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AN ONTOLOGY FOR NONLINEAR DYNAMICS

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Abstract. *Structural dynamics is one of the most important fields in engineering. In that context, most structures face the issue of nonlinearity, which is challenging to analyse because mathematics is based on linear foundations, and thus nonlinear methods are often lacking. Ontologies are a modular framework that detail the theory of what exists within a given field and how the entities within interconnect. Ontologies are useful for knowledge sharing and aiding automatic aggregation and analysis of information. To date, there is no ontology describing the domain of nonlinear dynamics. Therefore, this paper establishes a new ontology for the domain that provides data-based and analytical knowledge of the field. The ontology is designed with the goal of encompassing nonlinear system identification and exact solutions, but will naturally extend to fields such as structural health monitoring or structural dynamics broadly. In general, the ontology will help researchers to include non-linear effects in solving practical problems in structural dynamics, whilst also providing a knowledge base to help advance the theoretical basis of the field.*

Keywords: ontology, knowledge-base, database, nonlinear dynamics

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1 INTRODUCTION

Structural dynamics (or the theory of vibrations) is one of the most important fields of engineering. Understanding vibrations is vital to the design of lighter, greener and safer future structures. Almost all structures face the issue of nonlinearity, which is challenging to mathematically analyse because mathematics is largely built on linear foundations. Therefore, while methods of analysing linear systems have become well-established, general methods for nonlinear systems, as well as the collation of nonlinear research, are lacking.

Ontologies detail the theory of what exists within a field of study, providing a clear picture of different terms and how they interconnect [3]. They are a standard modular framework that encourages consistency in how data and information are described, both in natural language, and via logical definitions. This idea enables information originating from different sources (and described differently) to be more easily brought together and shared across different disciplines. In turn, they provide a powerful means to automatically gather and aggregate scientific information within a domain to advance understanding and development of analytical methods.

An ontology is a tree-like structure where each entity (node) is named, given a human-readable description and provided with any further details that aid its understanding. Any relationships between entities are also defined to ensure all future inferences are used in a consistent manner. An ontology is always incomplete (i.e. it will never represent all knowledge) but it is designed to be sufficient for the task for which it is made. It should also be extendable without having to remove or restructure existing entities, enabling additional entities to be added to help with new tasks. In addition, it should be possible to link it to other ontologies enabling a broader picture of how it fits in with other domains.

Ontologies have been used across a wide range of domains and applications, the most popular of which has been within biomedical research, where they have been useful for organising, annotating and analysing data from biological experiments. To collate these, the BioPortal provides a library of over 1000 ontologies, mostly within the biomedical domain, with the goal of sharing knowledge and data in an interoperable manner [8]. However, the BioPortal faces challenges with interconnecting ontologies of varying quality and completeness. To address this, the OBO Foundry, provide a library of biomedical ontologies that follow a set of principles to ensure all included ontologies are open and usable [4].

Ontologies on the topic of mathematics have also been published in the literature. For example, OntoMath [5, 6] is dedicated to describing mathematical knowledge for converting natural language mathematical text to a formal representation that can be used with computer algebra software. It is also used as a tool for teaching mathematics for automatic testing of knowledge and to provide recommended materials. In the context of dynamics, Tsialiamanis *et al.* [7] developed an ontology on structural health monitoring, to be used as a database of information on existing structures, and providing a framework around software that applies structural health monitoring methods. Antoniadou *et al.* [1] highlight the utility of developing an ontology for verification and validation in structural dynamics. Such an ontology would be an asset in determining which methods and models are best chosen for a given experiment, as well as being a helpful knowledge-base for verification and validation methods.

Butlin *et al.* [2] developed a mind-map model on the domain of nonlinear structural dynamics, capturing different perspectives across the field. The purpose of the model is to provide a tool for improving the dynamic performance of engineering structures and advancing the theory of the domain. The work of Butlin *et al.* is similar to an ontology in that it is a tree-like structure defining classes and their subclasses within the domain. However, it differs from an ontology in that it does not provide further definitions or descriptions of classes, nor does it provide axioms that restrict how inferences can be made.

To date, there is no ontology describing the domain of structural dynamics; this work aims to establish such an ontology, covering data-based and analytical knowledge in the field[1]. The ontology will be designed with the goal of encompassing nonlinear system identification and exact solutions, but will naturally extend to fields such as structural health monitoring or structural dynamics generally. To achieve the main goal, it will include mathematical expressions as objects to enable the automatic recovery of mathematical expressions from the web. In general, the ontology will help researchers to include nonlinear effects in solving practical problems in structural dynamics, whilst also providing a knowledge base to help advance the theoretical basis of the field. The ontology differs to the mindmap developed by Butlin *et al.* [2] by focussing initially on the study of equations and contexts relevant to nonlinear dynamics (without specifically relating to structural dynamics) and, as an ontology, it provides information on the connections between classes, detailing the restrictions on how instances within the domain can be used.

In future work, the ontology will be linked with computer algebra software to help with automatic analysis of dynamic equations. By describing an equation and its properties using the ontological entities, it will be simple to identify connections that group equations/systems with similar properties and assumptions, which can then be analysed together.

2 THE ONTOLOGY STRUCTURE

An ontology is a tree-structured method of knowledge representation where terms high-up in the tree are more general (e.g. *equation*) and terms further down are more specific (e.g. *partial differential equation*). Relationships between leafs of the tree are also be defined (e.g. *oscillation* has property *periodic*), enabling a detailed representation of the domain that captures important restrictions that any entities may have. This ontology therefore enables a description of dynamics models in an intuitive and explicit manner that is both human and computer readable. One of the key uses of any ontology is to assist with the representation of knowledge within a domain for the goal of acting as a communication medium between different people, therefore maximising knowledge sharing. The initial scope of the ontology begins with important concepts within dynamics, but is expected to evolve and expand as required for more specific problems within the domain.

To begin the top-level classes of the ontology are those considered the most important within dynamics. Fig. 1 shows all of the top-level classes currently included. These have been chosen as concepts typically required for an undergraduate course on dynamics. Each of these classes is further broken down in more specific subclasses.

Items within an ontology can be described by multiple terms, including highlighting the most common or preferred term. For example, the entity *DiracDeltaFunction* (under the superclass *Function*) is shown to have the alternative terms *Impulse Function* and *Unit Impulse*, with *Dirac Delta Function* being the preferred term. Providing alternative names helps reduce misunderstandings and ensure a shared meaning of terms. Terms are formally defined giving (where applicable) any mathematical definitions or preferred mathematical notations. These mathematical definitions are given in MathML format (the standard format understood by computer algebra applications). For example, Fig. 2 shows the natural language and mathematical definitions of *elastic energy*, as well as constraining that any instance must be of the data type

[1] Available online at `https://github.com/JosieMcCulloch/dynamics_ontology/`

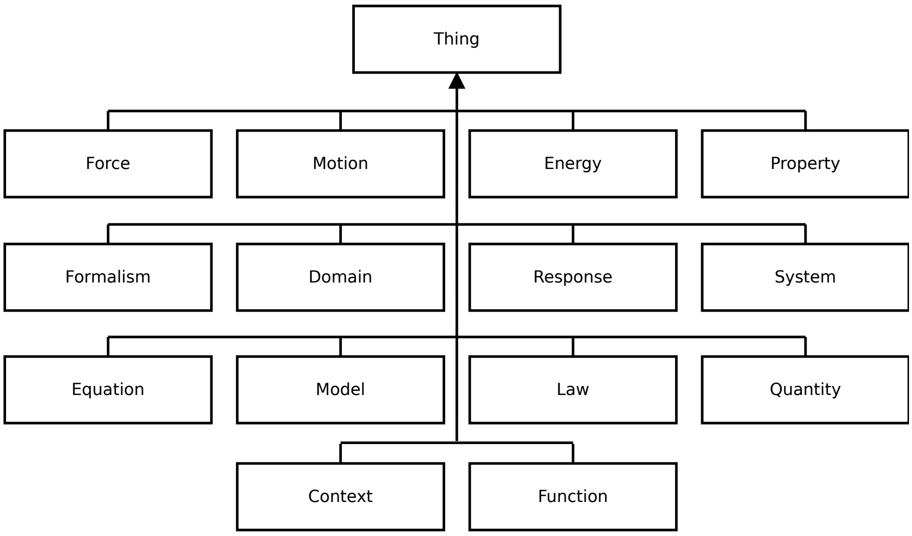

Figure 1: Nodes on the highest level of the ontology.

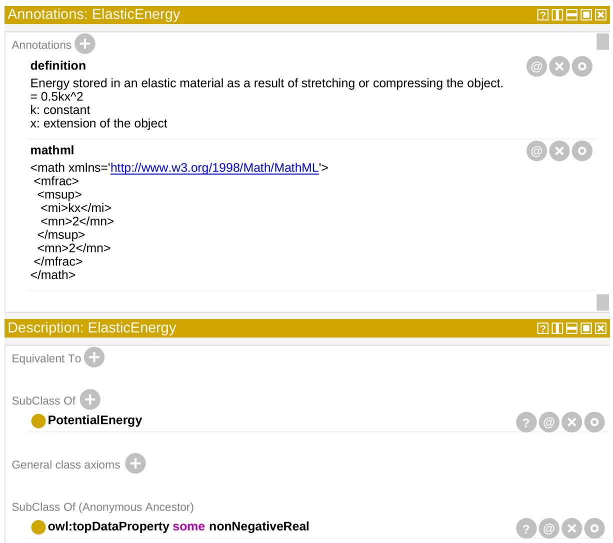

Figure 2: Annotations and descriptions applied to *ElasticEnergy*.

non-negative real.

As an ontology has a tree-like structure, most connections between two classes are an *is-a* relationship (i.e. a subclass is a specific type of its superclass). For example, Fig. 3 defines that a *constant* is a *quantity*. However, other types of connections are also included within ontologoies that further define and restrict how an instance of a class is to be used or understood. For example, Fig. 4 shows the class *conservation of linear momentum* is defined as a specific type of *law* that affects the *quantity* of *linear momentum*. The reverse connection *is affected by* is also defined to help make inferences easier with the ontology. Other details in the ontology include disjoint relationships (e.g. the property *passive* is disjoint with *forced*, ensuring that something cannot be accidentally labelled as both).

Any individual entity can be instantiated as an instance of a class. For example the *Fourier Transform* is assigned as an instance of the *TransformFunction* class (see Fig. 5), and the *Focker-Planck Equation* is assigned as an instance of a *PartialDifferentialEquation*.

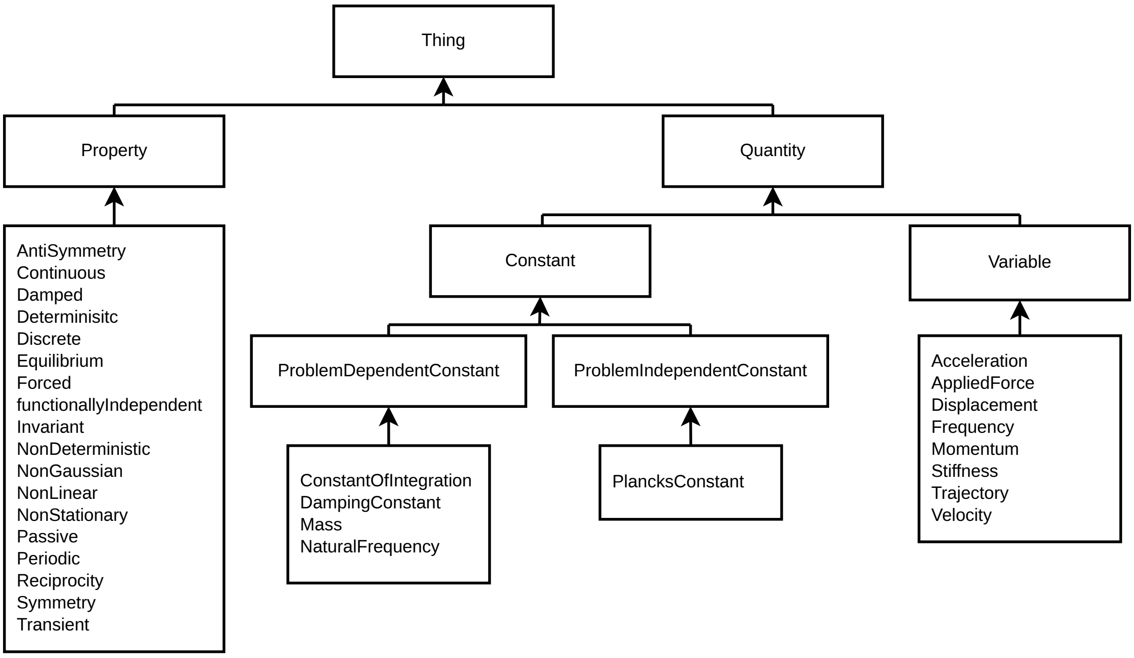

Figure 3: The *Property* and *Quantity* branches of the ontology in which each connection is an *is a* relationship.

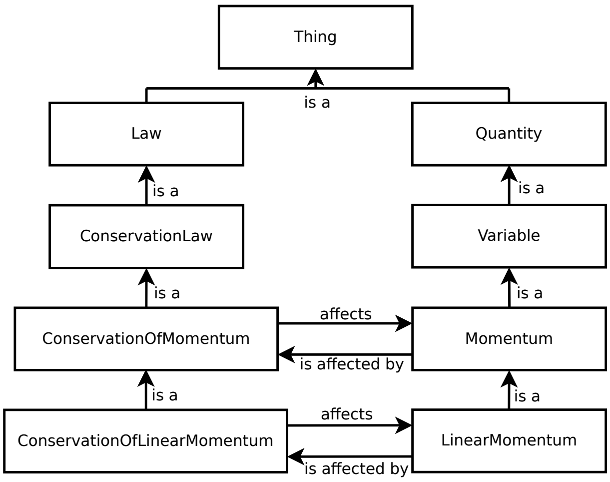

Figure 4: Connections applied to the law of linear momentum.

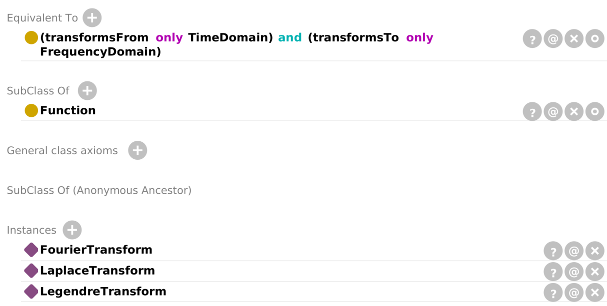

Figure 5: Descriptions applied to *TransformFunction*.

3 USES OF THE ONTOLOGY

One of the main uses of any ontology is as a tool to share knowledge of the domain. It enables those new to the domain to see which aspects are important, see their definitions, and understand how classes are connected. It can also help clear misunderstandings where multiple terms may be used to describe the same concept. In addition, by including definitions of equations and mathematical laws using the standard MathML syntax, the information on these classes is more readily available. The ontology can also act as a database of instances within the domain (e.g. listing specific solutions to equations as instances of the *solution* class of the ontology). This enables users to more easily find similar instances/data that may be useful to their research. As the ontology grows and more classes are added, this new information can be easily understood in the context of the existing structure of the ontology.

The ontology will be also be useful in solving problems in nonlinear dynamics. By including instances of equations and solutions that are described through the classes of the ontology (e.g. by detailing their domain, properties and quantities) it is possible to automate the grouping of similar dynamics problems and automate developing on them using computer algebra coupled with the ontology. By this, similarities not previously found between seemingly-different dynamics problems can be automatically searched and will be of importance in future research.

4 SUMMARY

This paper has introduced an ontology on the domain of nonlinear dynamics with the goal of knowledge sharing and linking with computer algebra systems. To begin, the ontology includes terms that are considered the most important across a broad range of dynamics problems. Over time these will be expanded to include classes that are specificaly useful for the goal of analysing approximate and exact solutiosn to equations within dynamics, as well as other research areas that may benefit (such as structural health monitoring).

Typically, an ontology is not isolated, but instead linked to other ontologies. This creates a bigger picture of how the domain relates to other domains, and reduces the need to reproduce class definitions that exist elsewhere. Several of the top-level classes lend well to sharing their definitions with other ontologies (e.g. *equation* and *function*) Therefore, in future such connections with existing ontologies will be carefully planned to ensure maximum usability of the new ontology.

The presented nonlinear dynamics ontology is available at `https://github.com/JosieMcCulloch/dynamics_ontology/` under the AGPL-3.0 license. As ontologies are intended to be useful for any research within their given domain, the authors encourage anyone to contribute to the presented ontology.

5 ACKNOWLEDGEMENTS

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