Ω.

2.3 Implementation strategy

The methodology follows an iterative approach, where the BESO algorithm progressively enhances the structural configuration through sensitivity analysis [20]. As illustrated in Figure 1, the optimization process consists of the following key steps:

- Define the design domain, material properties, fatigue parameters, and sustainability targets.
- Compute stress distributions under cyclic loading.
- Calculating embodied carbon based on material mass.
- Update the design domain iteratively using BESO.
- Terminate the process when constraints are satisfied, or further material removal is impractical.

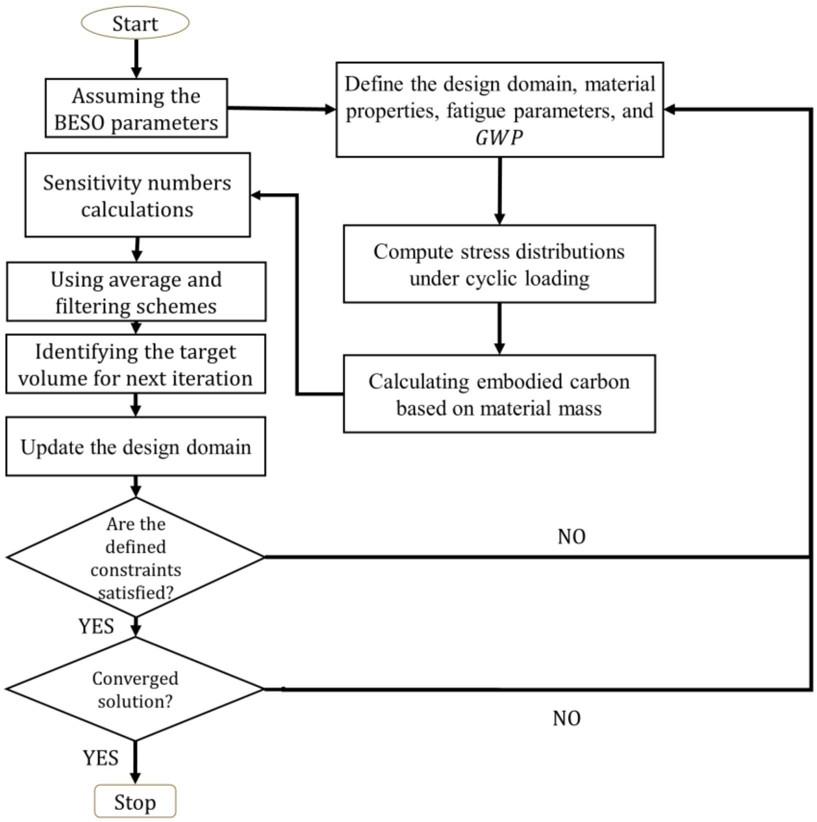


Figure 1: Flow of the algorithm

3 NUMERICAL EXAMPLES

This section explores the application of the developed BESO algorithm in topology optimization, incorporating sustainability, fatigue and volume constraints to ensure structural efficiency. The methodology is first validated through a benchmark case study, where a two-dimensional cantilever beam is optimized to assess the accuracy and robustness of the proposed approach, referencing the work of Habashneh and Rad [9]. To further evaluate its effectiveness, two additional numerical examples are analyzed: a simply supported beam (SSB) and a U-shaped plate, each designed to highlight different structural configurations and optimization challenges. Unless stated otherwise, the assumed material properties for all case studies are as follows: Young's modulus of $200\ GPa$, Poisson's ratio of 0.3, and a density of $7800\ kg/m^3$. The parameters σ_f' and b are assumed to be $580\ MPa$ and -0.262, respectively. A thickness of $10\ mm$ is considered for all examples. The influence of mesh resolution and analysis step size on the optimization results is thoroughly investigated, as these parameters significantly impact the accuracy, convergence behavior, and overall reliability of the computational simulations. To ensure consistency and validity, each numerical example in this study includes a detailed justification for the selected mesh size, highlighting its role in achieving a balance between computational efficiency and solution precision.

3.1 Cantilever beam

To validate the proposed methodology, a cantilever beam—widely used in the literature for benchmarking structural optimization techniques—is analyzed. The geometric configuration, loading conditions, and boundary constraints for the examined cases are illustrated in Figure 2. A concentrated force of $F = 100\ N$ is applied at the lower edge of the beam's free end. The computational model employs a finite element mesh comprising 2700 elements to ensure adequate resolution for topology optimization.

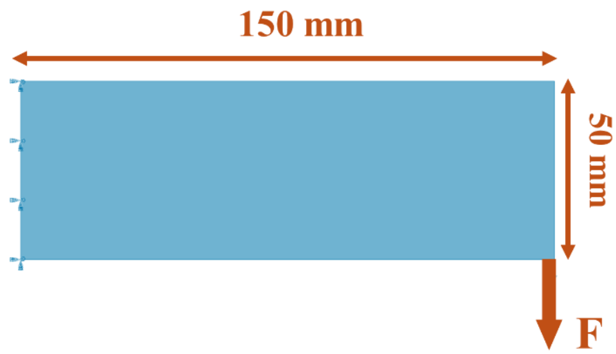


Figure 2: Considered Benchmark problem

The BESO parameters used in this study are as follows: the maximum allowable addition ratio of elements within the design domain AR_{max} is set at 2%, the minimum filter radius r_{min} is 8 mm, the evolutionary ratio (ER) is 2%, and the convergence criterion (τ) is 1%. The optimization process is subject to a volume fraction constraint of $V_f = 50\%$.

The obtained results are compared against the optimized configurations reported by Habashneh and Rad [9] to assess the accuracy and effectiveness of the proposed approach. Table 1 presents a comparative analysis of the optimized topologies generated by the proposed algorithm alongside the benchmark results. The close resemblance between the derived topologies and those from the reference study confirms the reliability of the developed method in producing structurally efficient and optimized configurations.

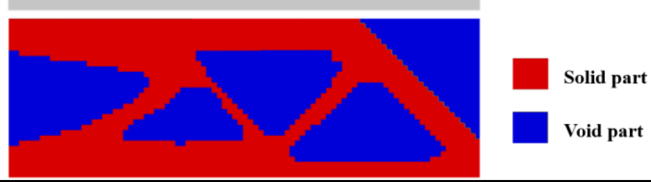
Algorithm	Optimized layout
Habashneh and Rad [9]	
Proposed work	

Table 1: Validation of the proposed algorithm.

3.2 Simply supported beam

The second numerical example presented in this research focuses on the optimization of SSB which is shown in Figure 3. The beam is subjected to a load (F) applied as a displacement with a magnitude of $8e - 3\ m$ at two positions at the top of the beam. The finite element mesh consists of 2400 quadrilateral (CPS4) elements, ensuring adequate resolution for capturing the structural response. The BESO parameters considered for this example are $ER = 1\%$, $AR_{max} = 1\%$, $r_{min} =$

$6\ mm$ and $\tau = 1\%$. The target volume fraction for the final material distribution is set at $V_f = 50\%$.

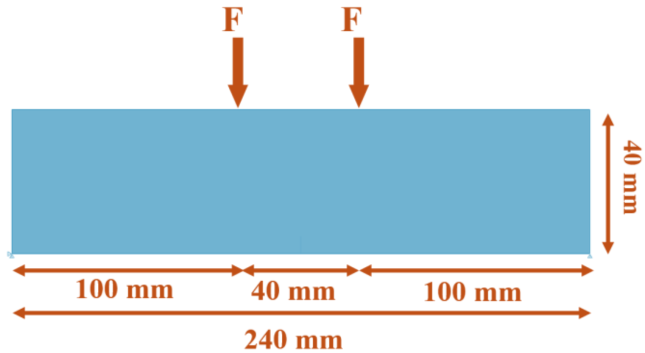


Figure 3: Considered simply supported beam

In this section, the problem of sustainable fatigue constrained topology optimization for simply supported beam is addressed. The optimal topology results are visualized in Figure 4. The top section represents the initial design domain, while the middle and bottom sections illustrate the progressive material removal and the emergence of the optimized structure. The red regions indicate retained material, whereas the blue regions denote removed material. The final optimized layout exhibits an efficient load-bearing truss-like configuration, enhancing structural performance while minimizing material usage. This optimization approach not only improves the structural efficiency but also considers fatigue constraints to ensure long-term durability under cyclic loading conditions. By strategically removing non-essential material, the design mitigates stress concentrations that could accelerate fatigue failure, thereby extending the service life of the structure.

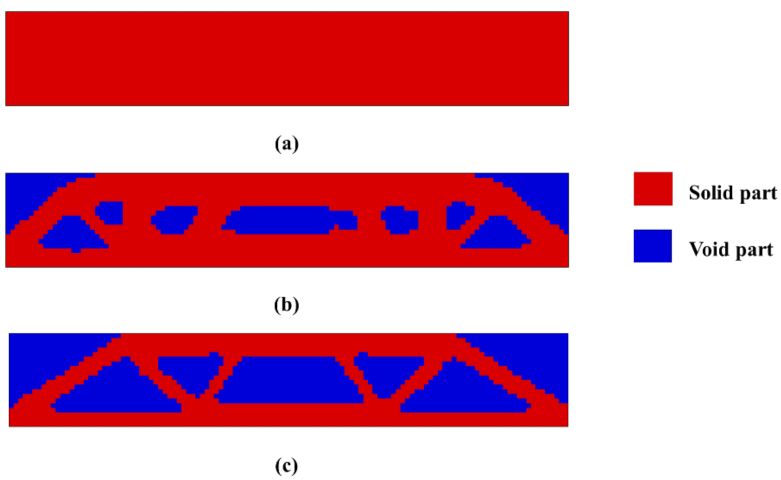


Figure 4: Material removal process leading to the optimized layout (a) Full domain (b) $V_f = 70\%$ (c) $V_f = 50\%$

3.3 U-shaped plate

The third numerical example in this study focuses on optimizing the structural layout of a U-shaped plate, as illustrated in Figure 5. The applied loading condition consists of a prescribed displacement (F) of $8e - 3\ m$, imposed at the midpoint of each right edge. The finite element model employs a discretized mesh consisting of 17,600 elements, ensuring sufficient resolution for capturing the structural response accurately. To govern the optimization process, the BESO parameters are defined as follows: ER of 1%, AR_{max} at 1%, r_{min} of 18 mm, and τ of 1%. The optimization is constrained by a final material volume fraction of $V_f = 50\%$, ensuring a balanced trade-off between structural efficiency and material utilization.

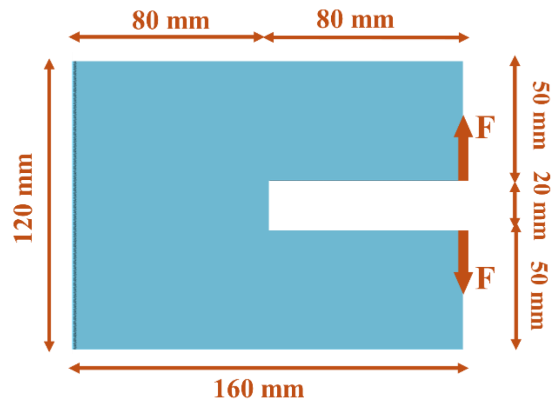


Figure 5: Considered U-shaped plate problem

In this section, the problem of sustainable fatigue constrained topology optimization for the U-shaped plate is addressed. The optimal topology results are visualized in Figure 6, showing how material is strategically removed to achieve an efficient structural layout. The first image represents the initial design domain, where the U-shaped cutout is embedded within a fully solid plate, providing the starting point for optimization. As the process advances, material is gradually eliminated based on the BESO algorithm, leading to an intermediate stage where voids start forming, and a more structured load-bearing pattern emerges. The final optimized design reveals an intricate truss-like configuration that efficiently distributes stresses while significantly reducing material usage. The retained red regions highlight the essential load paths, while the removed blue areas indicate non-essential material, ensuring an optimal balance between strength and weight reduction.

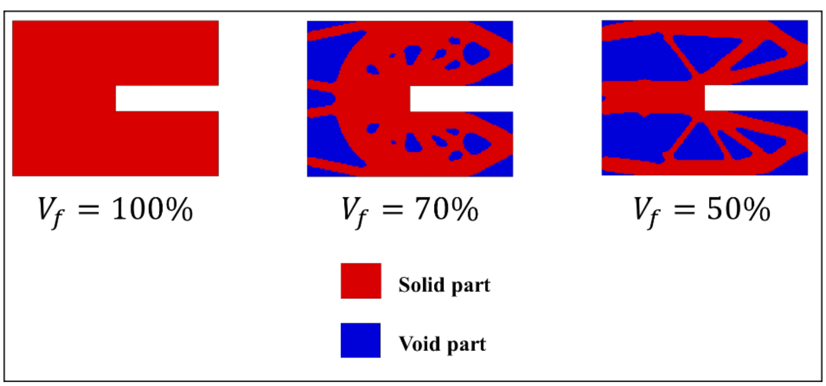


Figure 6: Material removal leading to the optimized layout

4 ASSESSING EMBODIED CARBON IN TOPOLOGY-OPTIMIZED DESIGNS

The primary source of CO_2 emissions in the steel industry stems from the production phase, which is highly energy-intensive due to the processes involved in extracting, refining, and manufacturing steel components. On average, the production of steel results in approximately 1.85 tons of CO_2 emissions per ton of steel produced. To evaluate the environmental impact of the structural configurations considered in this study, the CO_2 emissions from steel production were estimated based on the total material mass used in both the full design domain and the optimized layouts.

The total steel mass for each structural case was determined using the fundamental relationship:

$$Mass\ of\ Steel = V_0 \times \rho \tag{5}$$

The computed volume, mass, and optimized material utilization for each numerical example are summarized in Table 2 which represents the initial and optimized material volumes, along with their corresponding masses, for the cantilever beam, simply supported beam (SSB), and U-shaped plate.

Problem	$V_0\ (mm^3)$	Full domain mass (kg)	Volume of optimized layout	Mass of optimized layout (kg)
Cantilever beam	7.50×10^7	588.75	3.75×10^7	294.38
SSB	9.60×10^7	753.60	4.80×10^7	376.80
U-shaped plate	1.75×10^8	1373.75	8.75×10^7	686.88

Table 2: Volume and corresponding mass for the considered numerical examples.

The CO_2 emissions are determined by converting the mass of steel into tons and applying the standard emission factor of 1.85 tons of CO_2 per ton of steel. The corresponding calculation follows the equation:

$$CO_2\ Emissions\ (tons) = \frac{Mass\ of\ Steel\ (kg)}{1000} \times Emission\ Factor\ (tons\ CO_2\ /\ ton\ steel) \tag{6}$$

The optimization of structural layouts plays a crucial role in enhancing sustainability by reducing material consumption and minimizing CO_2 emissions associated with steel production. In this study, topology optimization was applied to three structural configurations—a cantilever beam, a simply supported beam, and a U-shaped plate—resulting in a significant reduction in material usage while maintaining structural efficiency. The initial and optimized layouts were assessed in terms of mass reduction and CO_2 emissions, where the standard emission factor of 1.85 tons of CO_2 per ton of steel was used to quantify the environmental impact. The comparative results are presented in Figure 7. The material volume constraint of 50% effectively leads to a proportional reduction in structural mass and corresponding CO_2 emissions. Specifically, the cantilever beam exhibited a decrease in mass from 588.75 kg to 294.38 kg, leading to a CO_2 reduction from 1.089 tons to 0.545 tons. Similarly, the simply supported beam experienced a mass reduction from 753.60 kg to 376.80 kg, lowering CO_2 emissions from 1.394 tons to 0.697 tons. The U-shaped plate, being the largest structural component analyzed, showed the most significant impact, with mass decreasing from 1373.75 kg to 686.88 kg, and CO_2 emissions dropping from 2.541 tons to 1.271 tons.

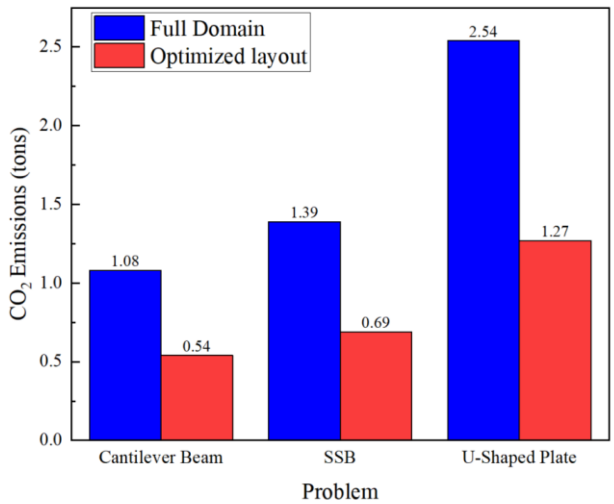


Figure 7: Emissions for considered problems

These findings highlight the direct correlation between topology optimization and sustainability, emphasizing that by eliminating non-essential material, the carbon footprint of steel structures can be substantially lowered. In practical terms, such reductions contribute to global efforts toward sustainable construction, resource efficiency, and environmentally friendly engineering practices [21,22].

5 CONCLUSIONS

This study presents a novel fatigue TO framework that integrates cyclic loading effects into the optimization process to enhance the durability, material efficiency, and sustainability of structural components. By incorporating fatigue constraints and embodied carbon assessments into the BESO methodology, the proposed approach offers a robust solution for developing lightweight and resilient structures subjected to repeated loading conditions.

Key findings from this research include:

- The methodology effectively minimizes fatigue damage accumulation by optimizing the structural layout while considering stress amplitudes and cyclic loading history. The use of Basquin's equation and Palmgren-Miner's rule in the TO framework ensures improved fatigue resistance.
- A significant reduction in material usage was achieved across all numerical examples, demonstrating the potential of topology optimization in reducing the carbon footprint of steel structures.
- The methodology was tested on three problems, cantilever beam, simply supported beam (SSB), and U-shaped plate. The optimized topologies exhibited improved fatigue resistance while achieving material savings.

The proposed framework provides a comprehensive solution for sustainable fatigue-based topology optimization, offering structural engineers a powerful tool for designing lightweight, durable, and sustainable load-bearing components. Further research is expected to focus on the integration of different materials in topology optimization to enhance fatigue life and sustainability, as well as leveraging artificial intelligence techniques to accelerate convergence and improve the computational efficiency of fatigue-based topology optimization.

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