

PARAMETRIC STUDY ON RETROFIT OF RC FRAMED BUILDINGS BY DISSIPATIVE TOWER

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Abstract

Italy is a country with high seismicity, but most of the existing buildings does not comply with current seismic standards. Indeed, they were designed before the enforcement of seismic standards or according to old seismic codes. A large number of these buildings is endowed with RC framed structure. These structures are inadequate in terms of lateral stiffness and strength to support seismic actions, cannot guarantee a ductile collapse mechanism, and thus result in safety risks to society. Therefore, these buildings need to be seismically retrofitted. In this paper, the addition of external dissipative towers (seismic-resistant steel towers equipped with Maxwell nonlinear fluid viscous dampers (FVDs)) to the existing RC building is proposed for retrofit. This technique is expected to reduce the seismic demand of the existing structure. The first goal of this study is to investigate and quantify the benefits gained by means of the dissipative towers for seismic retrofit of RC buildings. The second goal is to study, through parametric analysis, the influence of size and heightwise distribution of FVDs and of the stiffness of the steel tower, on the seismic response of the building to identify the combination of the features that lead to the best enhancement of seismic performance. To this end, a pilot building, representative of existing RC framed buildings, is designed and a numerical model of the building with and without damper-tower system is developed. Then, incremental nonlinear dynamic analyses are performed, and the results are used to determine the seismic performance in terms of fragility curves and mean annual frequency of exceedance of selected limit states. Comparing the results of systems with and without dissipative towers quantifies the achievable benefit, while the comparison of the results of systems retrofitted by different dissipative towers provides information on the influence of the design parameters.

Keywords: fluid viscous dampers, dissipative towers, RC frames, exiting buildings, nonlinear dynamic analysis.

1 INTRODUCTION

The high seismic risk in Italy, which is linked to both soil hazard and the construction vulnerability, is a nationwide issue, as demonstrated by recent dramatic seismic events. The latest ISTAT census shows that two-thirds of residential buildings do not comply with current seismic standards, because they were designed before the enforcement of seismic standards or in accordance with obsolete seismic codes. This poses a significant risk to public safety. Therefore, Italy's building stock is vulnerable, and the importance of new effective strategies to improve and adapt existing buildings to seismic conditions is evident from this brief reflection.

Among the seismic retrofitting strategies discussed in literature, this study investigates the addition of external seismic resistant steel towers, equipped with Maxwell nonlinear Fluid Viscous Dampers (FVD) (i.e. spring connected in series with a dashpot), to the existing RC framed structure. The dissipative towers are external reaction structures, with high lateral stiffness and strength, independent foundations, punctually and strategically placed on the perimeter of the building. The additional damper-tower systems sustain part of the horizontal seismic action and reduces accelerations and displacements of the RC structure. Figure 1 illustrates the schematic representation of the combined building-damper-tower system. As shown, the tower is connected to the existing building at floor level with horizontal supporting braces equipped with Fluid Viscous Dampers (FVDs). In this configuration, the FVDs's deformation are proportional to the floor displacements, and its efficiency is related to the stiffness of the external system.

In addition to reduce seismic demand, the dissipative tower offers other advantages. It enables quick and easy external installation, thus minimizing disruption to the occupants and allowing the building to remain operational, thereby reducing costs associated with activity interruptions. Furthermore, the system is local, meaning the design is not a function of the specific architectural and structural configuration of the RC building, which makes it ideal for standardization. Finally, the volume of the towers can be used to create new spaces (such as emergency stairs, elevators, installations, etc.), contributing to improved accessibility, functionality and architectural enhancement of the building.

The design of the described damper-tower system is complex, because many parameters influence the achieved seismic performance of the retrofitted building. The aim of this paper is to (i) investigate the effectiveness of dissipative towers for seismic retrofit of RC buildings and (ii) evaluate, the influence of size and height arrangement of FVDs, as well as the stiffness of the steel tower, on the seismic response of the building to discover the optimal combination of these parameters that results in the best seismic performance.

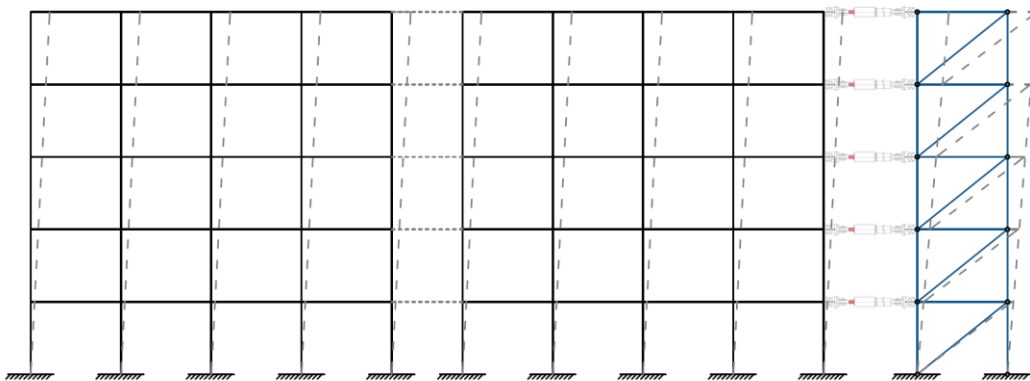


Figure 1: Building-damper-tower system

In order to evaluate the effect of the proposed strategy and compare the influence of the design parameters, the seismic response of a case study RC frame, representative of existing Italian buildings, has been assessed before and after retrofitting by dissipative towers. Twenty seismic upgrading configurations are designed for each plan direction and the related 2D finite element numerical models are developed in OpenSees. Incremental nonlinear Dynamic Analysis (IDA) is performed using a set of 30 artificial ground motions to determine the demand-to-capacity ratios of beams and columns, in terms of shear force and chord rotation. Hence, fragility curves and mean annual frequencies of exceedance are determined. Near Collapse (NC) and Significant Damage (SD) states are assumed as limit states that should not be exceeded.

2 DESIGN OF THE CASE STUDY RC FRAME

A pilot building is considered and is intended to be representative of the buildings constructed in Italy between the 1970s and 1990s. In order to reproduce the features and seismic deficiencies that characterize these buildings, it was designed according to the Old Italian practice and code [1], which only accounted for gravity loads (GL) and followed the allowable stress method.

The case study is a five-storey high reinforced concrete (RC) framed building (Figure 2), with an interstorey height of 3.20 m. The plan layout is rectangular, the dimensions are 28.50 x 15.50 m, with a total height of 16.0 m and an overall weight of 440.60 t. The floor is constituted by RC joists orientated along the Y-direction connected by a 4 cm thick RC slab. Beams and columns form frames primarily orientated along X-direction. Due to this, the building exhibits larger lateral stiffness and strength along the X-direction compared to the Y-direction; such a difference in the two directions of the building is a feature that often characterizes structures designed without following seismic prescriptions. The structural elements are located so that the distribution of stiffness and strength is symmetric, with respect to both X- and Y-directions, thus ensuring that the structural response to horizontal seismic forces is purely translational. The floors are supported by four-seven bay frames along the X-direction. Two outermost frames close the plan layout in the Y-direction.

To design the building, the internal forces are determined considering gravity loads only, based on the nominal values given in [2] and applied to the structural members according to the tributary area concept. The area of the column cross-section $A_{c,req}$ is sized considering axial force only. The minimum area of the longitudinal rebars is evaluated as the maximum value between $0.006 A_{c,req}$ and $0.003 A_c$, with A_c equal to effective cross-section area of the column. As a result, the cross-section of columns decreases from the bottom to the top of the building. Instead, internal forces resting on beams are determined considering the scheme of continuous beams and the boundary schemes of single span beams fixed at their ends. The minimum reinforcement assumed for the tension zone of beams is equal to 0.15% of their concrete cross-section. The cross-section of beams is equal to 30x60 cm at each floor. Stirrups with diameter of 8 mm are used for beams and columns. Spacing of stirrups in beams is equal to 150 or 200 mm at the ends, depending on the design shear force and 300 mm in the central part. Instead, spacing of columns is equal to 180 mm and is derived by the application of minimal requirements given by the code. The contribution of infill panels has been ignored because their mechanical properties are so poor that their structural contribution is considered negligible.

About materials, concrete C20/25 and steel grade Feb38K for rebars are used, consistently with the design practice of 1970s. The concrete characteristic compressive cylinder strength f_{ck} and the cubic one R_{ck} are equal to 20 and 25 MPa, respectively. Steel characteristic yield stress f_{yk} is equal to 380 MPa. Values of the allowable stress of concrete and steel are equal to 8.5 and 215 MPa, respectively.

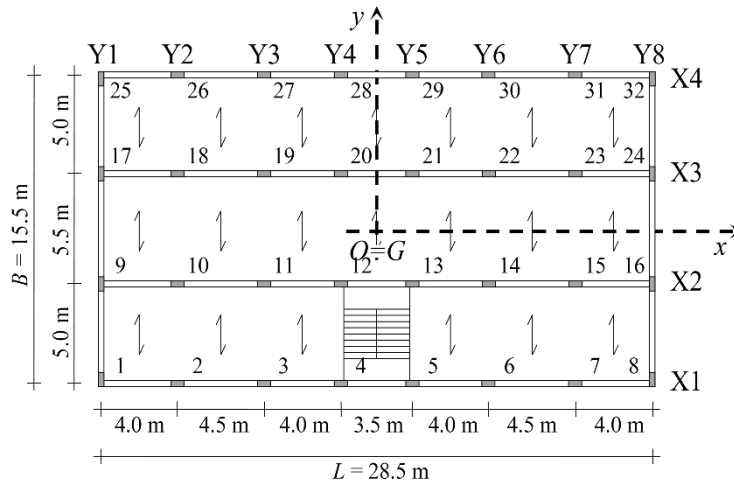


Figure 2: Plan layout of the case study building

3 NUMERICAL MODEL OF THE BUILDING-DAMPER-TOWER SYSTEM

The nonlinear behaviour of the case study before and after seismic retrofit with dissipative towers has been investigated by 2D finite element models implemented in OpenSees [3]. Due to the symmetrical distribution of mass and stiffness around the two principal axes of the building's plan layout, the case study building has been modelled as set of plane frames, that replicates half of the building, considering the seismic excitation that acts separately along the X- and Y-direction. Specifically, when seismic input is applied along the X-direction, the response of the building is simulated by a numerical model that includes frames X1 and X2, while in case of forces along the Y-direction the four frames Y1 to Y4 are modelled. Hereinafter, the two sets of frames are referred as X-frame and Y-frame models. A Rayleigh viscous damping is used and set at 5% for the first two modes of vibration. Since the concrete slab is assumed rigid in its own plane at each level, all the nodes of the same floor are constrained to have the same horizontal displacement through a rigid diaphragm. The masses are concentrated in the floors and are each equal to 220.4 t. A member-by-member modelling is adopted for the analysed frames. Columns and beams are modelled as *beamsWithHinges* elements, [4], with a linear elastic central portion and plastic hinges at the ends. The length of plastic hinges is equal to the depth of cross-section. Cross-section of columns and beams is discretized into fibres. In particular, the concrete part is subdivided into fibres having 5 mm depth and width equal to the width of the cross-section, while rebars are modelled as single fibres. *Concrete01* and *Steel02 uniaxialMaterials* are the constitutive laws for concrete and steel fibres, respectively. The confinement effect on the core of member cross-sections is neglected to replicate the lack seismic details of stirrups in members of RC structures designed without seismic provisions. The mechanical features are assigned in consistency with those used in the design. Since the floor constraint leads to unexpected axial forces in the fibre section of beams, a *zeroLengthElement*, with zero axial stiffness and high shear and flexural stiffness, is interposed between columns and beams' end to allow the beam axial deformation [5].

To evaluate the response of the structure after the introduction of dissipative tower, the numerical model has been updated by adding two plane steel towers, connected to the existing building at floor level with fluid viscous dampers. These are introduced in series to the train of plane frames X1 and X2 when seismic forces act along the X-direction, and to the train of plane frames Y1 and Y4 when seismic forces act along Y-direction.

The towers have the same interstorey height as the RC frames, and for each frame, the span length is equal to interstorey height. They are assumed to be hinged at the base. Since members of the steel towers are designed to remain elastic during the ground motion, an *elasticBeamColumn* element is used to model columns, beams and diagonals. Pin connections between members are simulated by springs with very low rotational stiffness.

The viscous damper is modelled using a *twoNodeLink* element with the *uniaxialMaterialViscousDamper*. Each damper is characterized by the elastic stiffness of a linear spring to model the axial flexibility, along with a damping coefficient and a velocity exponent, which are related by the force-velocity relationship (1):

$$F_d(t) = C_d |\dot{u}_d(t)|^\alpha \text{sgn}(\dot{u}_d(t)) \quad (1)$$

At each storey, the *twoNodeLink* element connects the master node of the building with the first node of the tower. In this configuration, one node has the same horizontal displacement of building's floor, by horizontal *equalDOF* constrains, while the second node is free from rigid diaphragm, allowing the tower to deform independently.

4 RESEARCH METHODOLOGY

To assess the benefits of dissipative towers for the seismic retrofit of RC buildings and identify the combination of the features that lead to the best enhancement in seismic performance, incremental nonlinear dynamic analyses are performed, considering the numerical models described in Section 3. Twenty seismic upgrading configurations are considered for each plan direction and reported in Tables 1 and 2.

In these tables, the first column indicates the floors where the dampers are installed; the first row lists the parameters being varied, and the corresponding values are shown in the row below. The cells at the intersection of these data points are interpreted as follows: for example, Dtw_B345 type B tower ($F_d = 400$ kN), with dampers located on floors 3, 4, and 5.

Location of dampers	F_d		
	A = 200 kN	B = 400 kN	C = 600 kN
4-5	Dtw_A45	Dtw_B45	Dtw_C45
3-5	Dtw_A345	Dtw_B345	Dtw_C345
3-4	Dtw_A34	Dtw_B34	Dtw_C34
1-5	Dtw_A12345	Dtw_B12345	Dtw_C12345

Table 1: Retrofit configurations with different damper force F_d

Location of dampers	k		
	B = 100%	D = 150%	E = 200%
4-5	Dtw_B45	Dtw_D45	Dtw_E45
3-5	Dtw_B345	Dtw_D345	Dtw_E345
3-4	Dtw_B34	Dtw_D34	Dtw_E34
1-5	Dtw_B12345	Dtw_D12345	Dtw_E12345

Table 2: Retrofit configurations with different lateral stiffness of steel towers

The first three configurations, from A to C, are obtained by setting the stiffness value to the minimum one ($k=100\%$) required to satisfy the strength verifications and by varying the base shear (F_d) of the bare building absorbed by the dampers (from 200 kN to 600 kN). This is distributed, over the height to the dampers according to the first vibration mode of the structure in the considered direction, through the equation (2):

$$F_d^i = F_d \frac{\phi_i^1}{\sum_{i=1}^n \phi_i^1} \quad [\text{FB1}] \quad (2)$$

Where F_d^i is the base shear ratio absorbed by the dampers on the i -th floor

F_d is the base shear amount of the bare building absorbed by the dampers

ϕ_i^1 is the eigenvector associated to the first vibration mode

The other configurations, B, D and F, are obtained by fixing $F_d = 400 \text{ kN}$, and increasing the stiffness of the towers, without changing the size of the dampers. In each configuration, non-linear dampers with $\alpha = 0.15$ are adopted. The damper size is assumed to be constant at every level and the C_d constants are determined by inverting the equation (1).

The seismic excitation is simulated by a set of 30 artificial ground motions, each with a total duration of 30.5 seconds. These motions are compatible with the EC8 elastic spectrum for soil type C and characterized by 5% damping ratio. The reference PGA for soil type A equal to $0.35g$ is adopted to simulate the seismic excitation corresponding to a return period $T_R = 475$ years [7]. This set of ground motion is scaled from PGA 0.05 g to 0.60 g in steps of 0.05 g .

The structural response, before and after the seismic upgrading, is first investigated in terms of storey drift demand Δ . In particular, the maximum values of storey drift are evaluated for each ground motion and then averaged over the thirty inputs for each PGA. Furthermore, the seismic performance of the RC frame, is evaluated with reference to the NC and SD limit states. For a given limit state, the demand-to-capacity ratios (D/C) of columns and beams are evaluated in terms of shear force (V_{Ed} / V_{Rd}) and chord rotation ($\vartheta / \vartheta_{LS}$). The shear strength capacity is calculated according to EC2 [8]. The chord rotation capacity of columns ϑ_{NC} is assumed equal to the ultimate value of chord rotation ϑ_{um} calculated according to the equation provided by EC8 [9] for the NC limit state, while at SD limit state, the chord rotation capacity ϑ_{SD} is taken equal to 75% of ϑ_{NC} . Both the shear strength and chord rotation capacities are determined at each step of the time-history analysis and compared to the corresponding demands. As for the suitability of the steel tower's cross-sections, the attainment of either instability or yielding of all members is checked. Stability (SI) and Resistance (RI) indexes are derived based on the relevant provisions stipulated in Eurocode 3 [10]. Both the considered indexes are determined considering the combined effect of axial internal forces and bending moments. Stability and resistance indexes lower than 1.0 identify members satisfying the related verifications.

The seismic performance of the analysed frames is evaluated by (i) the construction of fragility curves and (ii) the determination of the mean annual frequency of exceedance λ_{LS} of selected limit states. The fragility function describes the probability of exceedance of the limit state (LS) at different PGAs and is defined by a lognormal cumulative distribution function (3):

$$P(C|IM = x) = \Phi\left(\frac{\ln x / \theta}{\sigma}\right) \quad (3)$$

where IM is the ground motion intensity measure (here assumed as PGA)

θ is the meaning of the $\ln$ of the values of PGA corresponding to the considered LS

σ is the standard deviation of the $\ln$ of PGA corresponding to the considered LS

Fragility curves allow for a first quantification of the maximum performance the case study can provide, regardless of the seismic hazard of the site. In this regard, the PGA corresponding to a 50% probability of exceedance of the relevant limit state is conventionally assumed to be the PGA capacity. The performance obtained is considered adequate if the PGA capacity is not lower than the PGA value established by the code for the related verification.

Finally, the seismic performance of the RC frame also analysed in terms of mean annual frequency of exceedance λ_{LS} of the SD and NC limit states, which is calculated as (4):

$$\lambda_{LS} = \int_0^\infty p_{LS}(s) \cdot \left| \frac{d\bar{\lambda}_s(s)}{ds} \right| ds \approx \sum_{i=1}^n p_{LS}(s_i) |\Delta \bar{\lambda}_i| \quad (4)$$

where $p_{LS}(s)$ is the probability of exceedance of the considered limit state (fragility curve)
 $\bar{\lambda}_s(s)$ is the probability of exceedance of the assigned PGA (seismic hazard curve)

The mean seismic hazard function is obtained by the hazard corresponding to four values of PGAs, which are characterized by probabilities of exceedance P_{VR} of 2, 10, 20 and 50% in a reference period of 50 years. The PGA for $P_{VR} = 10\%$ is assumed to be 0.35 g, and PGAs corresponding to the other values of P_{VR} are calculated as $PGA = PGA_{10\%} \times (P_{VR}/10)^{1/3}$ according to EC8 [10]. The values of $\bar{\lambda}_s$ corresponding to T_R are calculated as $\bar{\lambda}_s = 1/T_R$. The four couples of values (PGA, $\bar{\lambda}_s$) are reported in a PGAs Vs. $\bar{\lambda}_s$ coordinate system and fitted by a quadratic function in a logarithmic space [11]. The values of the mean annual frequency of exceedance λ obtained for the SD and NC limit states of the cases study are compared to the relevant limit values (λ_{req} [EMM2]) reported in the Italian technical document CNR-DT 212/2013 [11] for existing buildings belonging to class II (ordinary buildings).

5 SEISMIC PERFORMANCE BEFORE AND AFTER SEISMIC UPGRADING

In the following sections, the seismic performance of the original case study frame is compared to that of the structure upgraded by dissipative towers. For a better explanation of the results, the paragraph 5.1 presents the results obtained from the first three retrofit configurations (A to C), where the damper heightwise distribution and F_d rates vary, with equal stiffness k (Table 1). Then, paragraph 5.2 presents the ones obtained from the other configurations (B, D and E), where the damper distribution and tower stiffness change, with equal F_d , Table 2. For the sake of brevity, only the results obtained for the X-direction are presented in this paper.

5.1 Influence of tower strength and heightwise distribution of FVDs

Figure 3 shows the ratio λ/λ_{req} of the mean annual frequency of exceedance (λ) to the maximum acceptable mean annual frequency of exceedance (λ_{req}) at SD and NC limit states. For the RC frames in the original configuration (red bar), the ratio λ/λ_{req} exceeds unity at both SD and NC limit states, which is due to the ductile failure of columns. In each retrofit configuration, the introduction of the dissipative towers reduces the mean annual frequency of exceedance well below the required value at both SD and NC limit states, by up to 95%, confirming the effectiveness of the retrofit approach. A common trend is the reduction of the ratio λ/λ_{req} [EMM3] as the base shear F_d sustained by the dampers increases (from configuration A to C), which in turn increases the force transmitted to the towers and the design strength of the steel tower.

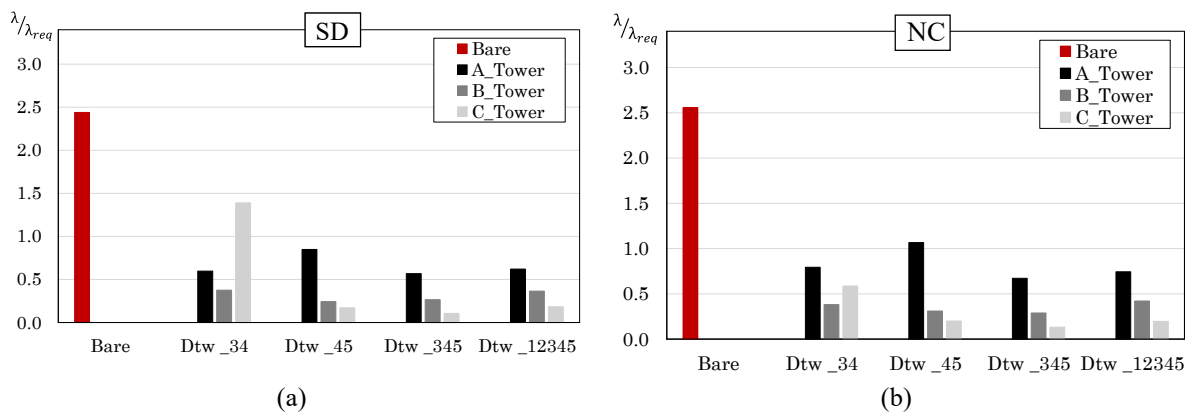


Figure 3: Ratio of mean annual frequency of exceedance of SD (a) and NC (b) limit states for equal $k\%$

Figure 4 compares the heightwise distribution of the pilot building before seismic upgrading (red line) with the configurations enhanced by dissipative towers for a PGA value of 0.27g [EMM4]. Figures 4a, 4b, 4c and 4d group the results based on damper's height distribution. The results show that the frame in the original configuration exhibits the largest drift demand at the fourth storey, where the formation of a soft storey collapse mechanism occurs. In each retrofit configuration, the introduction of the dissipative towers results in a significant reduction of the drift demand. This benefit increases from configuration A to C as the tower strength increases, and the drift demand reduces up to almost 75% compared to the original building.

The arrangement of the dampers significantly influences the effectiveness of the intervention. Installing dampers at all floors (D_{tw_A12345} , D_{tw_B12345} and D_{tw_C12345}) or creating a buffer strip around the weak storey (D_{tw_A345} , D_{tw_B345} and D_{tw_C345}), provides similar results; this is because the dampers on the bottom levels dissipate less, due to the lower velocity variation. The addition of dampers to the weak and upper floors (D_{tw_A45} , D_{tw_B45} and D_{tw_C45}), on the other hand, leads to a similar drift, but moves the soft storey collapse mechanism from the fourth to the third level. Finally, adding dampers only on the weak floor and the floor below (D_{tw_A34} , D_{tw_B34} and D_{tw_C34}) leads to the increase in the upper floors. Therefore, configuration D_{tw_345} is the best performing, as it provides the best results with the fewest elements (i.e. lowest costs).

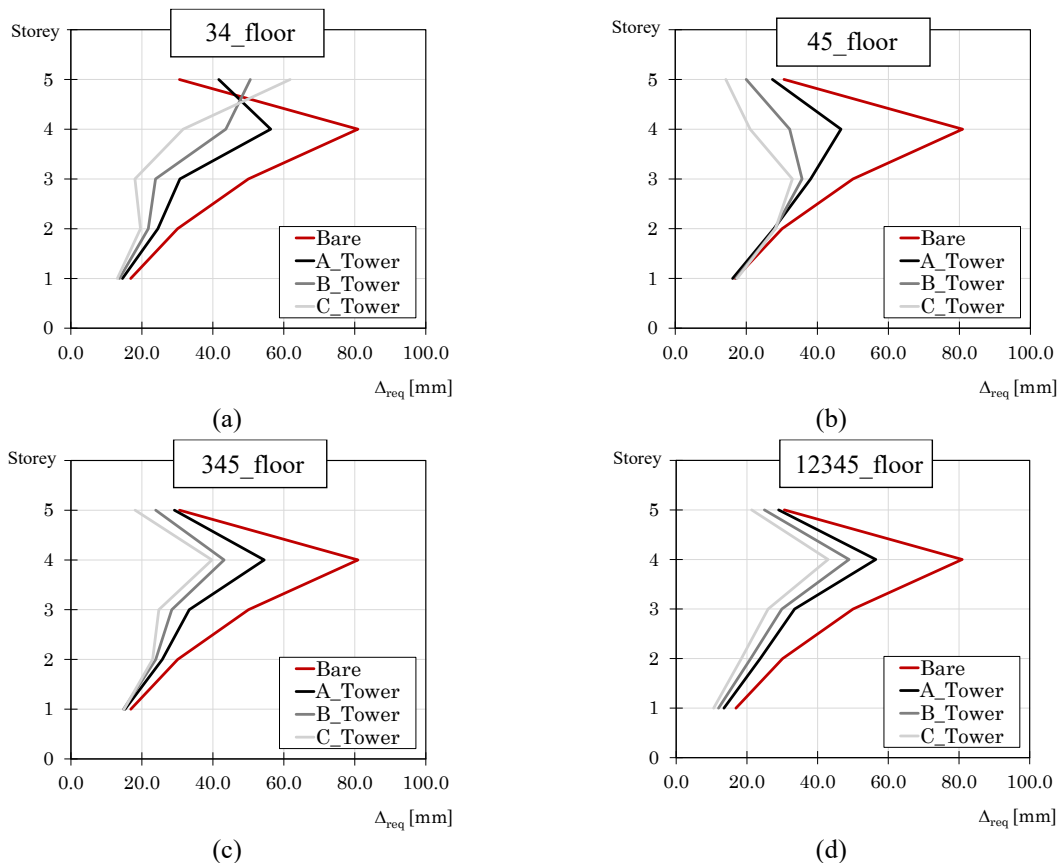


Figure 4: Distribution along the height of the drift demand for each height damper configuration for equal k%

5.2 Influence of tower stiffness and heightwise distribution of FVDs

The second comparison is made in terms of tower's stiffness, for the same base shear ($F_d=400\text{kN}$) sustained by the dampers. Figure 5 shows the ratio λ/λ_{req} for both the considered limit states. The red bars represent the λ/λ_{req} of the configuration before retrofit. The histograms again confirm the validity of the intervention, providing ratios lower than unity, and

demonstrate that as the tower's stiffness increases, thus the contrast opposed to floor displacements, the λ/λ_{req} decreases, but only slightly.

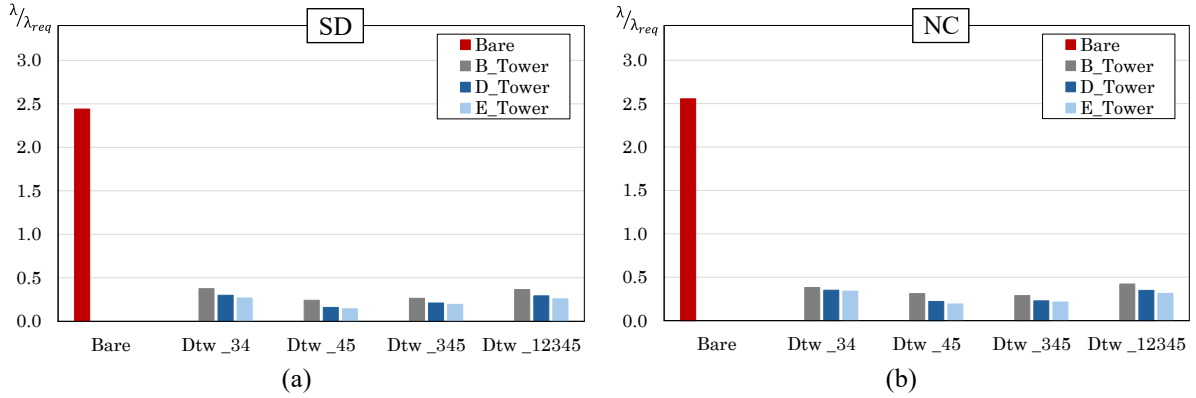


Figure 5: Ratio of mean annual frequency of exceedance of SD (a) and NC (b) limit states for equal F_d

This trend is confirmed by the heightwise distribution of the storey drift demand along the height shown in Figure 6. It reveals a slight reduction in the drift demand, as the tower's stiffness increases, from B to E towers, with [FB5] a reduction of up to almost 20% compared to the B-retrofit configuration with $k=100\%$. As in the previous section, and for the same considerations, the best heightwise distribution of FVDs is the 345_floor configuration, which creates a buffer strip around the weak storey. Therefore, the stiffness of the tower's members appears to be unaffected by the storeys where the dampers are inserted.

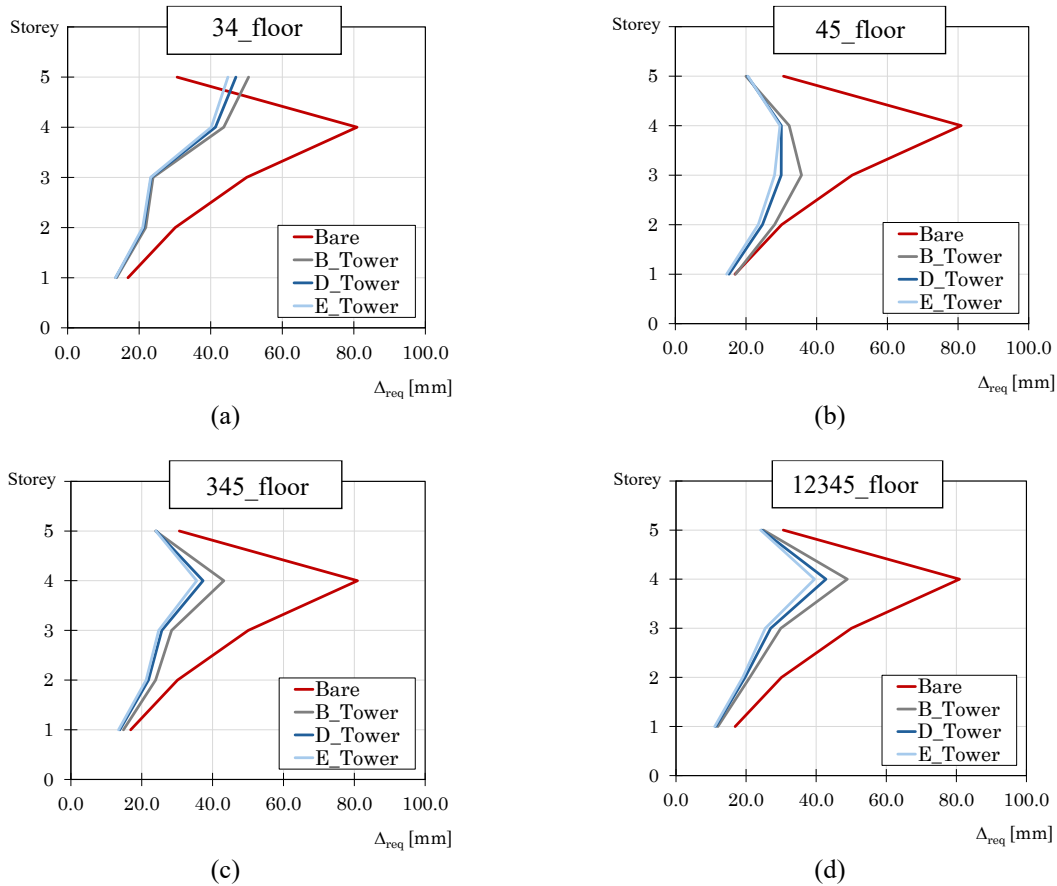


Figure 6: Distribution along the height of the drift demand for each height damper configuration equal F_d

6 CONCLUSION

This paper analyses the benefits that can be gained by adding external steel dissipative towers equipped with FVDs. It also investigates, through parametric analysis, the influence of the height distribution of FVDs, as well as lateral strength and stiffness of the steel towers, on the seismic response of the building-damper-tower system. To this end, a case study of an RC-framed building is designed by old building codes and without considering seismic provisions. Hence, a 2D numerical model of this building, in both its original and upgraded configurations, is created in OpenSees.

The seismic response of the case study buildings, before and after seismic retrofit, is assessed in terms of mean annual frequency of exceedance of the NC and SD limit state. The parametric analysis included twenty different retrofit configurations for each in-plan direction and confirmed the effectiveness of the proposed technique. Indeed, all the retrofit configurations led to mean annual frequency of exceedance λ lower than that of the original building. In most of the cases, λ was reduced below the target values given in [11] for the SD and NC limit states.

On the other hand, the results show that important role of the heightwise distribution of the FVDs, which significantly influences the system's response. In particular, the best allocation of FVDs was the one that follows the drift distribution of the original building, creating a buffer strip around the weak storey. An improper choice of floors where FVDs are located, as in the 34 floor configuration, could reduce the effectiveness of the intervention^[FB6]. These results outline the importance of a reliable assessment tool to detect the weak storey of the building and provide guidelines on the proper design the seismic upgrading intervention. Likewise, a significant improvement in the performance of the building occurs as the resistance of the tower, and therefore the size of the dampers, increases. Finally, a design that provides the steel towers with strength sufficient to sustain the forces transmitted by the FVDs also provides them with lateral stiffness sufficient to exploit the seismic upgrading potential of FVDs. Increasing the tower's stiffness above this value did not lead to a further substantial improvement in the response of the building-tower system.

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