Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2025) 4644-4654

ECCOMAS
Proceedia

COMPDYN 2025
10th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15-18 June 2025

KINEMATICALLY-INDUCED BENDING OF BRIDGE PIERS ON CAISSON FOUNDATIONS

Raffaele Di Laora[1], Raffaele Cesaro[1], and Emmanouil Rovithis[2]

[1] Department of Engineering, University of Campania "Luigi Vanvitelli"
81031 Aversa (CE), Italy
e-mail: raffaele.dilaora@unicampania.it, raffaele.cesaro@unicampania.it

[2] Department of Civil Engineering, Democritus University of Thrace
Xanthi, Greece
erovithis@civil.duth.gr

Abstract

Bridge piers in seismic areas are usually supported by deep foundations, such as piles or caissons. Given the large flexural stiffness of the deep foundations, they are not able to adapt to the short wavelengths in the surrounding soil, experiencing a motion which differs, at high frequencies, from the free-field one. In addition, caisson foundations, due to their lower rotational stiffness as compared to pile foundations, are more prone to experience a kinematically-induced rotational component of the motion, which does not exist in free-field conditions. In this case a kinematic bending arises in the pier owing to the rotational constraint exerted by the pier-to-deck joint.
This work proposes an analytical solution for assessing such kinematic bending moments, as function of excitation frequency, soil and pier properties, as well as rotational stiffness of foundation and top restraint.
Furthermore, a closed-form analytical solution is provided to quantify the magnitude on the kinematic bending moment compared to the inertially-induced one. A focus on the role played by the parameters involved is also presented.

Keywords: Bridge Piers, Earthquakes, Caissons, Seismic Design.

ISSN:2623-3347 © 2025 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2025.
doi: 10.7712/120125.12764.25729

1 INTRODUCTION

Deep foundations are typically employed to support bridge piers in seismic areas, such as piles or caissons, to withstand the significant forces experienced during seismic events.

These deep foundations possess large flexural stiffness, and as a consequence, they are unable to adapt to the short wavelengths in the surrounding soil during the passage of seismic waves. As a result, at higher frequencies, the motion experienced by the foundations diverges from the free-field motion.

Among the various foundation types, caissons exhibit lower rotational stiffness compared to pile foundations, making them more susceptible to a kinematically-induced rotational component of motion, which does not occur under free-field conditions. This rotation leads to the development of a kinematic bending moment in the pier, attributed to the rotational constraint imposed by the pier-to-deck joint.

This effect is typically neglected in current practice and only a few scientific studies have highlighted this phenomenon, often through FEM analysis [3, 4].

In light of the above, this work proposes an analytical solution to assess the kinematic bending moment as a function of excitation frequency, soil and pier properties, and the rotational stiffness of both the foundation and top restraint. The proposed solution is derived under the assumption of visco-elastic behavior of the soil. Furthermore, a closed-form analytical solution is provided to quantify the magnitude of the kinematic bending moment in comparison to the inertially-induced moment.

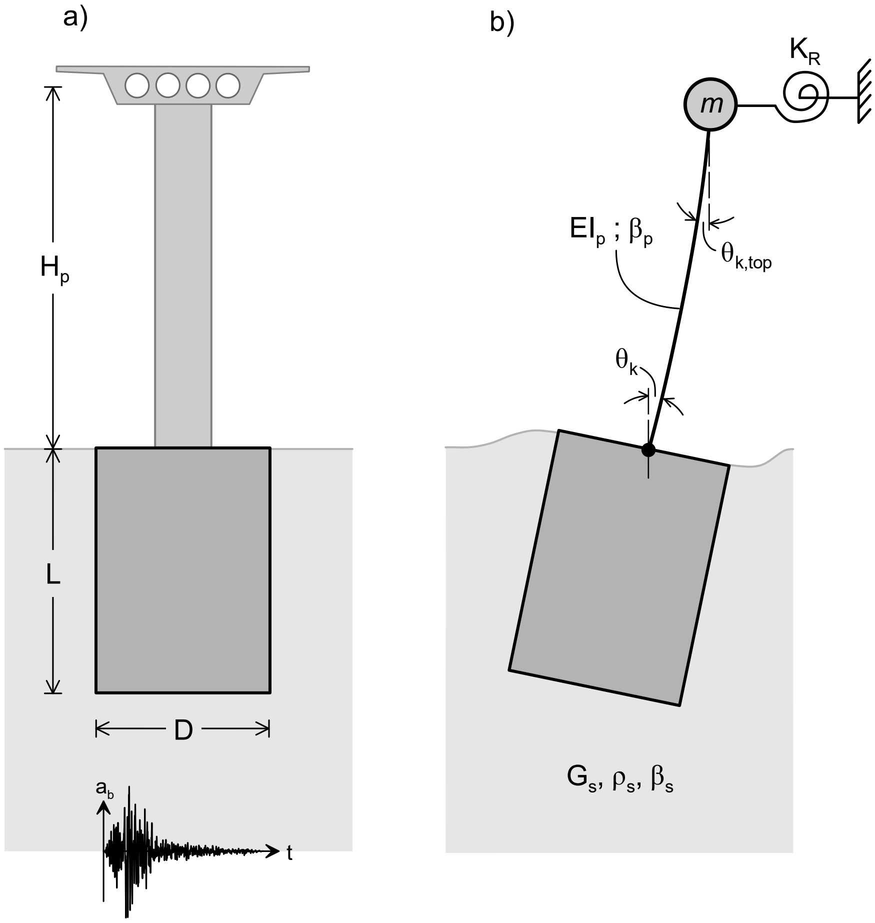

Figure 1: Problem under investigation.

2 ANALYTICAL SOLUTION FOR THE KINEMATIC BENDING MOMENT

The caisson, which consists of a rigid massive, embedded foundation can be modelled through a viscoelastic complex impedance matrix $\tilde{\boldsymbol{K}}$ which represents globally the stiffness and damping properties of the soil–foundation system. Focusing on the lateral response, i.e. neglecting the vertical impedance, the following matrix is obtained:

$$\tilde{\boldsymbol{K}} = \begin{bmatrix} \tilde{K}_H & \tilde{K}_{HM} \\ sym & \tilde{K}_M \end{bmatrix} \tag{1}$$

where, $\tilde{K}_H$, $\tilde{K}_M$ and $\tilde{K}_{HM}$ are the impedances referring respectively to the swaying, rocking and coupled swaying-rocking modes of vibration. For a given mode of vibration, the corresponding complex impedance is equal to:

$$\tilde{K} = \tilde{K}(\omega) + i\omega C(\omega) = K \cdot \tilde{k}(\omega) + i\omega\left[C_{rad}(\omega) + C_{hyst}(\omega)\right] \tag{2}$$

where, ω is the circular excitation frequency (= $2\pi f$), K is the static stiffness, $\tilde{k}(\omega)$ is the dynamic stiffness coefficient, $C_{rad}(\omega)$ is the radiation damping and $C_{hyst}(\omega)$ is the hysteretic amping. Very recently Conti and Di Laora [1] proposed simplified expressions for the estimation of impedance functions, based on rigorous elastodynamic FEM analysis.

The corresponding complex compliance matrix is equal to:

$$\tilde{\boldsymbol{C}} = \tilde{\boldsymbol{K}}^{-1} = \begin{bmatrix} \tilde{C}_U & \tilde{C}_{U\theta} \\ sym & \tilde{C}_\theta \end{bmatrix} \tag{3}$$

where, $\tilde{C}_U$, $\tilde{C}_\theta$ and $\tilde{C}_{\theta U}$ are the compliances referring respectively to the swaying, rocking and coupled swaying-rocking modes of vibration.

Under seismic excitation, the passage of seismic waves through the ground induces deformations in the soil mass. The caisson embedded in the soil, owing to its large flexural stiffness, is not able to follow the surrounding soil deformations with increasing excitation frequency, i.e. with decreasing soil wavelength. As a consequence, the soil-caisson interaction produces a motion modification respect to the free-field signal. Furthermore, due to their lower rotational stiffness as compared to pile foundations, caissons are more prone to experience a kinematically-induced rotational component of the motion, which does not exist in free-field conditions. Thus, in absence of the pier, the caisson would have a kinematic horizontal displacement u_{k0} and rotation θ_{k0} at its top.

On the other hand, the pier, owing to the rotational spring at the top, applies upon the caisson a (*kinematic*) moment M_k which causes a displacement and rotation, of apposite sign component to u_{k0} and θ_{k0}, equal to:

$$\Delta u_k = M_k \cdot \tilde{C}_{U\theta} \tag{4}$$

$$\Delta\theta_k = M_k \cdot \tilde{C}_\theta \tag{5}$$

The kinematic moment is proportional to the rotation of the pier top $\theta_{k,top}$ and the stiffness of the rotational spring K_R that represents the rotational constraint exerted by the pier-to-deck joint:

$$M_k = \theta_{k,top} \cdot K_R \tag{6}$$

It can be observed that the difference of the rotation between top and bottom of the pier is related to the moment M_K, which is constant along the pier, so:

$$\theta_k - \theta_{k,top} = \frac{M_K \cdot H_p}{EI_p} \tag{7}$$

where EI_p / H_p represents the rotational stiffness of the pier. Note that the net base rotation θ_k may be expressed as the kinematically – induced rotation θ_{k0} after subtracting the base rotation $\Delta\theta_k$ due to the restrain imposed to the pier by the rotational spring at its top. Hence, the consideration of Equations (5) and (6) yields:

$$\theta_k - \theta_{k,top} = \theta_{k0} - \Delta\theta_k - \theta_{k,top} = \theta_{k0} - M_K \cdot \tilde{C}_\theta - \frac{M_K}{K_R} \tag{8}$$

Upon combining Equations (7) and (8), it is possible to obtain the kinematically-induced bending moment as function of the rotational component of the Foundation Input Motion (FIM), the rotational caisson compliance, the stiffness of the rotational springs, the pier height and its flexural stiffness:

$$M_k = \frac{\theta_{k0}}{\tilde{C}_\theta + \frac{1}{K_R} + \frac{H_p}{EI_p}} \tag{9}$$

The rotational component of the FIM it can be obtained from the horizontal free-field displacement at the surface u_{ff0} and the so-called rotational kinematic interaction factor I_θ:

$$\theta_{k0} = \frac{I_\theta \cdot u_{ff0}}{L} \tag{10}$$

The kinematic interaction factor I_θ can be computed through the analytical solution proposed by Conti and Di Laora [1].

Figure 2 shows the theoretical results for a massive rigid cylindrical caisson with density ρ_f = 2.5 Mg/m^3 and diameter D = 5 m, supporting a pier with flexural stiffness EI_p = 120000 MN/m^2, embedded in a homogenous soil with density ρ_s = 1.8 Mg/m^3, Poisson's ratio ν_s = 0.4, damping ratio β_s = 0.05 and shear wave velocity V_s = 100 m/s. The soil is assumed to behave as a linear viscoelastic material, such that is characterized by a complex shear wave velocity $V_s^* = V_s(1+2i\beta_s)^{0.5}$ where i is the imaginary unit. From Figure 2a it can be observed that as the slenderness of the caisson increases, the maximum kinematic bending moment increases for a first range of slenderness and then starts to decrease. This behavior can be attributed to a dual effect: for a fixed soil wavelength, increasing the depth of the caisson (at constant diameter) may induce greater rotations due to the displacements profile acting on it; conversely, as the length increases, the rotational stiffness of the foundation increases. Therefore, it can be argued that for a constant diameter, with the increase in the caisson length, the beneficial effect resulting from the increase in the rotational stiffness prevails over the other, leading to a progressively lower maximum kinematic bending moment.

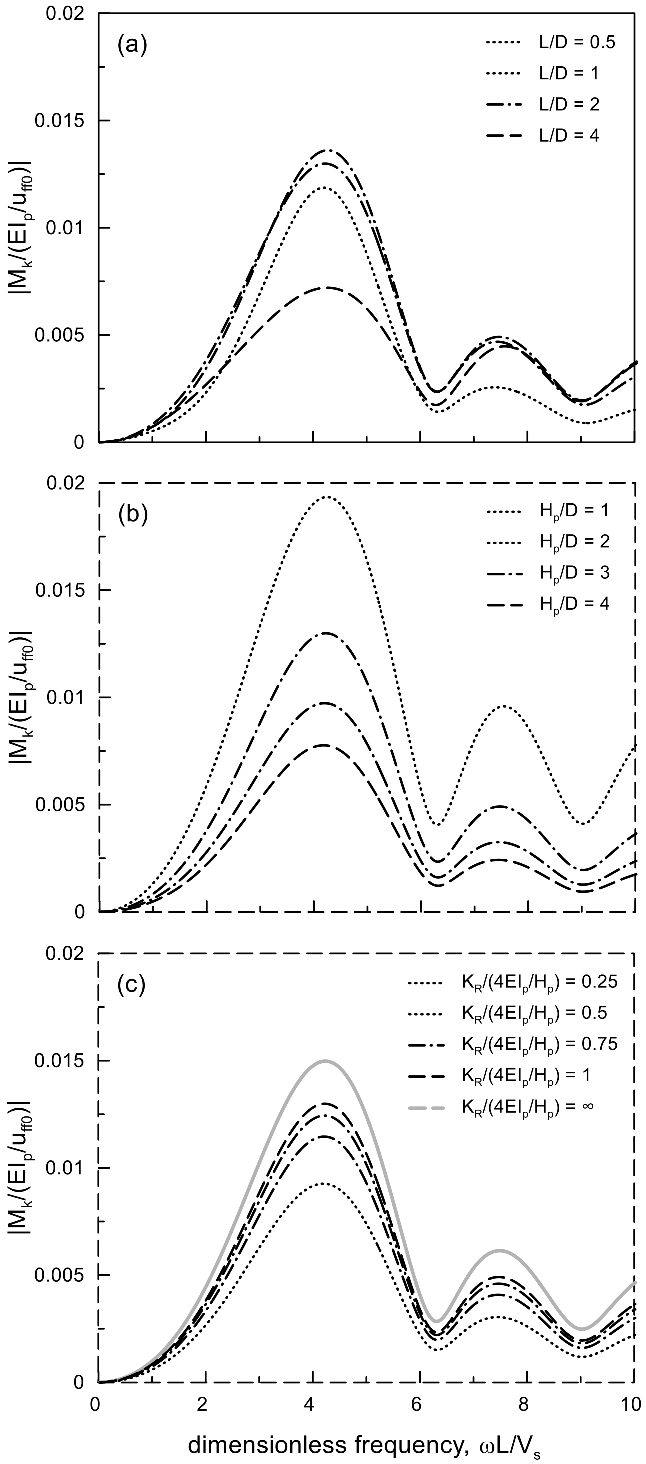

Figure 2: Kinematically-induced bending moment as function of dimensionless frequency for ρ_f = 2.5 Mg/m^3, ρ_s = 1.8 Mg/m^3, ν_s = 0.4; β_s = 0.05, D = 5 m, EI_p = 120000 MN/m^2 and V_s = 100 m/s; (a) H_p/D = 2 and $K_R/(4EI_p/H_p)$ = 1; (b) L/D = 2 and $K_R/(4EI_p/H_p)$ = 1; (c) H_p/D = 2 and L/D = 2.

Figure 2b shows that the maximum kinematic bending moment is inversely proportional to the pier height since if the pier height decreases its rotational stiffness increases.

Furthermore, as is shown in Figure 2c, the maximum kinematic bending moment increases as the rotational spring stiffness ratio $K_R/(4EI_p/H_p)$ increases, but the influence of this parameter decreases progressively, until it reaches an asymptotic value, this result is in agreement with the findings by Mucciacciaro et al. [2].

3 COMPARISON WITH THE INERTIAL-INDUCED BENDING MOMENT

The bending moment at pier bottom due to the inertial force F_{in} (= $m\ a_{top}$) associated to the mass m at pier top, is equal to (See Appendix A):

$$M_{in} = F_{in} \cdot H_p \cdot \left[1 - \frac{1}{2(1+\Omega)}\right] \tag{11}$$

where Ω is a stiffness dimensionless factor:

$$\Omega = \frac{EI_p}{H_p \cdot K_R} \tag{12}$$

The ratio between the kinematic and the inertial moment at pier bottom is as follows:

$$\frac{M_k}{M_{in}} = \frac{\theta_{k0}}{\tilde{C}_\theta + \dfrac{(1+\Omega)}{\Omega \cdot K_R}} \cdot \frac{2\cdot(1+\Omega)}{F_{in} \cdot H_p \left[2\cdot(1+\Omega) - 1\right]} \tag{13}$$

The inertial forces F_{in} is equal to the product between the mass m and the structural acceleration a_s, so it can be expressed as [3]:

$$F_{in} = m \cdot a_s = m \cdot F(\omega) \cdot a_{ff0} \cdot \left(I_u - \frac{H_p}{L} \cdot I_\theta\right) \tag{14}$$

where a_{ff0} is the free-field acceleration, I_u is the kinematic interaction factor associated to the translational displacement of the foundation and $F(\omega)$ is the transfer function of the pier-caisson system. Given that the free field acceleration a_{ff0} can be expressed as:

$$a_{ff0} = -\omega^2 \cdot u_{ff0} \tag{15}$$

Substituting equations 10, 15 and 14 into equation 13, the following expression is obtained:

$$\frac{M_k}{M_{in}} = -\frac{1}{\omega^2} \cdot \frac{2\cdot(1+\Omega)\cdot I_\theta}{L \cdot \left[\tilde{C}_\theta + \dfrac{(1+\Omega)}{\Omega \cdot K_R}\right] \cdot m \cdot F(\omega) \cdot \left(I_u - \dfrac{H_p}{L} \cdot I_\theta\right) \cdot H_p \left[2(1+\Omega) - 1\right]} \tag{16}$$

The transfer *F(ω)* function is equal to:

$$F(\omega) = 1 + \omega^2 \cdot H(\omega) \cdot m \tag{17}$$

Neglecting the caisson contribution on the inertial response of the system, the function *H(ω)* is equal to:

$$H(\omega) = \frac{1}{\tilde{k}_s - m \cdot \omega^2} = \frac{1}{k_s \cdot \left(1 + 2 \cdot i \cdot \beta_p\right) - m \cdot \omega^2} \tag{18}$$

where, $\tilde{k}_s$ is the translational dynamic stiffness of the pier, k_s is the translational static stiffness and β_p is the structural damping ratio. The static stiffness of the pier is equal to (See Appendix B):

$$k_s = \frac{12 \cdot EI_p}{H_p^3} - \frac{36 \cdot \left(EI_p\right)^2}{H_p^4} \cdot \left(K_R + \frac{4 \cdot EI_p}{H_p}\right)^{-1} \tag{19}$$

Taking into account the influence of the foundation in the inertial response, *H(ω)* can be written as:

$$H(\omega) = \frac{1}{\left(\dfrac{1}{\tilde{k}_s} + \dfrac{1}{\tilde{k}_c}\right)^{-1} - m \cdot \omega^2} \tag{20}$$

where $\tilde{k}_c$ is the translational dynamic stiffness of the caisson:

$$\tilde{k}_c = \left(\frac{1}{\tilde{K}_H - \tilde{K}_{HM}^2 \cdot \tilde{K}_M^{-1}}\right)^{-1} \tag{21}$$

Figure 3 shows the ratio between the kinematic and the inertial moment at pier bottom for the theoretical cases analyzed above. In particular, it is assumed that the mass of the superstructure m is equal to five times the mass of the caisson. It can be observed that for same “extreme” cases, the kinematically-induced bending moment may be much larger than the initial moment, although this effect is typically neglected in current practice. However, it should be noted that even for cases where the inertial effect is predominant, the ratio M_k/M_{in} is considerable, suggesting that the kinematic effect should always be taken into account.

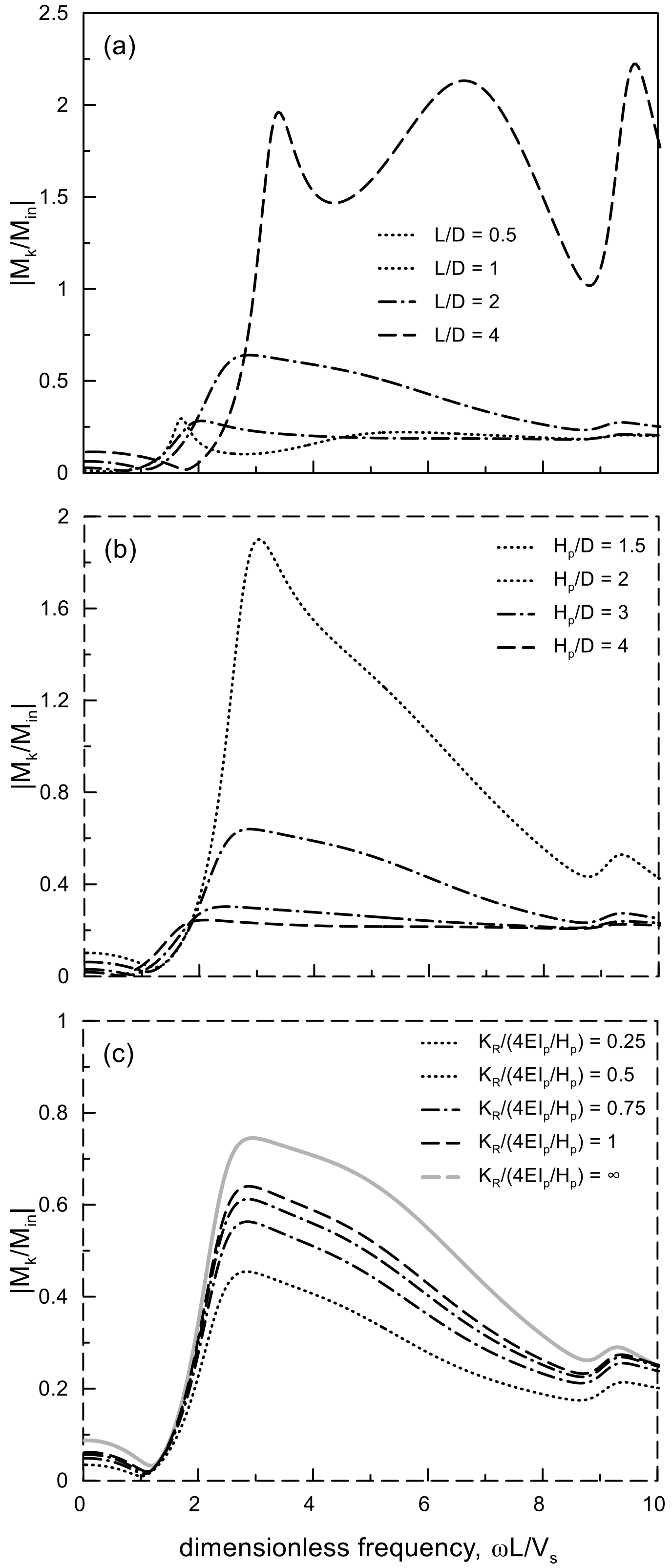

Figure 3: Ratio between the kinematically and inertially-induced bending moment as function of dimensionless frequency for ρ_f = 2.5 Mg/m^3, ρ_s = 1.8 Mg/m^3, v_s = 0.4; β_s = 0.05, D = 5 m, EI_p = 120000 MN/m^2 and V_s = 100 m/s; (a) H_p/D = 2 and $K_R/(4EI_p/H_p)$ = 1; (b) L/D = 2 and $K_R/(4EI_p/H_p)$ = 1; (c) H_p/D = 2 and L/D = 2.

4 CONCLUSIONS

In this work, an analytical solution is derived for assessing the kinematically-induced bending moment arising in bridge piers supported by caisson foundations. This phenomenon is due to the rotational constraint exerted by the pier-to-deck joint, which counteracts the rotational component of the Foundation Input Motion. The kinematically-induced component results from the modification of the free-field input motion caused by the presence of the caisson.

The proposed solution is based on the hypothesis of visco-elastic behavior of the soil.

Furthermore, after deriving a simple expression for assessing the inertial component of the bending moment arising at the pier, it has been shown that the kinematically-induced component may be significant, although it is typically neglected in current practice.

Further investigations should focus on validating the proposed model and examining the phase lag between the two components of the bending moment, thus understanding how they combine in time domain.

APPENDIX A - DERIVATION OF EQUATION 11

With reference to the conceptual scheme depicted in Figure A1, the bending moment at pier bottom M_{in}, is equal to:

$$M_{in} = F_{in} \cdot H_p - K_R \cdot \theta_{top} \tag{22}$$

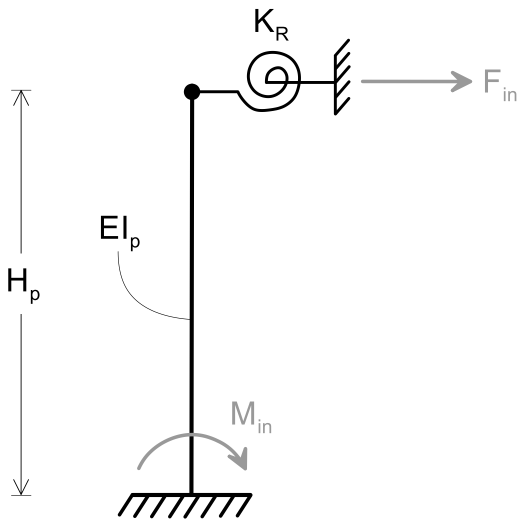

Figure A1: Conceptual model adopted for the derivation of the inertial bending moment at pier bottom.

The rotational spring at pier top applies a bending moment with opposite sign compared to the moment arising from the initial force acting at pier top, thus:

$$\theta_{top} = \frac{F_{in} \cdot H_p^2}{2 \cdot EI_p} - \frac{K_R \cdot \theta_{top} \cdot H_p}{EI_p} \tag{23}$$

therefore,

$$\theta_{top} \cdot \left(1 + \frac{K_R \cdot H_p}{EI_p}\right) = \frac{F_{in} \cdot H_p^2}{2 \cdot EI_p} \tag{24}$$

thus,

$$\theta_{top} = \frac{F_{in} \cdot H_p^2}{2 \cdot EI_p} \cdot \left(1 + \frac{K_R \cdot H_p}{EI_p}\right)^{-1} = \frac{F_{in} \cdot H_p^2}{2} \cdot \frac{1}{EI_p + K_R \cdot H_p} \tag{25}$$

Substituting equation 15 into equation 12, the inertial moment as function of the pier height, the pier flexural stiffness and the rotational spring stiffness is obtained:

$$M_{in} = F_{in} \cdot H_p - K_R \cdot \left(\frac{F_{in} \cdot H_p^2}{2} \cdot \frac{1}{EI_p + K_R \cdot H_p}\right) = F_{in} \cdot H_p \cdot \left[1 - \frac{1}{2\left(1 + \frac{EI_p}{H_p \cdot K_R}\right)}\right] \tag{26}$$

APPENDIX B - DERIVATION OF EQUATIONS 19 AND 21

With refence to the system in figure A1 the stiffness matrix is equal to:

$$\boldsymbol{K_s} = \begin{bmatrix} k_{s,H} & k_{s,HM} \\ sym & k_{s,M} \end{bmatrix} = \begin{bmatrix} \frac{12 \cdot EI_p}{H_p^3} & -\frac{6 \cdot \left(EI_p\right)^2}{H_p^4} \\ sym & K_R + \frac{4 \cdot EI_p}{H_p} \end{bmatrix} \tag{27}$$

The compliance matrix is:

$$\boldsymbol{C_s} = \begin{bmatrix} C_{s,u} & C_{s,u\theta} \\ sym & C_{s,\theta} \end{bmatrix} = \boldsymbol{K_s}^{-1} = \frac{1}{k_{s,H} \cdot k_{s,M} - k_{s,HM}^2} \begin{bmatrix} k_{s,M} & -k_{s,HM} \\ sym & k_{s,H} \end{bmatrix} \tag{28}$$

It follows that, the translational stiffness results to be:

$$k_s = \left(C_{s,u}\right)^{-1} = \left(\frac{k_{s,M}}{k_{s,H} \cdot k_{s,M} - k_{s,HM}^2}\right)^{-1} = \left(\frac{1}{k_{s,H} - k_{s,HM}^2 \cdot k_{s,M}^{-1}}\right)^{-1} \tag{29}$$

In the same way, the complex translational stiffness of the caisson at its top is equal to:

$$\tilde{k}_c = \left(\frac{1}{\tilde{K}_H - \tilde{K}_{HM}^2 \cdot \tilde{K}_M^{-1}}\right)^{-1} \tag{30}$$

REFERENCES

[1] R. Conti & R. Di Laora. Substructure method revisited for analyzing the dynamic interaction of structures with embedded massive foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, *148*(6), 04022029, 2022.

[2] R. Di Laora, Y. Grossi, L. de Sanctis & G. M. Viggiani. An analytical solution for the rotational component of the Foundation Input Motion induced by a pile group. *Soil Dynamics and Earthquake Engineering*, *97*, 424-438, 2017.

[3] M. Mucciacciaro, N. Gerolymos & S. Sica. Seismic response of caisson-supported bridge piers on viscoelastic soil. *Soil Dynamics and Earthquake Engineering*, 139, 106341, 2020.

[4] A. Zafeirakos, & N. Gerolymos. Towards a seismic capacity design of caisson foundations supporting bridge piers. *Soil Dynamics and Earthquake Engineering*, 67, 179-197, 2014.