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NUMERICAL INSIGHTS ON ACTIVE MASS DAMPER PERFORMANCE IN FRAME STRUCTURES

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Abstract. *This study presents a numerical approach to evaluate the effects of mass dampers, with a specific focus on the applications of Active Mass Dampers (AMDs) to framed structures. The research highlights the differences between classical and non-classical damping analysis, demonstrating that the classical approach is valid only for proportional damping. In contrast, for structures with localized damping contributions, such as AMDs, a methodology accounting for non-classical damping characteristics is necessary. As show by the numerical analysis, AMDs significantly alter the dynamic response of the structure, affecting both natural frequencies and damping coefficients. Moreover, the presence of an AMD induces a phase shift between the movements of different floors, a phenomenon not captured by classical modal analysis.These findings emphasize the importance of adopting appropriate procedures for analyzing structures equipped with active damping devices. The proposed approach provides an efficient tool for assessing the impact of such systems without relying on time-domain simulations.*

Keywords: Active Mass Damper, Vibration Control, Non-classically Damped Structures.

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1 INTRODUCTION

There are several types of mass dampers, the first idea of mass damper was proposed in 1911 by Hermann Frahm [1] to control the rolling of ships. This first mass damper model was be called passive mass damper (PMD). In the first half of the 20^{th} century, PMD was studied to ensure optimal design and it began to be applied to buildings in the last decades of the 20^{th} century [2]. This system has been widely used in skyscrapers to control wind vibrations due to its features, like the ability to operate without external energy [3]. In small buildings it has not found widespread use for two main reasons. The first is the requirement of large space to accommodate the machinery. The second is that in small buildings the wind action is much less influential and the system would be used to control seismic actions. However, PMD is effective only in the tuning frequency [4, 5, 6].

With the aim of extending the tuning as much as possible, the system was developed supporting variable stiffness and variable damping. This version of the machinery is called semi active tuned mass damper (SAMD) [7, 8]. Unfortunately, SAMD has not found wide use in constructions due to the complexity of the mechanism [2].

Another type of mass damper is the active mass damper (AMD). In this case, the mass is connected to the building with an actuator that produces the control force thanks to the signals of the control algorithm. This system is very efficacious, since it can work at every frequency and it is small, but requires energy to function [3]. For these reasons, it is typically avoided in skyscrapers due to the constant requirement of energy to control the wind action. However, its functioning is perfect for controlling seismic actions and traffic-induced vibrations [9] in small buildings.

The last type of mass damper is the hybrid mass damper (HMD). This system is the union between SMD and PMD. HMD is configured like a passive system with the addition of actuators to improve its performance [2]. The difference between SAMD and HMD is that in the first the active components change the stiffness and damping characteristics while in the second one the active parts are the actuators that can act on the mass.

To recap, PMD, SAMD and HMD systems are ideal for protecting large buildings, in terms of height and mass, from wind actions, with HMD also effective against seismic actions, whereas AMD is ideal for protecting lightweight buildings from seismic actions.

The common feature of all these systems is that they are generally applied to the highest floor to have the maximum efficiency. For this reason, they provide a damping contribution that is concentrated in only one part of the building. This peculiarity requires a devoted analysis procedure, since the total damping can not be expressed as linear combination of mass and stiffness. These situations are generally indicated as non-classically damped structures [10]. In this work we present a methodology for the analysis of non-classically damped structures, as an alternative to the finite method with time history analysis [11] or the Boundary Element Method (BEM) [12, 13] when the damping is due to the presence of an AMD on only one floor.

2 METHODOLOGY

For a multi degree of freedom (MDF) system subjected to external dynamic force $\mathbf{p}(t)$ the equation of motion can be stated as:

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{p}(t) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are respectively the mass and stiffness matrices [14], $\mathbf{C}$ is the damping matrix and $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the dynamic response of the structure in terms of displacement, velocity and acceleration vectors respectively. In the case of classical damping, is possible to calculate the $\mathbf{C}$ matrix by using the Rayleigh method [10, 15]:

$$\mathbf{C} = \alpha\,\mathbf{M} + \beta\,\mathbf{K} \tag{2}$$

where:

$$\alpha = \zeta_1 \frac{2\,\omega_1\,\omega_2}{\omega_1 + \omega_2};\;\; \beta = \zeta_2 \frac{2}{\omega_1 + \omega_2}$$

and ω_1, ω_2 are the first two pulsations of the building and ζ_1, ζ_2 are the structural damping coefficients assumed equal to known values. Otherwise, it is possible to calculate the $\mathbf{C}$ matrix by assigning known damping coefficients for all modes with:

$$\mathbf{C} = \mathbf{M} \sum_{i=0}^{n-1} \alpha_i \left[\mathbf{M}^{-1}\,\mathbf{K}\right]^i \tag{3}$$

Within the classical procedure, it is not possible to consider the damping of the mass dampers. To add their contribution it is necessary to know the position and the damping capacity of the machine. From these two parameters it is possible to write the $\mathbf{C}_{AMD}$ matrix which contains the damping values C_{AMD_n} of the machine in relation to the degrees of freedom of the structure, where the damping is defined as: $C_{AMD_n} = f_{AMD}\,\dot{u}^{-1}$. The total damping matrix of the structure will become the sum of the previously calculated structural damping contribution $\mathbf{C}_{STC}$ and the $\mathbf{C}_{AMD}$ matrix:

$$\mathbf{C} = \mathbf{C}_{STC} + \mathbf{C}_{AMD} \tag{4}$$

To calculate the damping contribution for all modes, it is necessary recalculate the eigenvalues also considering the damping matrix. The eigenvalue problem becomes:

$$\left(\boldsymbol{\lambda}^2\,\mathbf{M} + \boldsymbol{\lambda}\,\mathbf{C} + \mathbf{K}\right)\boldsymbol{\Psi} = 0 \tag{5}$$

It is possible to convert the system of N second-order differential equations into a system of $2N$ first-order differential equations by defining the matrices $\mathbf{A}$ and $\mathbf{B}$:

$$\boldsymbol{\lambda}\,\mathbf{A}\,\boldsymbol{\kappa} + \mathbf{B}\,\boldsymbol{\kappa} = 0 \tag{6}$$

when $\mathbf{A}$ and $\mathbf{B}$ are written as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix};\;\; \mathbf{B} = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \tag{7}$$

$\boldsymbol{\lambda}$ is a vector of complex eigenvalues with N pairs of terms (for a total of $2N$ values) and $\boldsymbol{\kappa}$ is the eigenvector matrix.

If the damping ζ_n corresponding to the n^{th} couple is lower than 1, $\zeta_n < 1$, it will be made up of complex and conjugate values: $\lambda_n, \bar{\lambda}_n$. Otherwise, if the damping ζ_n corresponding to the n^{th} couple is higher than 1, $\zeta_n > 1$, it will be made up of two different real values: λ_n, λ_m.
To generalize the procedure, the second terms of the couple can be indicated as λ_c regardless of whether it is complex ($\bar{\lambda}_n$) or real (λ_m).
λ_n, λ_c are defined as:

$$\lambda_n, \lambda_c = -\zeta_n\,\omega_n \pm \omega_{Dn} \tag{8}$$

in which $\boldsymbol{\omega}_D$ are the pulsations according for damping and is defined as:

$$\omega_{Dn} = \omega_n \sqrt{\zeta_n^2 - 1} \tag{9}$$

where $\boldsymbol{\omega}$ is the undamped pulsations and $\boldsymbol{\zeta}$ is the damping coefficient. The following relations can be derived:

$$\omega_n = \sqrt{\lambda_n \, \lambda_c} \tag{10}$$

$$\zeta_n = -\frac{\lambda_n + \lambda_c}{2\sqrt{\lambda_n \, \lambda_c}} \tag{11}$$

$$\omega_{Dn} = \frac{\lambda_n - \lambda_c}{2} \tag{12}$$

Similarly, the eigenvectors matrix is also double sized, where each columns corresponds to a eigenvalue and the rows are duplicated according to the following definition:

$$\boldsymbol{\kappa} = \left\{ \begin{array}{c} \boldsymbol{\lambda\psi} \\ \boldsymbol{\psi} \end{array} \right\} \tag{13}$$

The free vibrations of the structure in time function can be calculated through:

$$\mathbf{u}_i(t) = \sum_{n=1}^{N} \mathbf{u}_n(t) \tag{14}$$

For dampings lower than one ($\zeta_n < 1$), $\mathbf{u}_n(t)$ is defined as:

$$\mathbf{u}_n(t) = e^{-\zeta_n \, \omega_n \, t} \left[\beta_n \cos\left(\omega_{Dn} \, t\right) - \gamma_n \sin\left(\omega_{Dn} \, t\right)\right] \tag{15}$$

whereas for dampings higher than one ($\zeta_n > 1$), $\mathbf{u}_n(t)$ is defined as:

$$\mathbf{u}_n(t) = e^{-\zeta_n \, \omega_n \, t} \left[\beta_n \cosh\left(\omega_{Dn} \, t\right) - \gamma_n \sinh\left(\omega_{Dn} \, t\right)\right] \tag{16}$$

where for both:

$$\beta_n = B_c \, \psi_c + B_n \, \psi_n \tag{17}$$

$$\gamma_n = B_c \, \psi_c - B_n \, \psi_n \tag{18}$$

with B_n and B_c defined as:

$$B_n = \frac{\lambda_n \, \boldsymbol{\psi}_n^T \mathbf{M}\mathbf{u}(0) + \boldsymbol{\psi}_n^T \mathbf{C}\mathbf{u}(0) + \boldsymbol{\psi}_n^T \mathbf{M}\dot{\mathbf{u}}(0)}{2\,\lambda_n \, \boldsymbol{\psi}_n^T \mathbf{M}\boldsymbol{\psi}_n + \boldsymbol{\psi}_n^T \mathbf{C}\boldsymbol{\psi}_n} \tag{19}$$

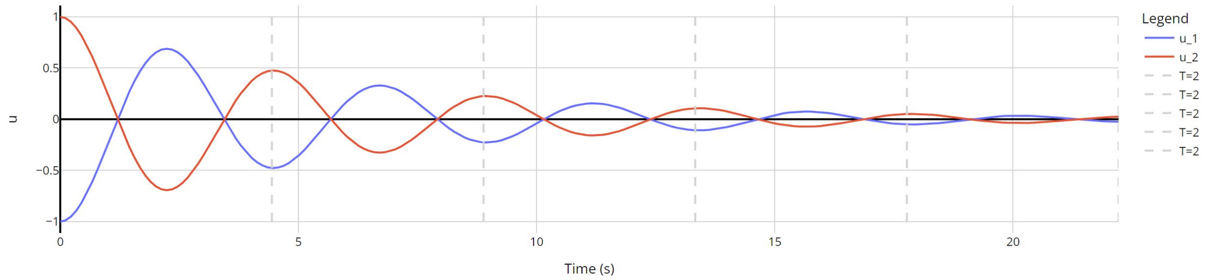
$$B_c = \frac{\lambda_c \, \boldsymbol{\psi}_c^T \mathbf{M}\mathbf{u}(0) + \boldsymbol{\psi}_c^T \mathbf{C}\mathbf{u}(0) + \boldsymbol{\psi}_c^T \mathbf{M}\dot{\mathbf{u}}(0)}{2\,\lambda_c \, \boldsymbol{\psi}_c^T \mathbf{M}\boldsymbol{\psi}_c + \boldsymbol{\psi}_c^T \mathbf{C}\boldsymbol{\psi}_c} \tag{20}$$

By introducing ϕ_i as the eigenvectors associated to the classically damped procedure and by assigning $\mathbf{u}(0) = \boldsymbol{\phi}_i$ and $\dot{\mathbf{u}}(0) = 0$, one can obtain the free vibrations in time function starting from each natural mode of the undamped structure but considering the contribution of the nonclassical damping. B_n and B_c then become:

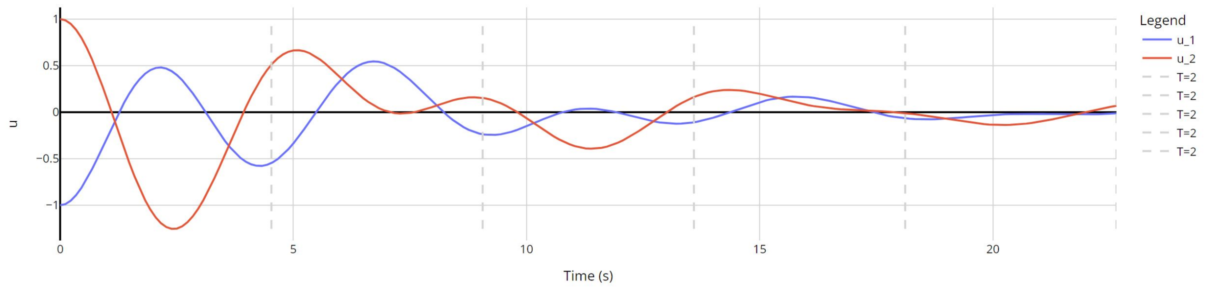
$$B_n = \frac{\lambda_n \, \boldsymbol{\psi}_n^T \mathbf{M}\boldsymbol{\phi}_i + \boldsymbol{\psi}_n^T \mathbf{C}\boldsymbol{\phi}_i}{2\,\lambda_n \, \boldsymbol{\psi}_n^T \mathbf{M}\boldsymbol{\psi}_n + \boldsymbol{\psi}_n^T \mathbf{C}\boldsymbol{\psi}_n} \tag{21}$$

$$B_c = \frac{\lambda_c \, \boldsymbol{\psi}_c^T \mathbf{M}\boldsymbol{\phi}_i + \boldsymbol{\psi}_c^T \mathbf{C}\boldsymbol{\phi}_i}{2\,\lambda_c \, \boldsymbol{\psi}_c^T \mathbf{M}\boldsymbol{\psi}_c + \boldsymbol{\psi}_c^T \mathbf{C}\boldsymbol{\psi}_c} \tag{22}$$

At this point it is important to note that the building is subjected to phase shift. This denotes a fundamental difference with classically damped structures. During the oscillation, a classic damped building reaches the initial configuration when the displacement is zero between two maximal values. Otherwise, a non-classically damped building never reaches the initial configuration, but when a point of the structure reaches the initial position (zero displacement) the other parts keep a residual displacement. This characteristic of the non-classic damping building is fundamental because it implies that the deformed shape changes continuously, thus the structural behavior cannot be predicted through the classical procedure.



(a) Structure with proportional damping.



(b) Structure with concentrated damping.

Figure 1: Conceptual comparison of free vibration for the second mode in a two-dimensional two-plane frame. u_1 and u_2 are the displacements of the first and second floor respectively. Image generated trough an in-house Python code.

This fact is visible if one compares the free vibrations for two identical buildings with the two types of damping, as in Fig:1. In the classical damped building the functions representing the displacement $\mathbf{u}$ for each point of the structure intersect where the displacement is zero. For the non-classically damped building the intersections of the functions correspond to not null displacement.

3 NUMERICAL RESULTS

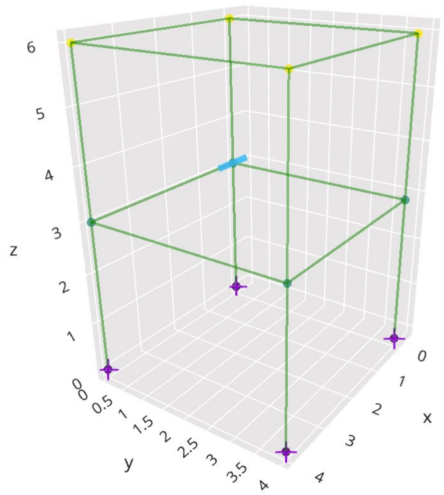
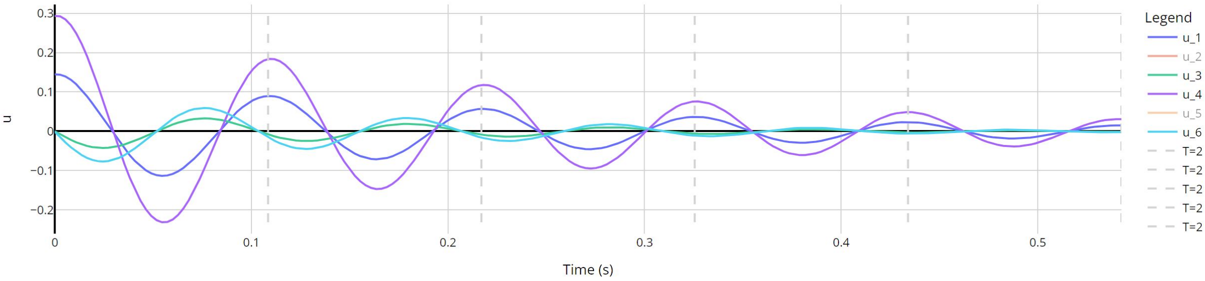


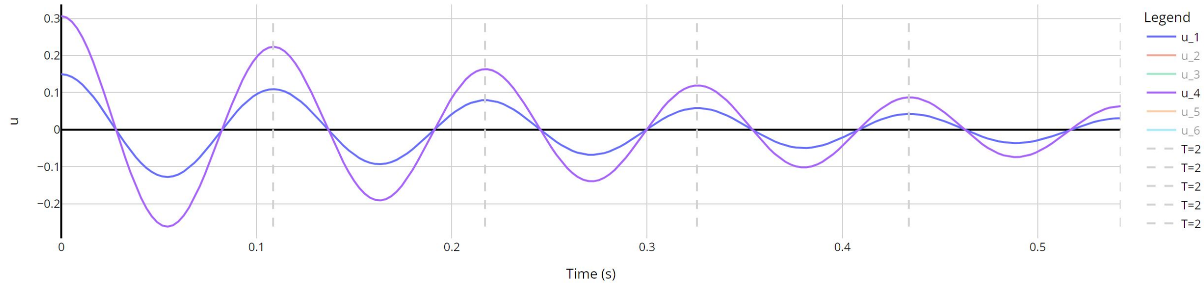
Figure 2: 3D model of the structure. In azure is represented the AMD. Image generated trough an in-house Python code.

To demonstrate the applicability of this methodology and its effectiveness, a first application is illustrated below. The structure (Fig:2) is a regular 3D frame with squared plan of 2 floors. The sides are 4 m and the height is 3 m for each floor. The columns and beams are made of reinforced concrete (C28/35) with cross-section 0.5 $\times$ 0.3 m. Fixed constraints are applied at the bases of the columns and a rigid diaphragm is considered at the floor levels. The stiffness and mass matrices $\mathbf{K}$ and $\mathbf{M}$ are assembled according to the Timoshenko beam theory.

An AMD is now considered at the first floor. The contribution is associated to the mode having coordinates $(0, 0, 3)$, thus representing a translational and rotational damping for the first floor. The entire structure will not be affected in the 1^{st} and 4^{th} modes (characterized by displacement only in y direction), but the other modes will be completely different. The analyses have been performed through an in-house Python code that implements the presented methodology. In the numerical analysis, AMD is considered as a damping concentrated at one point of the structure with $C_{AMD_n} = 150$ kg/s.
In fig: 3 the free vibrations of the 2^{nd} mode are visible (similar to the 5^{th} mode). In the case without AMD there are only displacements along the x direction and since the structure is regular they are equal to the 1^{st} mode. Given the positioning of the AMD in the non-classical structure, rotations and phase shift occur.
In fig: 4 are visible the free vibrations of the 6^{th} mode (similar to the 3^{rd} mode). In the case without AMD there are only rotations. Given the positioning of the AMD in the non-classical structure, x-displacements and phase shift appear.

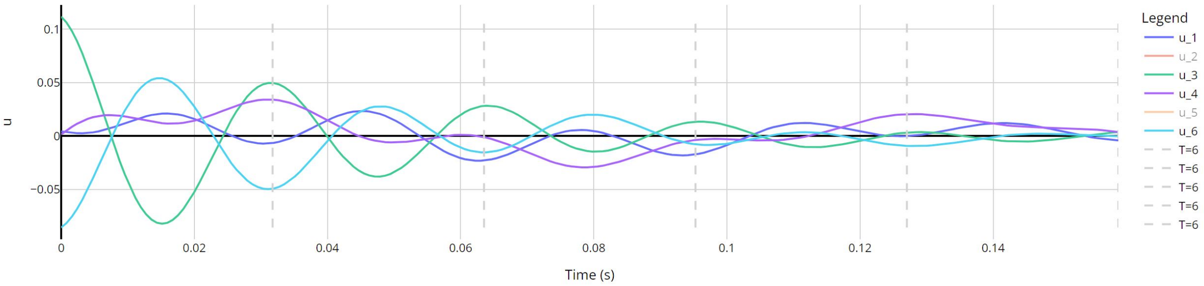


(a) Structure with AMD.

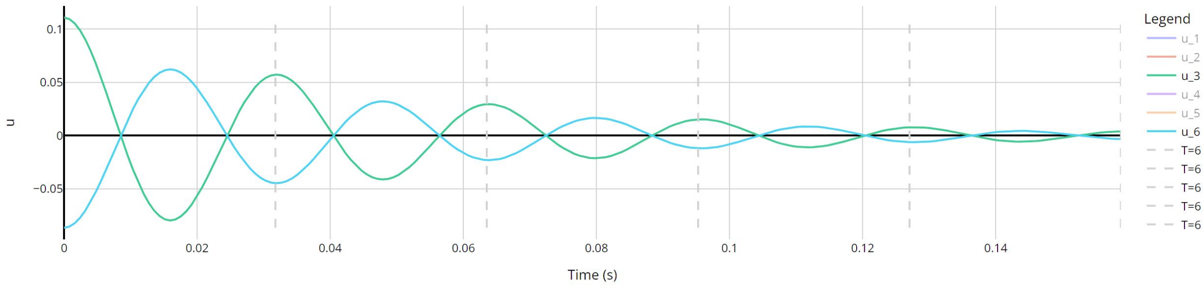


(b) Structure without AMD.

Figure 3: Comparison of free vibrations for the second mode of the structure in Fig 2. u_1, u_2, and u_3 are the displacements in x and y directions and the rotation in z axis of the first floor respectively, u_4, u_5, and u_6 are the displacements in x and y directions and the rotation in z axis of the second floor respectively. Image generated trough an in-house Python code.



(a) Structure with AMD.



(b) Structure without AMD.

Figure 4: Comparison of free vibrations for the sixth mode of the structure in Fig 2. u_1, u_2, and u_3 are the displacements in x and y directions and the rotation in z axis of the first floor respectively, u_4, u_5, and u_6 are the displacements in x and y directions and the rotation in z axis of the second floor respectively. Image generated trough an in-house Python code.

Table 1, table 2 and table 3 show the the results obtained performing the non-classic procedures and the classic one compared with the results calculated without the AMD:

Pulsation ω

	T1	T2	T3	T4	T5	T6
no AMD	42.73870	57.90913	63.06051	128.53066	193.64124	197.83455
Non-Standard	42.73870	58.38964	62.64585	128.53066	194.89493	196.23693
Standard	42.73870	57.90913	63.06051	128.53066	193.64124	197.83455

Table 1: Pulsations of the structure calculated with classic eigenvalues for standard procedure and with eq: 10 for the non-classical one.

Pulsation ω_D

	T1	T2	T3	T4	T5	T6
no AMD	42.68524	57.83670	62.97901	128.18379	192.62039	196.75148
Non-Standard	42.68524	58.20579	62.49141	128.18379	192.65787	195.13123
Standard	42.68524	57.72846	62.90374	128.18379	191.9022	196.32851

Table 2: Damped pulsations of the structure calculated with eq: 9 for standard procedure and with eq: 12 for the non-classical one.

Damping ζ

	T1	T2	T3	T4	T5	T6
no AMD	0.05000	0.05000	0.05082	0.07341	0.10254	0.10449
Non-Standard	0.05000	0.07929	0.07017	0.07341	0.15107	0.10600
Standard	0.05000	0.07893	0.07046	0.07341	0.13371	0.12315

Table 3: Damping coefficients of the structure calculated with Rayleigh method for standard procedure and with eq: 11 for the non-classical one.

The numerical examples showed that the classical and the non-classical procedures provided identical results for the case of classical damping. However, differences are detected when a mass damper contribution is taken into account, the classical procedure is not able to estimate the behavior of the structure with mass damper and consequently also the characteristic of the sollecitations. Irregular buildings cannot be approximated as simple oscillators.

4 CONCLUSION

This study presented a numerical approach to evaluate the contribution of mass dampers, specifically focusing on AMDs in framed structures, highlighting the differences between classical and non-classical damping analysis. The results obtained demonstrate that the classical approach is suitable only for proportional damping, while for structures with localized damping contributions, such as AMDs, it is necessary to adopt a methodology that takes into account the non-classical nature of the set-up. The numerical analysis highlighted how the use of an AMD significantly influences the structure's dynamics, modifying both the natural pulsations and the damping coefficients. Furthermore, it has been demonstrated that the presence of an AMD introduces a phase shift between the movements of the different floors, an aspect that cannot be predicted using classical modal analysis methods. These results underline the importance of adopting appropriate procedures to study structures equipped with active damping devices. The proposed approach represents an effective tool to quickly assess the impact of such devices, without the need for time domain simulations.

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