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CALIBRATION OF A DAMPING LAW TO PREDICT PROBABILISTIC SEISMIC CAPACITY BY NONLINEAR STATIC ANALYSIS

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Abstract

Under strong earthquakes, real structures generally experience significant dynamic response and their members perform in the nonlinear range of behaviour. To this end, nonlinear dynamic analysis is widely recognised as the most accurate tool to predict all the aspects of the structural response demanded by the single ground motions. However, many uncertainties affect the seismic response of the structure and the related performance. Among these uncertainties, the feature of the ground motions is the most important one. Hence, probabilistic approaches for the evaluation of the seismic performances, based on Incremental Dynamic Analysis and fragility curves, has recently gained popularity in the scientific community. On the other hand, computational cost of these analyses is still too high to be suitable for professional purposes. For this reason, nonlinear static methods of analysis still represent a compromise between accuracy and computational burden. Unfortunately, the capacity assessed by nonlinear static methods of analysis often underestimates that predicted by probabilistic approaches and nonlinear dynamic analysis. This is due to the fact that nonlinear static methods of analysis do not account adequately for the increase of energy dissipation induced by the structural damage. In this framework, a new damping law is formulated, calibrated and incorporated in the nonlinear static method of analysis to predict accurately the capacity of RC framed structures determined by the probabilistic approach. To this end, a set of frames representative of buildings with RC framed structure have been designed, numerically modelled and assessed by incremental nonlinear dynamic analysis and nonlinear static analysis.

Keywords: nonlinear static analysis, RC frames, seismic assessment, probabilistic approach, fragility curves.

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1 INTRODUCTION

Scientific community has been devoting increasing efforts to develop methods of analysis aimed at predicting the seismic performance of existing structures. Given that real structures cannot keep an elastic behaviour under strong earthquakes, a reliable estimate of their seismic performance requires the determination of the inelastic deformation experienced by structural members during such events. To this end, the nonlinear dynamic analysis is considered a powerful tool. Furthermore, nowadays it is widely recognised that a proper assessment of structures should provide a measure of the actual seismic performance in terms of probability of exceedance of the considered limit state (for instance, structural collapse). Based on these considerations, Incremental Dynamic Analysis (IDA) and the related collapse fragility curves are increasingly popular in structural assessment [1]-[3]. A fragility curve specifies the probability of collapse as a function of ground motion intensity measure, such as the Peak Ground Acceleration (PGA). In addition, an estimated fragility function can be combined with a ground motion hazard curve to compute the mean annual frequency of collapse (reliability analysis) [4].

Even though the superiority of IDA and reliability analysis over the classical methods of analysis is recognised, there are several critical issues related to the use of this type of analysis. Among the others, there are the difficulties related to the definition of site-specific accelerograms that properly represent the expected seismic excitation, the need of a correct modelling of the nonlinear cyclic behaviour of structural elements, the considerable computing time to carry out IDA, the large amount of data produced and the extensive post-processing work necessary for the interpretation of the results. Hence, this approach is considered accessible only for experts in seismic engineering and classical methods of analysis are still in use and allowed by seismic codes, for instance the EuroCode 8 (-EC8) [5]-[6]. Among these methods, the nonlinear static method of analysis is the best compromise between the needing of a tool that estimates the inelastic deformations of structures subjected to earthquake with good accuracy and an acceptable computational burden. Among the approaches available in scientific literature, the Capacity Spectrum Method (CSM), proposed by Freeman [7], and the N2 Method, proposed by Fajfar [8], are the most popular.

Unfortunately, the nonlinear static method of analysis does not provide a probabilistic measure of the actual level of protection of the structure as the IDA and reliability analysis. Indeed, in literature, the nonlinear static method of analysis is employed just to estimate the maximum response observed in nonlinear dynamic analysis. Based on this consideration, this study proposes a new equivalent damping law designed to estimate the seismic risk of the building within a probabilistic framework. The proposed damping law is formulated, calibrated and incorporated into the nonlinear static method of analysis according to the CSM format to predict accurately the seismic performance of RC framed structures as determined by IDA and reliability analysis. To this end, a set of frames representative of building structures constructed in Italy in the seventies is defined. Each case study frame is assessed using both IDA and nonlinear static method of analysis. Capacity criteria provided by EuroCode 2 [9] are used for the member verification. In both cases, the outcomes of the analyses are summarised in the related PGA-capacity. The proposed equivalent damping law is calibrated to minimise the differences in terms of PGA-capacity between the two methods. Finally, the effectiveness of the CSM equipped with the proposed damping law is investigated by comparing its accurateness in predicting the PGA-capacity with that of other nonlinear static methods available in literature.

2 CASE STUDY FRAMES AND NUMERICAL MODELING

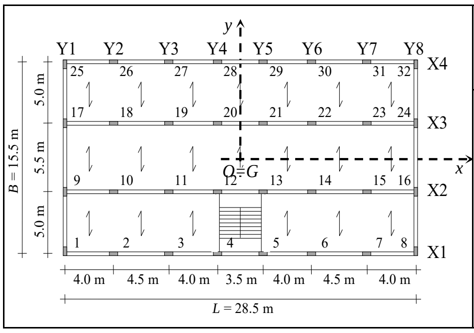
A set of case study frames representative of the buildings with RC framed structure constructed in Italy in the 1970s is considered. The nonlinear numerical model for IDA and

PushOver (PO) analysis is built in OpenSees environment [10]. Artificial accelerograms are used to simulate the earthquake excitation in IDA.

2.1 Analysed frames

The analysed case study frames are extracted from buildings with same plan layout (Fig. 1) but different number of storeys (3, 4, 5, 6 and 7 storeys). In order to reproduce the features and seismic deficiencies that characterize RC buildings structures constructed in the 1970s, structural members are arranged according to the old Italian practice and designed according to the building code enforced in Italy in 1974 [11]. The plan layout is rectangular, the dimensions are 28.5x15.5 m, and the weight of the deck is 440.6 t. The deck is constituted by RC joists orientated along the *y*-direction connected by a 4 cm thick RC slab. Beams and columns form frames primarily oriented along *x*-direction. Inter-storey height is equal to 3.20 m for all the storeys and all the buildings. The decks are supported by four-seven bay frames along the *x*-direction. Two outermost frames close the plan layout in the *y*-direction. The structural elements are located so that the distribution of stiffness and strength is symmetric, with respect to both *x*- and *y*-directions, thus ensuring that the structural response to horizontal seismic forces is purely translational.

The design of the structural members is based on the allowable stress method. The concrete characteristic compressive cylinder strength f_{ck} and the cubic one R_{ck} are equal to 20 and 25 MPa, respectively. Steel characteristic yield stress f_{yk} is equal to 380 MPa. Values of the allowable stress of concrete and steel are equal to 8.5 and 215 MPa, respectively. Internal forces are determined considering gravity loads only, based on the nominal values given in [12], and applied to the structural members according to the tributary area concept. The area of the column cross-section $A_{c,req}$ is sized considering axial force only. The minimum area of the longitudinal rebars is evaluated as the maximum value between 0.006 $A_{c,req}$ and 0.003 A_c, with A_c equal to effective cross-section area of the column. As a result, the cross-section of columns decreases from the bottom to the top of the building. Figure 1 shows the column cross-sections of the 7-storey building. Cross-sections of the buildings with lower number of storeys can be obtained eliminating the cross-sections of the storeys in excess from the bottom. Internal forces resting on beams are determined considering the scheme of continuous beams and the boundary schemes of single span beams fixed at their ends. The cross-section of beams is equal to 30x60 cm at each floor. The minimum reinforcement assumed for the tension zone of beams is equal to 0.15% of their concrete cross-section. Stirrups with diameter of 8 mm are used for beams and columns. Spacing of stirrups at beam ends is 150 or 200 mm, depending on the design shear force, and 300 mm in the central part. Instead, spacing of columns is 180 mm and is derived by the application of minimal requirements given by the code.



Storey	1, 8 25, 32	2, 7 26, 31	3, 6 27, 30	4, 5 28, 29	9, 16 17, 24	10, 15 18, 23	11, 14 19, 22	12, 13 20, 21
7th	30x30	30x30	30x30	30x30	30x30	30x30	30x30	30x30
6th	30x30	30x30	30x30	30x30	30x30	30x30	30x30	30x30
5th	30x30	30x30	30x30	30x30	30x30	30x40	30x35	30x35
4th	30x30	30x35	30x35	30x30	30x35	30x50	30x45	30x40
3rd	30x30	30x40	30x35	30x35	30x45	30x65	30x55	30x50
2nd	30x35	30x50	30x45	30x40	30x55	30x75	30x65	30x60
1st	30x35	30x60	30x50	30x45	30x65	35x75	30x75	30x70

Figure 1: Plan layout of the 7-storey case study building and size of column cross-sections

2.2 Numerical model

The nonlinear response of the analysed buildings subjected to a seismic excitation in the *x*-direction has been evaluated by 2D finite element models. Due to the virtual symmetrical distribution of mass and stiffness about the *x*-axis, each case study building has been modelled as set of plane frames that replicates half of the building. Specifically, the response of the building is simulated by a numerical model that includes frames X1 and X2. A Rayleigh viscous damping is used and set at 5% for the first two modes of vibration for nonlinear dynamic analysis. Since the concrete slab is assumed rigid in its own plane at each level, all the nodes of the same floor are constrained to have the same horizontal displacement by a rigid diaphragm. The masses are concentrated in the floors and are each equal to 220.3 t. A member-by-member modelling is adopted for the analysed frames. Columns and beams are modelled as *beamsWithHinges* elements [13], with a linear elastic central portion and plastic hinges at the ends. The length of plastic hinges is equal to the depth of cross-section. Cross-section of columns and beams is discretized into fibres. In particular, the concrete part is subdivided into fibres having 5 mm depth and width equal to the width of the cross-section, while rebars are modelled as single fibres. *Concrete01* and *Steel02 uniaxialMaterials* are the constitutive laws for concrete and steel fibres, respectively. The confinement effect on the core of member cross-sections is neglected to replicate the lack of seismic details of stirrups in members of RC structures designed without seismic provisions. The mechanical features are assigned in consistency with those used in the design. Since the floor constraint leads to unexpected axial forces in the fibre cross section of beams, a *zeroLenghtElement*, with zero axial stiffness and high shear and flexural stiffness, is interposed between columns and beams' end to allow the beam axial deformation [14].

2.3 Accelerograms for IDA

Nonlinear dynamic analyses are performed based on a set of 30 artificial ground motions adopted to simulate the seismic excitation with a return period $T_R = 475$ years. The selected accelerograms have a total duration of 30.5 s and are compatible with the EC8 elastic spectrum for soil type C and characterised by 5% damping ratio, a reference value of PGA for soil type A equal to 0.35g. Hence, PGA is scaled with values ranging from 0.05 g to 1.00 g, in step of 0.05 g.

3 PROPOSED EQUIVALENT DAMPING LAW

For each case study frame, 8 values of storey drift capacity ranging from 0.5% to 4% of the inter-story height, in step of 0.5%, are considered. This range includes, in the parametrical investigation, frames with low, medium, and high ductility. Each frame is evaluated using IDA and considering the different levels of ductility. Hence, the results from the IDA are used to define the related PGA-capacity. The seismic response of the frame is evaluated also by PO analysis. The equivalent damping ratio to be incorporated into the nonlinear static analysis is determined in order to replicate the PGA-capacity provided by the IDA. Finally, the results obtained for all the case study frames are fitted by means of an analytical damping law.

3.1 Incremental Dynamic Analysis

The procedure of CNR-DT 212/2013 guidelines [2] is used to assess each analysed frame. The storey drift demand determined by IDA is checked and the collapse value of PGA is identified when the demand to capacity ratio in terms of drift reaches the value of one. These results are used to determine the fragility curves of the structure [4] defined by a lognormal cumulative distribution function that describes the probability of exceedance of p_{CO} at different values PGA=x:

$$P(C|PGA = x) = \Phi\left(\frac{\ln x/\theta}{\sigma}\right) \tag{1}$$

were the parameters θ and σ are the mean and standard deviation of the logarithms of the values of PGA at frame collapse. The seismic hazard of the site is quantified by its mean hazard curve that returns the annual frequency of exceedance λ_S of different values of PGA. To derive this curve, four different probabilities of exceedance P_{VR} (5, 10, 63 and 81%) in a reference period of time of 50 years are considered. Second, the return periods T_R corresponding to the above probabilities of exceedance are determined according to EC8 [5]. The value of PGA corresponding to P_{VR} = 10% is assumed equal to 0.35 g and the PGA values corresponding to the other probabilities of exceedance are determined according to EC8 [5]. The four couples of values (PGA, λ_S=1/T_R) are plotted in a λ_S Vs PGA coordinate system. Finally, they are fitted by a linear function in the logarithmic space to derive the analytical seismic hazard function

$$\overline{\lambda}_s = k_0 e^{(-k_1 \ln PGA)} \tag{2}$$

where the values of parameters k_0 and k_1 are determined minimizing the sum of the squares of the residuals between the previously determined points and those provided by the analytical function.

The fragility function and the seismic hazard function are combined together in the following integral to compute the mean annual frequency of collapse λ_{CO}

$$\lambda_{CO} = \int_0^{+\infty} p_{CO}(s) \left|\frac{d\overline{\lambda}_s(s)}{d\,s}\right| d\,s \cong \sum_{i=1}^{n} p_{CO}(s_i) \left|\Delta\overline{\lambda}_i\right| \tag{3}$$

For a given fragility curve, the values of λ_{CO} depends on the seismic hazard function. Hence, for each frame and level of ductility capacity, the above defined hazard function is scaled until the mean annual frequency of collapse λ_{CO} equates the maximum acceptable value λ_{CO}^{Max} specified by the CNR-DT 212/2013 guidelines for existing structures for near collapse limit state, which is equal to 2.3x10^{-3}. The value of PGA corresponding to the earthquake return period stipulated by the Italian building code [15] for the near collapse verification is determined from this hazard curve and assumed as PGA-capacity of the frame.

3.2 Nonlinear static analysis

The seismic response of the case study frames is determined also by the nonlinear static method of analysis following the approach of the Capacity Spectrum Method (CSM) [7], which idealises the structure by an equivalent overdamped elastic Single-Degree Of Freedom (SDOF). To this end, a displacement-controlled PO analysis of the frame is carried out considering a load pattern proportional to the related first mode of vibration. For each step of the analysis, the drift demand is evaluated and compared to the drift capacity. Collapse is attained when the drift demand becomes equal to the capacity. The results of this analysis are summarised in the base shear Vs. roof displacement relationship (performance curve) stopped at the attainment of the frame collapse. Then the performance curve is scaled dividing the coordinates of by the modal participation factor of the first mode of vibration and the related top floor component. The stiffness of the equivalent SDOF is calculated as the slope of the line that passes from the origin of the reference system and the collapse point. The equivalent damping ratio ξ_{eq} takes into account the energy dissipation derived from the nonlinear behaviour of the structure and can be related to the PGA-capacity of the structure. Indeed, the PGA-capacity can be assumed as the value of PGA that characterizes the overdamped elastic spectrum passing from the collapse point of the scaled performance curve. The procedure proposed in [15] is used for this end. Hence, for each analyzed frame, it is derived the equivalent damping ratio ξ_{eq} that returns a PGA-capacity based on PO analysis equal to that based on IDA and determined as in Section 3.1.

Since, ξ_{eq} is traditionally related to the ductility capacity of the system, the performance curve is transformed into an equivalent elastic-perfectly plastic relationship and the ductility capacity of the frame is determined as the ratio of the top displacement at collapse to that corresponding to the yielding. The idealised bilinear relationship is defined by the yield displacement and strength, which are determined by two equivalence conditions:

1. the areas under the original and idealized curve within the range of interest are the same;
2. the linear branch of the idealized curve intersects the performance curve for a value of base shear equal to 60% of the yield strength.

3.3 Calibration and validation of the damping law for PGA-capacity prediction

For each analysed frame and drift capacity, the values of ductility capacity μ and the related equivalent damping ratio ξ_{eq} evaluated in Section 3.2 are plotted in Figure 2. Here, each red point represents the ductility capacity μ and the related equivalent damping ratio ξ_{eq} of the single case. It is evident that ξ_{eq} increases as μ increases. Furthermore, the points are distributed in a narrow band. Based on these results, the factors ξ_{eq} is expressed through logarithm function of μ. The coefficients of the function are determined by minimizing the standard deviation of the differences between the values of ξ_{eq} provided by the logarithm function and those obtained by the numerical investigation. Since equivalent damping ratio ξ_{eq} includes also the inherent viscous damping in the elastic range ξ_0, a lower bound equal to ξ_0 (generally, 5% for RC framed structures) is set on ξ_{eq}.

$$\xi_{eq} = 0.6421\ln(\mu) - 0.2419 \geq 0.05 \tag{3}$$

To investigate the effectiveness of the proposed damping law, the PGA-capacity of each frame is determined by the nonlinear static method of analysis with the equivalent damping ratio provided by Equation (3). The PGA-capacity of the analysed frames is determined also by three other approaches based on the nonlinear static methods available in literature, i.e. the N2 method proposed by Fajfar [8] and the Capacity Spectrum method applied once with the equivalent damping law proposed by Freeman [7] and again with the equivalent damping law by Rosenblueth [17]. The PGA-capacity based on IDA, determined in Section 3.1, is assumed as target to be predicted. Hence, the errors committed by the four methods in predicting the PGA-capacity provided by IDA are calculated.

Figure 3 shows and compares the errors in predicting the PGA-capacity using the proposed methods and those available in literature. Drift capacities equal to 1.5% and 3.5% of the inter-

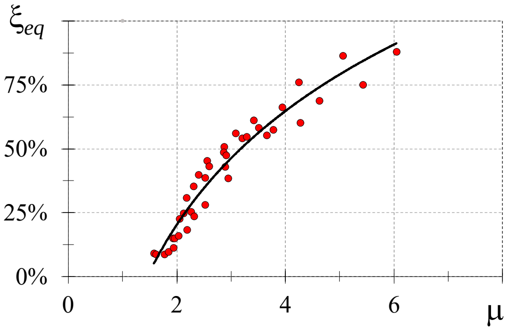


Figure 2: Determination of the analytical equation for ξ_{eq}

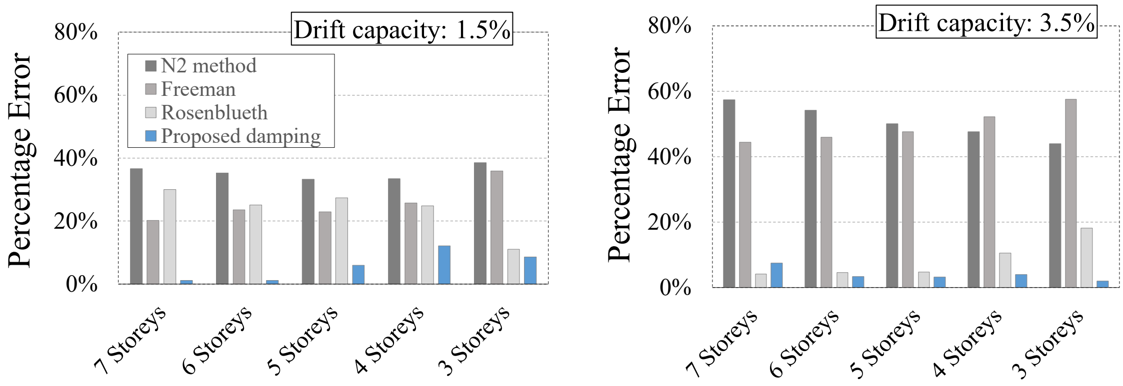


Figure 3: Error committed in prediction of PGA-capacity, comparison between different nonlinear static methods.

storey height are considered to represent frames that experience moderate and large ductility demand at collapse limit state, respectively. The N2 method and the CSM method with the equivalent damping ratio proposed by Freeman predict a PGA-Capacity quite different from the target value, independently of number of storeys and value of ductility demand. With few exceptions, the error committed by these two methods is about 35% and 50% for frames with moderate and large engagement in nonlinear range of their behaviour, respectively. When the PGA-capacity is predicted by the Rosenblueth's damping law, the error is significant and generally similar to that observed for the N2 method and the CSM with Freeman equivalent damping ratio for frames with moderate ductility demand. The prediction improves for frames that experience large ductility demand. In this case, the Rosenblueth's damping law leads to errors ranging from 4% to 20% depending on the number of storeys of the frame. The proposed damping law is proved to be the most accurate and reliable in predicting the target PGA-capacity. Indeed, it provides a prediction of the PGA-capacity based on IDA with an error always lower that 10%, regardless of the frame's engagement in nonlinear range of behaviour and number of storeys.

4 CONCLUSION

This paper investigates the possibility of estimating the output of IDA and reliability analysis of a building with RC framed structure by means of the nonlinear static method of analysis. To this end, the outcome of "IDA + reliability analysis" is condensed in the related PGA-capacity, i.e., the value of PGA corresponding to the earthquake return period for the near collapse limit state verification. This value is obtained from the hazard curve that determines a mean annual frequency of collapse λ_{CO} equal to the maximum acceptable threshold λ_{CO}^{Max}. Hence, the nonlinear static method of analysis in the format of CSM is equipped with an equivalent damping ratio that makes the resulting PGA-capacity equal to that provided by "IDA + reliability analysis", here assumed as target PGA-capacity. The parametrical investigation carried out in the paper led to the formulation and calibration of the analytical law for the determination of this equivalent damping ratio. Furthermore, it also proved the effectiveness of the proposed nonlinear static method against classical methods already present in literature and enforced by seismic codes. In summary, the results of the numerical investigation carried out with RC frames with different number of storeys (from 3 to 7) and drift capacity (from 0.5% to 4.0% of the interstorey height) shows that:

- the N2 method and the CSM with the Freeman equivalent damping ratio estimate the target PGA-capacity with a significant percentage error, up to 60%, regardless of number of storey and drift capacity of the frame;

- the CSM with the Rosenbluth equivalent damping ratio returned a better estimation of the target PGA-capacity for frames with large drift capacity, while it was deficient for frame with low drift capacity, when the percentage error was in the range 20-30%;
- the CSM with the proposed equivalent damping ratio provided a suitable estimation of the target PGA-capacity in all the examined cases; indeed, only in one case the percentage error was equal to 12%, while it was lower than 10% in all the others.

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