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AVERAGE RESPONSE OF SINGLE DEGREE OF FREEDOM SYSTEMS FOR ESTIMATING THE NONLINEAR RESPONSE OF STRUCTURES

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Abstract

The development of efficient proxies that allow approximating the nonlinear dynamic response of buildings is a paramount step in estimating seismic risk. Classical methodologies have been oriented to analyse the causal relationship between intensity measures (IM) and engineering demand parameters (EDP) extracted from nonlinear models. In this research, it has been proposed an innovative procedure for estimating this nonlinear behaviour by using the average response of a set of single-degree-of-freedom (SDoF) systems. This approach allows capturing the time-history response of a complex structure with less computational effort though high efficiency. In order to prove this, it has been estimated the maximum global drift ratio, MGDR, of more than one-thousand multi-degree of freedom systems representing Reinforced Concrete structures, whose number of stories vary from 3 to 13, by using nonlinear dynamic analysis, NLDA. Then, it has been calculated the MGDR by using both the time history response of a unique SDoF, with the fundamental period of the structure, and the average response of several oscillators, as proposed in this research. After employing cloud analysis, it has been observed that the MGDR, calculated using NLDA, is better approximated by using the average response of several SDoFs than a single SDoF. Similar conclusions can be drawn if analysing the nonlinear dynamic response of more than one-thousand steel frame structures, whose number of stories vary from 3 to 20.

Keywords: Nonlinear dynamic response, seismic risk, single degree of freedom systems, cloud analysis.

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1 INTRODUCTION

The main purpose of seismic risk assessment is to know the probability of occurrence of expected damages in civil infrastructures. Proper assessment and design methodologies require knowledge of how seismic actions, in terms of intensity measures (IMs), affect structural response. Variables representing structural response are commonly known as engineering demand parameters (EDPs). Several IMs have been proposed as proxies to predict the seismic performance of buildings. In this respect, a relevant milestone has been the response spectra theory; see for instance [1-4]. Based on this, *IM*s considering the damped response of single-degree-of-freedom systems (SDoF) were introduced. This knowledge, combined with the increasing capacity of computers and the collection of strong motion databases, have allowed to perform probabilistic calculations which had previously been unthinkable. In this way, the statistical analysis of several spectrum-based IMs and EDPs (obtained through advanced nonlinear models) has been recently performed [5-6]. The most efficient IMs are generally employed for deriving fragility functions.

In this research, it has been proposed an innovative approach for estimating the dynamic response of structures by using proxies calculated as the averaged time-history response of a set of single-degree-of-freedom (SDoF) systems. This approach allows capturing the time-history response of a complex structure with less computational effort though high efficiency. Specifically, to account for plastic damage and the participation of higher modes, the dynamic response has been approximated by averaging the time-history evolution of a set of SDoFs around the fundamental period of the simulated structures. This period has been estimated through classical relationships in terms of power functions, which depend on the height of the structure, see for instance [7]. This approach allows more efficient estimations of the nonlinear dynamic response of buildings. In order to prove this, it has been estimated the maximum global drift ratio, MGDR, of more than one-thousand multi-degree-of-freedom, MDoF, RC models, whose number of stories vary from 3 to 13, by using nonlinear dynamic analysis, NLDA, (Further details can be found in Vargas-Alzate et al. 2022). Then, it has been calculated the MGDR by using both the time history response of a unique SDoF, with the fundamental period of the structure, and the average response of several oscillators, as proposed in this research. It has been observed that the MGDR, calculated using NLDA, is better approximated by using the average response of several SDoFs than a single SDoF. Similar conclusions can be drawn if analysing the nonlinear dynamic response of more than one-thousand steel frame structures, whose number of stories vary from 3 to 20, and are subjected to a large set of ground motion records.

2 PROBABILISTIC GENERATION OF STRUCTURAL MODELS

Uncertainties govern the estimation of seismic risk. In terms of the structural characterization for large population of buildings, the acting loads and mechanical properties of the materials are the most important ones. In addition, information regarding the geometrical distribution of buildings is also required. In the framework of the KaIROS project (https://cordis.europa.eu/project/id/799553), a set of computer programs to generate structural models considering such uncertainties have been developed. This software has been employed in this article to perform the calculations. Specifically, two sets of probabilistic numerical models representing the behavior of RC and steel moment frame structures, SMF, are used as a test bed. A more detailed discussion of the generation of these models is provided below.

2.1 Reinforced concrete structures

Gravity loads and mechanical properties.

As commented above, to properly analyze the probabilistic seismic behavior of a group of buildings, it is necessary to consider the aleatory uncertainties regarding input variables. Concerning gravity loads and mechanical features of the materials, the following properties have been considered as random variables: live loads, *LL*, permanent loads, *DL*, compressive strength of concrete, *fc*, tensile strength of steel, *fy*, elastic modulus of concrete, *Ec*, and elastic modulus of steel, *Es*. A continuous Gaussian distribution has been assumed for these variables. Mean values, μ, standard deviations, σ, and coefficient of variation, c.o.v. of each variable are shown in Table 1. It is worth mentioning that properties assigned to LL correspond to a fraction (33%) of the design value (Eurocode 8).

	RC frames		
	μ (kPa)	σ (kPa)	c.o.v
LL	1	0.1	0.1
DL	4.5	0.18	0.04
fc	2.5E4	2.5E3	0.1
fy	4.6E6	2.3E5	0.05
Ec	2.3E7	1.84E6	0.08
Es	2.18E8	1.09E7	0.05

Table 1 Mean values, μ, standard deviations, σ, and coefficient of variation, c.o.v. of the assumed random variables

Geometrical properties

Concerning geometrical properties, the following random variables are considered: number of stories, N_{st}, number of spans, N_{sp}, story height, H_{st}, and span length, S_l. N_{st} and N_{sp} follow a uniform discrete distribution in the interval (3, 13) and (3, 6), respectively; H_{st} and S_l are distributed uniformly in the interval (2.8, 3.2) m and (4, 6) m, respectively.

In order to assign the cross-section dimensions of the first story columns, the following equation has been used:

$$W_c \text{ or } D_c = c_1 * \ln(N_{st}) + c_2 * S_l + c_3 * \Phi_{1,0} + c_4 \qquad \text{Eq. 1}$$

where c_i are coefficients that may be adjusted depending on the data distribution of the analyzed area. For this study, $c_1 = 0.4$, $c_2 = 0.05$, $c_3 = 0.01$ and $c_4 =$-0.35; $\Phi_{1,0}$ represents the standard normal distribution. Note that columns are not necessarily square, that is, one random sample is generated for the width, W_c, and one for the depth, D_c, of the element (Eq. 1). For upper stories, the size of columns decreases systematically by 5 cm every three stories. Values generated are rounded to the nearest multiple of 5 cm to be consistent with real dimensions of RC elements.

The width of beams, W_b, depends on the number of stories of the building model and on the span length, and has been calculated using the following equation:

$$W_b = b_1 * N_{st} + b_2 * S_l + b_3 \qquad \text{Eq. 2}$$

where b_i are coefficients that depend on the characteristics of the study zone. For the hypothetical case study, $b_1 = 0.01$, $b_2 = 0.02$ and $b_3 = 0.19$. The depth of the beams, D_b, is obtained using the following equation:

$$D_b = g_1 * N_{st} + g_2 * S_l + g_3 \quad \text{Eq. 3}$$

where $g_1 = 0.01$, $g_2 = 0.05$ and $g_3 = 0.17$. Notice that no random term is considered for beams.

As pointed out above, in the framework of the KaIROS project, a software package has been developed for probabilistic seismic analyses by using as structural software the Ruaumoko program [8]. This allows to generate probabilistic MDoF systems representing the behavior of RC frame buildings; this module has been easily adapted to simulate the seismic response of other structural types located in other seismic environments (Pinzón et al, 2020). Using this algorithm, 1160 structural models have been generated (this number is related to the availability of records within the ground motion database, as will be shown later).

The modified Takeda hysteresis law has been used to represent the in-cycle behavior of the structural elements (beams and columns). This law allows considering strength degradation due to large deformations and excessive number of inelastic cycles. To do so, a strength degradation function, which depends on the ductility reached by the structural elements as well as a degrading coefficient associated to the number of inelastic cycles has been considered in the hysteretic model [8]. For the stiffness degradation, the degrading factor has been fixed to 0.4. The yielding surfaces are defined by the bending moment-axial load interaction diagram for columns and bending moment-curvature for beams. P-Delta effects and the large-displacement theory have been considered in the numerical model, as the structures analysed may experience deformations beyond the elastic range, sometimes even exceeding their capacity to withstand gravity loads.

Coefficients considered in Eq. 1 to Eq. 3, as well as the mechanical property values (Table 1), have been selected to approximate the expected behavior of RC structures located in earthquake-prone regions. In order to validate the generated models, their fundamental periods have been compared with those computed by means of the classical rough approximation $T_1 = N_{st}/10$. Figure 1 (Left) shows the evolution of T_1 as a function of the number of stories for the 1160 building models; T_1 values agree with those expected for the building typology analyzed. Figure 1 (Right) shows a sketch of one hundred building models generated according to the conditions explained above.

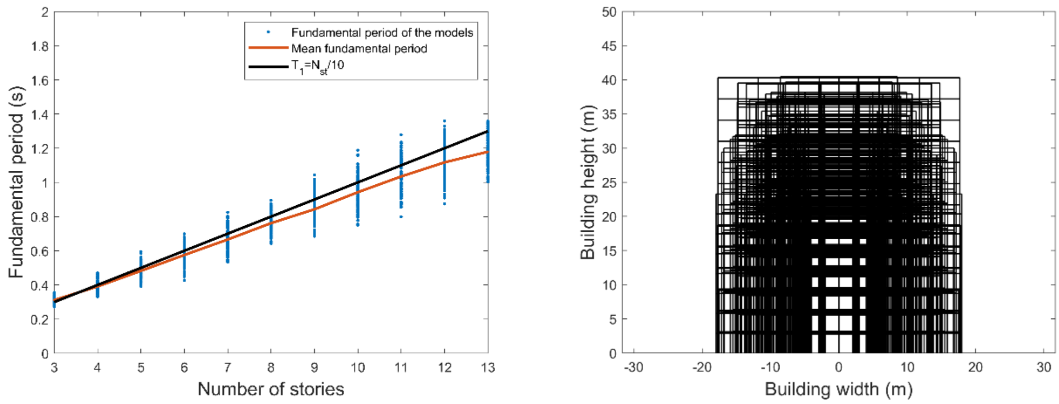

Figure 1 a) Variation of the fundamental period as a function of the number of storeys; b) 100 building models generated with KaIROS software.

2.2 Steel structures

Gravity loads and mechanical properties.

Mean values for live, LL, and permanent, DL, loads have been defined according to the NTC-RCDF (2017). These variables are assumed Gaussian with mean, μ, standard deviation, σ, and coefficient of variation (σ/μ) values shown in Table 1. Because of its large scattering, a higher coefficient of variation has been assigned to LL than to DL. However, only a fraction (30%) of the total design load has been considered for LL.

	SMF frames		
	μ (kPa)	σ (kPa)	σ/μ
LL	1	0.1	0.10
DL	5	0.2	0.04
fy	4.2e5	2.1e4	0.05
Es	2.1e8	0.84e7	0.04

Table 2 Statistical parameters considering in the generation of the probabilistic models

Because of the high-quality control related to the steel production, low variability is expected in the mechanical properties of this material. Anyhow, it has been preferred to take it into account within the probabilistic model. Accordingly, the tensile strength, *fy*, and the elastic modulus, *Es*, of the steel have been considered as random variables. Since these variables are highly correlated, a coefficient of correlation equal to 0.8 has been considered when sampling them; continuous Gaussian distributions have been assumed to represent these variables. Nonlinear behaviour is allowed at the end of beams and columns by considering a bilinear hysteresis rule. The strength and ductility parameters have been calculated according to the modified Ibarra-Medina-Krawinkler Model [9-10].

Geometrical properties

It has been generated 1200 structural steel models whose number of stories, N_{st}, number of spans, N_{sp}, story height, H_{st}, and span length, S_l, were considered as random variables. N_{st} and N_{sp} are uniform discrete distributions within the intervals (3, 20) and (3, 6), respectively. H_{st} and S_l are uniformly distributed in the interval (3.3, 3.7) and (5, 7) m, respectively.

The structural elements presented in Table 3 have been adopted to assign the cross-sectional properties of the structural elements of the first story depending on N_{st}. For upper stories, the column and beam types reduce systematically each 3 stories to the next section with lower properties according to Table 3. Hence, 1200 structural models have been generated. This number depends on the ground motion records available in the database, as shown below. Figure 2 (left) shows one hundred of the generated models.

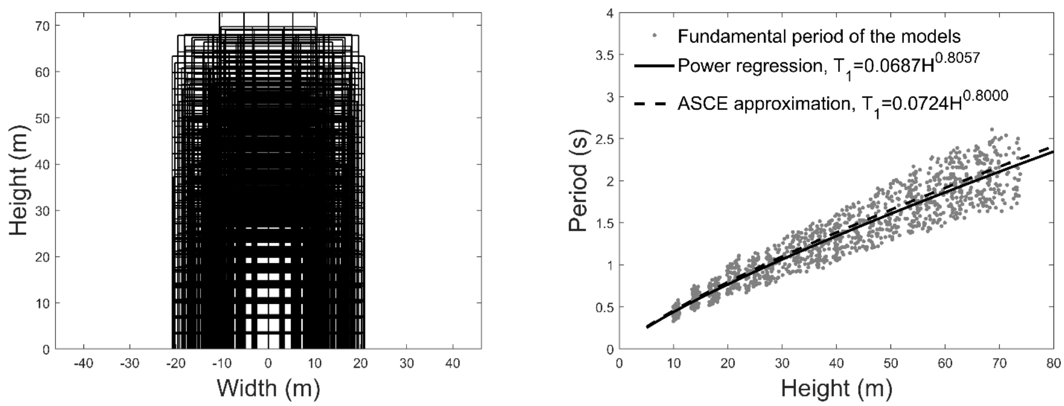

Figure 2 Left: One-hundred structural models. Right: Fundamental period vs height of the models

The structural sections presented in Table 3, and the mechanical property values shown in Table 3, have been adjusted so that the generated models approximate the dynamic properties of SMF structures located in earthquake-prone areas. This can be verified by comparing the fundamental period of these models with the following approximation provided by ASCE 7-16 (2017):

$$T_1 = 0.0724H^{0.8} \quad \text{Eq. 4}$$

where T_1 represents the fundamental period and H the height of the building in meters. Figure 2 (Right) shows the evolution of T_1 as a function of H; T_1 values observed in the generated models are in agreement with those expected for the typology analyzed.

N_{st}	Column	Beam	N_{st}	Column	Beam
3	W18x119	W14x68	**12**	W30x173	W18x97
4	W18x119	W14x74	**13**	W30X191	W18x106
5	W27x161	W14x74	**14**	W30X191	W18x106
6	W27x161	W14x74	**15**	W30X191	W18x106
7	W27x161	W16x89	**16**	W30X191	W18x119
8	W27x161	W16x89	**17**	W33X201	W18x119
9	W30x173	W16x89	**18**	W33X201	W18x119
10	W30x173	W18x97	**19**	W33X201	W21x132
11	W30x173	W18x97	**20**	W33X201	W21x132

Table 3 Steel W sections assigned to the first-floor elements

3 SEISMIC HAZARD CHARACTERIZATION

Seismic hazard is another important variable to properly estimate seismic damage and risk. Actually, the main source of uncertainty when assessing the seismic response of civil structures is the variability of ground motions. The randomness in both seismic actions and, consequently, in the structural response, increases alongside with the severity of the earthquake. Nowadays there are excellent accelerometric databases at global, regional and country scale that have allowed to quantify uncertainties in seismic hazard. For instance, the *Instituto Nazionale di Geofisica e Vulcanologia*, INGV, provides the Engineering Strong Motion Database (ESM) and includes a link to the most important accelerometric databases in the world. This database has been employed to characterize seismic hazard in RC structures. For the steel structures, a large database of 1277 records (2554 accelerograms from two horizontal components, North-South and East-West) of the Mexico City area from years 1960 to 2014 has been used in this study. All records have a Peak Ground Acceleration (PGA) greater than 10 cm/s^2, and moment magnitude varying in a range from 3 to 8.2. These seismic actions characterize the hazard of the city, which is dominated by the effects of the soil amplification, leading to increased intensity values at specific periods as well as long-duration acceleration records.

There are several methodologies to properly select ground motion records from a database that, for instance, are consistent with a specific site-dependent spectral shape. For the purpose of this study, the most important requirement is to have enough ground motion records which are scaled (if necessary) to analyze the generated structural models at different performance levels. Nonetheless, it is also important to avoid excessive scaling of the records so that bias introduced will be negligible. According to this, the main steps followed to pick and set up two suites (one for RC and one for steel structures) of accelerograms are described in the following:

STEP 1. Choose the strong motion database
STEP 2. Identify the *IM* for scaling the ground motion record set
STEP 3. Define suitable intensity bands for the selected *IM*
STEP 4. Compute the selected *IM* for each ground motion record
STEP 5 Sort the records in descending order as a function of the calculated *IM* values
STEP 6. If needed, the earthquake record with the highest *IM* value is scaled so that it falls into the highest intensity band; if this record naturally fulfills the interval condition, it is not scaled; this step is repeated until having enough records belonging to this interval
STEP 7. STEP 6 is repeated for each interval considered.

It seems reasonable to select an *IM* highly correlated with the structural response (STEP 2) so that, by increasing this variable, the structural response is expected to do so as well. In this respect, several researchers have proven that it is more efficient to use an *IM* based on the geometric mean of spectral acceleration values around the fundamental period, Sa_{avg}, than only using the spectral acceleration value associated to this period, $Sa(T_1)$. Analogous to this concept, the arithmetic mean of spectral acceleration values around the fundamental period, *AvSa*, has been considered herein as *IM* to select and scale ground motion records. It has been preferred to use the arithmetic mean than the geometric one since this *IM* tends to be more correlated to the *MIDR*, as will be shown later.

3.1 Seismic hazard for reinforced concrete structures

Because of the probabilistic approach adopted in this study, there is not a single structural model but a group of them which, in addition, have highly variable fundamental periods. Therefore, the period range for averaging the spectral ordinates of the IM is established from the dynamic properties of the entire population of buildings. In this way, for RC structures, groups of earthquakes are selected whose mean spectral acceleration, in the interval (0.2–1.6) s, lies in a band limited by two intensity levels. These limits correspond to 0.7 times the minimum and 1.12 times the maximum period observed in Figure 1; after several runs, they are the ones minimizing the dispersion in most of the IM-EDP clouds analyzed. Note that these limits tend to be broader when analyzing structures sharing similar number of stories. The intensity levels defining the upper and lower limits of each band range from 0.035 to 0.7 g at intervals of 0.035 g (i.e., 20 bands are defined). The highest limit has been selected so that approximately 10% of the models exceed their capacity to withstand gravity loads.

By means of the selection and scaling process described above, the horizontal component related to the east-west direction of 1160 earthquake records (58 records per band) of the ESM database have been obtained (the entire database contains 1163 ground motion records). Figure 3 (Right) shows the response spectra of the selected 1160 records.

3.2 Seismic hazard for steel structures

Because of the probabilistic nature of the generated models, there is a set of fundamental periods ranging from 0.4 to 2.6 sec (see Figure 2, right). This may complicate the definition of the interval for averaging spectral acceleration values. To address this issue, this period range is defined from the fundamental periods of the entire population of buildings. Hence, the ground motions are selected given that the arithmetic mean of spectral acceleration values is calculated in the interval (0.23–2.92) sec. The intensity levels that limit the intensity bands are 0.05 to 1 g, every 0.05 g. As the number of ground motion records available within the analysed database is 1277, the horizontal components related to the east-west direction of 1200 ground motion records (60 records per interval) are scaled to meet the criteria explained above. Figure 3 (Right) shows the response spectra of the scaled ground motion records. It can be seen the large dispersion in terms of the frequency content of this set of records.

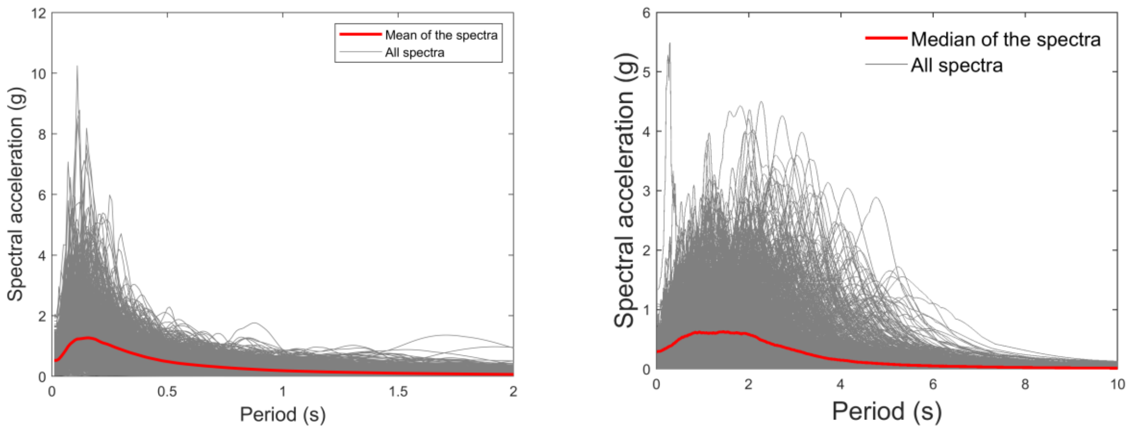

Figure 3 Left: Response spectra of the 1160 selected and scaled accelerograms. Right: Response spectra of the 1200 selected and scaled ground motion records.

4 DESCRIPTION OF THE SURROGATE MODEL

4.1 Surrogate model based on the combined response of single degree of freedom systems.

To analyse the expected damage of the generated buildings, it is necessary to consider several sources of uncertainty. However, the more complex the structural model, the more computation time is required to extract probabilistic information from it. Simplifying the structural model reduces the computational effort, allowing large set of building models to be analyzed in a fraction of time. In this sense, one of the most simplified representations of a building to perform dynamic calculations is the SDoF system. It allows calculating time-history responses in an easy way, given a determined fundamental period and damping. However, the response of a unique SDoF neither considers the loss of stiffness due to plastic damage nor the participation of higher modes. In the development of advanced IMs for estimating EDPs, these two shortcomings have been addressed by averaging spectral quantities around the fundamental period of the building.

In this research is proposed to approximate the dynamic response of the buildings by averaging the time-history response of a set of SDoF systems in the interval (0.1*T*, 1.8*T*); *T* represents the fundamental period of the analysed building in the n direction. The dynamic response associated with each oscillator has been estimated by means of the dynamic equilibrium equation for SDoF systems:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}_g(t)$$

Eq. **¡Error! No hay texto con el estilo especificado en el documento.**-1

where $\ddot{u}(t)$, $\dot{u}(t)$ and $u(t)$ are the spectral acceleration, velocity and displacement time history responses of the SDoF, respectively; $\ddot{u}_g(t)$ is the acceleration ground motion; m, c, and k represent the mass, damping, and stiffness of the system, respectively. Thus, the spectral time history response of a building is estimated as follows:

$$\bar{\ddot{u}}(t) = \frac{1}{p}\sum_{i=1}^{p} \ddot{u}_{(i)}(t, T_i)$$

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$$\bar{\dot{u}}(t) = \frac{1}{p}\sum_{i=1}^{p} \dot{u}_{(i)}(t, T_i)$$

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$$\bar{u}(t) = \frac{1}{p}\sum_{i=1}^{p} u_{(i)}(t, T_i)$$

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where $\bar{\ddot{u}}_n(t)$, $\bar{\dot{u}}_n(t)$, and $\bar{u}_n(t)$ are the average response of p SDoFs; $\ddot{u}_{(i)}(t, T_i)$, $\dot{u}_{,i)}(t, T_i)$ and $u_{(i)}(t, T_i)$ are the responses of the SDoFs with periods T_i, within the interval (0.1T, 1.8T).

4.2 Maximum global drift ratio as EDP

Several EDPs can be extracted from the averaged time-history responses described in the previous section. For example, the maximum global drift ratio of a structure, MGDR, which is an EDP widely used in seismic risk estimations employing the capacity spectrum method, can be estimated according to the following equation:

$$MGDR = PF_1 \frac{max(\bar{u}(t))}{\mathrm{H}}$$

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where PF_1 is the load participation factor; H is the height of the building. It is worth mentioning that PF_1 vales have been obtained from the MDoF models.

4.3 Results

By employing nonlinear dynamic analysis, the time-history responses of the MDoFs models have been calculated. Then, the MGDR for both cases (RC and SMR) have been calculated. Results in which the maximum displacement at the roof divided by the height of the structure is greater than 0.03 have been excluded from the statistical analysis. Figure 4 shows histograms of the MGDR for both type of structures. It can be seen that the MGDR values for SMF structures are the ones exhibiting the highest dispersion. Finally, it has been analyzed the bivariate distribution between the MGDR estimated via NLDA and by considering two types of proxies: i) The response of a unique oscillator whose fundamental period corresponds with the one of the MDoF model; ii) the approach proposed in this research (See Eq. 4.5).

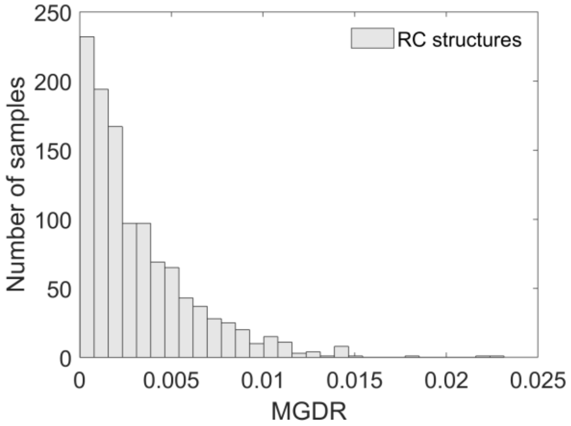

Figure 4 MGDR values for both set of structural types

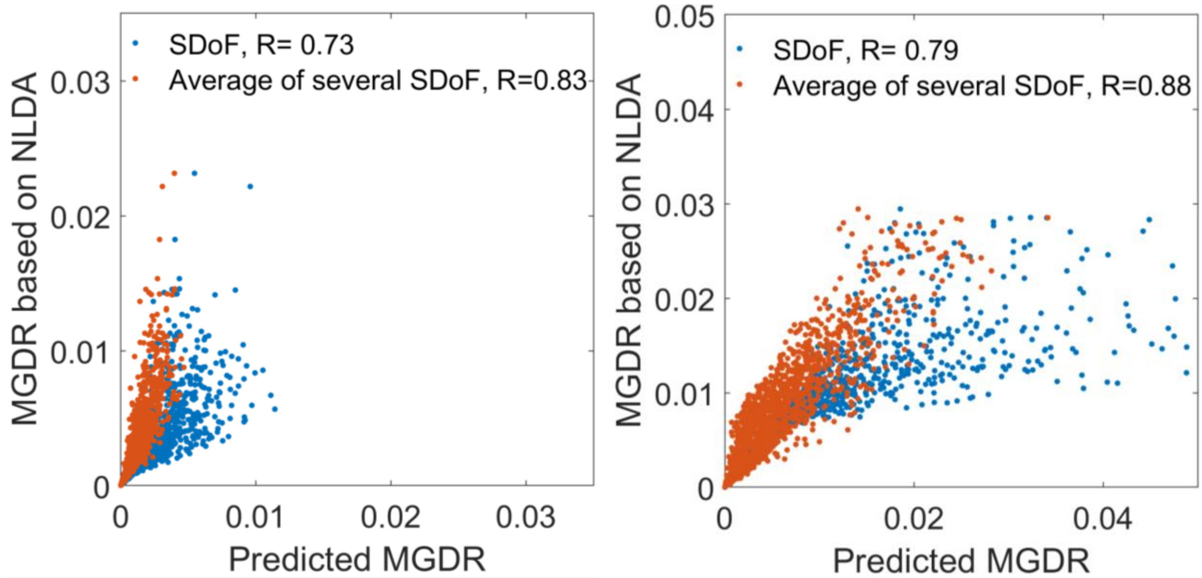

Figure 5 Estimations of the MGDR for RC and SMF structures

5 CONCLUSIONS

In this research, it has been developed simplified proxies for predicting the nonlinear response of MDoF systems that represent the behavior of buildings. Particularly, it has been analyzed the nonlinear response of RC and SRM structures by considering two different seismological environments. These proxies are obtained by considering the average response in time of several oscillators whose periods are close to the fundamental period of the simulated MDoF. In order to analyse the efficiency of the proxies, it has been estimated the maximum global drift ratio, MGDR, of more than one-thousand MDoF systems representing Reinforced Concrete structures, whose number of stories vary from 3 to 13, by using nonlinear dynamic analysis, NLDA. Then, it has been calculated the MGDR by using both the time history response of a unique SDoF, with the fundamental period of the structure, and the average response of several oscillators, as proposed in this research. After analyzing the bivariate distributions between the results stemming from NLDA and the simulated proxies, it has been observed that the MGDR, is better approximated by using the average response of several SDoFs than a single SDoF. Similar conclusions can be drawn if analyzing the non-linear dynamic response of more than one-thousand steel frame structures, whose number of stories vary from 3 to 20.

The high efficiency exhibited by the simplified proxies proposed herein has important practical implications for seismic risk assessment and structural design. Since these IMs exhibit strong correlations with MGDR, they provide a more reliable basis for predicting structural performance under seismic loads. This efficiency is especially beneficial when designing structures or developing fragility functions, as it reduces dispersion in IM-EDP relationships, leading to more accurate estimations of damage probabilities.

ACKNOWLEDGMENTS

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