Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2025) 1059-1069

ECCOMAS
Proceedia

COMPDYN 2025
10th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15-18 June 2025

A NOVEL HETEROGENEOUS LIMIT ANALYSIS TOOL WITH STEREOTOMY FOR THE ASSESSMENT OF MASONRY CURVED STRUCTURES

Natalia Pingaro[1], Mohammad Pourfouladi[1], and Gabriele Milani[1]

[1] Politecnico di Milano
Piazza Leonardo da Vinci, 32, 20133, Milano
e-mail: natalia.pingaro@polimi.it, mohammad.pourfouladi@polimi.it, gabriele.milani@polimi.it

Abstract

Recent advances in computational tools and experimental validation are driving innovations in the preservation and analysis of double curvature masonry structures. With such a need in mind, this study presents an integrated workflow combining parametric modeling (PoliBrick plugin) and Finite Element Limit Analysis (FELA), focusing on failure mechanism and collapse multiplier triggered in curved masonry structures subjected to vertical loads. The PoliBrick plugin, developed for Rhino software by two of the authors, is used to parametrize and generate three-dimensional masonry assemblages of bricks where stereotomy is accounted for, which are then integrated in finite element limit analysis. Limit Analysis is dealt with the Upper Bound (UB) theorem and by means of a heterogeneous approach, where blocks are assumed infinitely resistant, and joints are reduced to interfaces with rigid-perfectly plastic behavior. While current confidence in vertical load resistance for modern buildings does not directly apply to historic masonry structures, limit analysis provides a practical, rapid estimation of collapse loads. To validate it, a benchmark study is presented, by analyzing a masonry arch for which previously numerical and experimental results are present in the literature. Results obtained show the efficiency and robustness of the proposed method in predicting the ultimate load-bearing capacity of curved masonry structures, by highlighting the functionality of the PoliBrick plugin in UBLA (Upper Bound Limit Analysis). The framework proposed provides enhanced accuracy for limit analysis, revealing insights into failure behavior under gravitational loads.

Keywords: Masonry Preservation, Parametric Modeling, PoliBrick, Limit Analysis, Masonry Arches

ISSN:2623-3347 © 2025 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2025.
doi: 10.7712/120125.12471.25837

1 INTRODUCTION

The architectural heritage, particularly in Italy, is characterized by numerous masonry arches and vaults, which can exhibit structural safety issues under gravity loads. Such load carrying elements are commonly found in historical buildings such as castles, palaces, and churches, making their assessment essential. Although curved historical structures are designed to efficiently transfer vertical loads through their geometry, some cases have revealed potential problems [1]. A key factor influencing their structural behavior is stereotomy, which plays a crucial role in load-bearing capacity. The arrangement of bricks not only affects masonry strength and load diffusion but also may significantly contribute to the overall stability of historical constructions [2].

Shortly after Heyman's seminal study on arches [3], limit analysis (e.g., [4] and [5]) rapidly became the preferred method for reliably estimating the ultimate load-bearing capacity of masonry curved structures. Its effectiveness lies in the ability to replicate the common failure mechanisms observed in such structures, where flexural hinges develop, leading to crack propagation according to an associated flow rule. This approach, which applies external loads statically, can be handled using either the static theorem, providing a lower-bound estimate of the collapse multiplier, or the kinematic theorem, leading to an upper-bound solution [6]-[7]. The actual failure load is identified finding the solution that meets the conditions required by both theorems, including equilibrium, compatibility, plastic strain rate flow rule, and material admissibility. Differently from the static approach, the kinematic theorem is generally more intuitive and easier to apply [8]. Moreover, it can be used manually with a single Lagrangian variable to analyze predefined failure mechanisms or integrated with Finite Elements (FEs) for a more detailed assessment [9]-[10].

Despite significant advancements in limit analysis research, no existing subroutines can automatically generate a curved geometry, at the same time accurately reproducing the stereotomy, prodromic for the generation of a mesh to be used in a limit analysis software. By doing so, apart from drastically reducing the computational burden needed in the geometry generation, the plastic interfaces between the blocks would precisely align with the mortar joints, leading to a more accurate prediction of both the failure mechanism and the collapse load.

To cope with such a need, this paper presents the first results obtained for a case of technical interest, applying PoliBrick plugin, implemented in Grasshopper within the Rhinoceros environment [11], to automatically generate the mesh to be used directly in a Finite Element Limit Analysis (FELA) software based on the kinematic theorem [9]-[10]. The proposed approach is first described theoretically and then validated through a benchmark masonry arch in-situ tested [12], for which a broad blend of numerical results is available for comparison [13]-[14].

2 NUMERICAL APPROACH

A numerical method based on the Upper Bound Limit Analysis (UBLA) combined with FEs is proposed for analyzing collapse masonry curved structures, where also concentrated vertical loads can be considered. The proposed approach uses the same mathematical formulation discussed in [9]-[10], where a classic limit analysis problem is deduced making the following assumptions: (i) the structure is discretized using infinitely resistant hexahedral and wedge elements, and (ii) interfaces, where inelastic deformations are lumped, are modeled with quadrilateral elements exhibiting a rigid-perfectly plastic behavior with infinite ductility. In solving this limit analysis problem, both the collapse load and the failure mechanism are determined. The problem could be approached using either kinematic or static theorems. Based on the previous assumptions, plasticity occurs at a finite number of interfaces, which results in the upper and lower bounds coinciding. Bearing in mind such important property, the kinematic approach is adopted in this work, as it is more straightforward, although the static method could theoretically be derived from the formulation of the self-dual linear programming problem.

For a detailed numerical treatment, the reader may refer to [9] and [10].

In the present research PoliBrick plugin [11] is used to generate the mesh. Such a tool represents a significant advancement in the parametric modeling of masonry structures, particularly for complex geometries such as single and double curvature vaults. Comprising by only six components and developed within the Rhino-Grasshopper environment, PoliBrick simplifies the generation of intricate brick patterns - including basket weave, stretcher bond, and herringbone bond - on free-form surfaces by introducing, for the first time, an alphabet-based approach. Its parametric framework leverages Non-Uniform Rational B-Splines (NURBS) to tessellate surfaces into patches, enabling precise brick placement through algorithmic curve generation and collision detection using R-tree algorithms. The plugin exports node coordinates, element typologies (e.g., Hexa8), and interface nodes as text files (.txt), which can be directly imported into the UBLA software used in this paper to predict active collapse mechanism and load-bearing capacity. This capability addresses a critical gap with respect to existing literature. For the sake of brevity, the reader is referred to [11] to have an insight about PoliBrick plugin implementation.

The discretization method employed to address the integration of the UBLA code with stereotomy is illustrated in Figure 1.

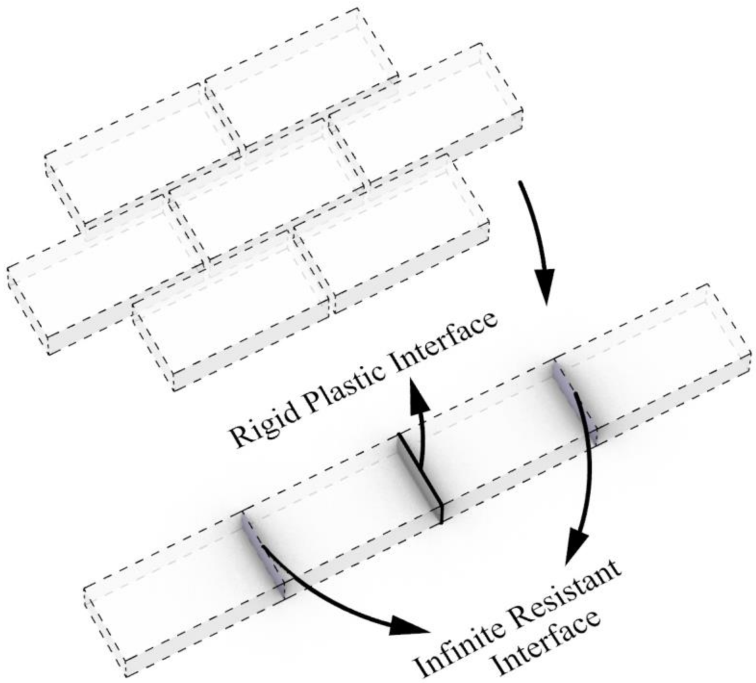

Figure 1: From actual stereotomy to that employed in the UBLA code obtained from PoliBrick

Finally, unlike the numerical codes presented in [9] and [10], the limit analysis in this study incorporates not only Hexa8 elements but also Wedge6 elements, allowing for a broader representation of surface geometries, as shown in Figure 2.

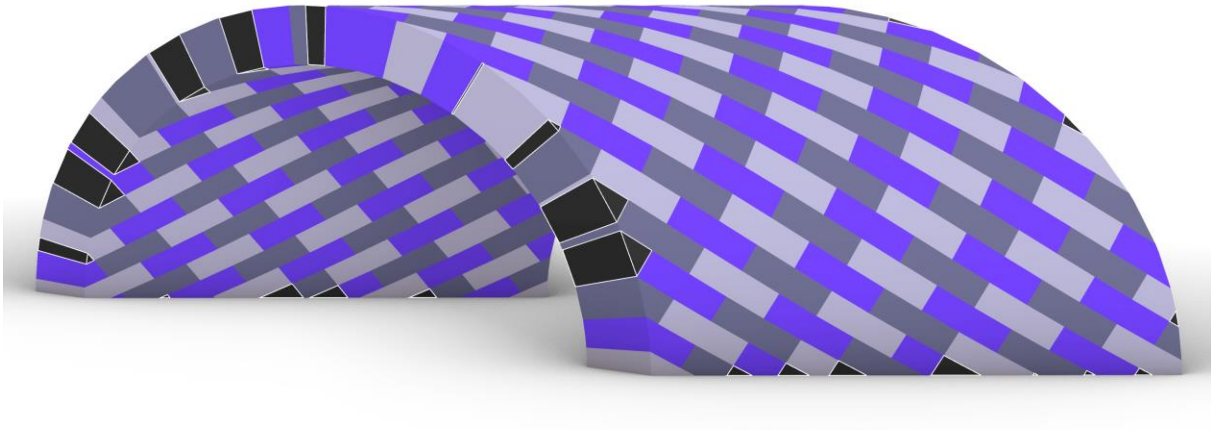

Figure 2: Example of a geometry in which Wedge6 elements – highlighted in black color – are necessary

3 VALIDATION

To benchmark the approach presented in Section 2, an ancient masonry arch located near Novi Ligure, Italy, built in the early 20th century [12], is analyzed. As depicted in Figure 3 the arch has a span of *2555 mm*, a rise of *830 mm*, a depth of *250 mm*, and an in-plane thickness of *120 mm*. The structure consists of Italian clay bricks (*55 × 120 × 250 mm*) laid in alternating stack and header bond patterns, with lime mortar joints approximately *10 mm* thick. Such a curved structure was tested in situ, see [12], where a concentrated vertical load applied at one-fourth of the arch span was progressively increased up to failure. Apart experimental data, previous numerical results exist, some of them obtained by means of either a lower bound limit analysis approach [13] or simple truss 2D finite element models [14].

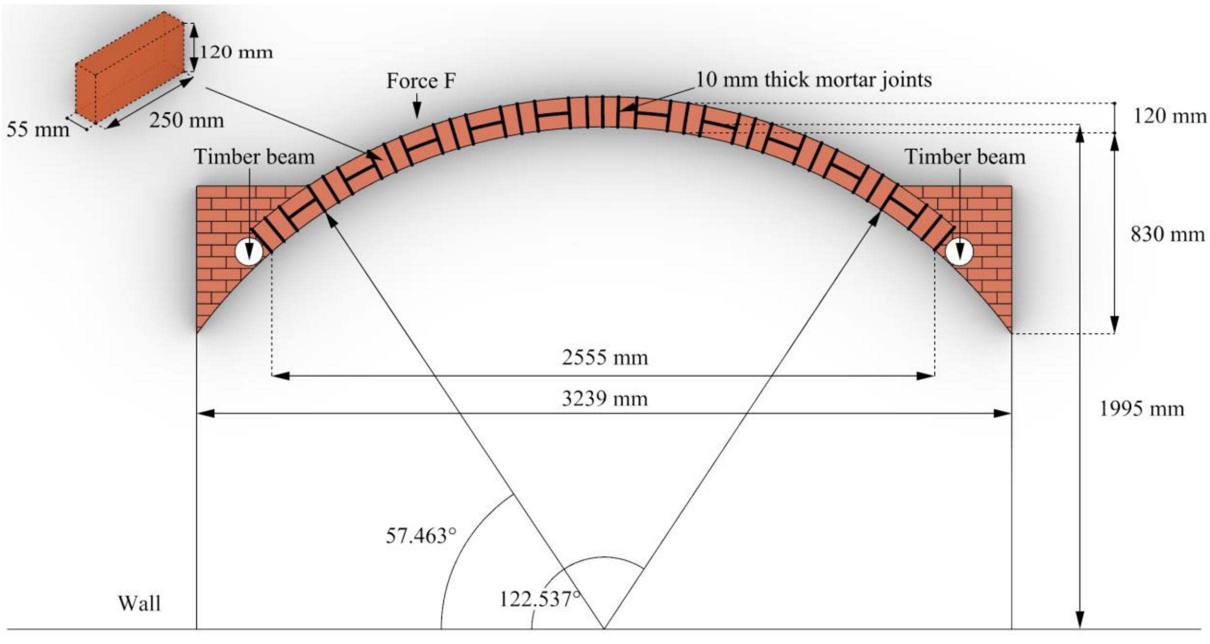

Figure 3: Geometry of the arch examined

The procedure implemented here begins with a geometric parameterization of the arch in PoliBrick plugin, where key architectural dimensions are specified in input, such as brick dimensions, arch thickness, rise, radius, and layer configuration. As clearly visible in Figure 3, the structure consists of two superimposed layers of masonry blocks. The stereotomy generation process employs an advanced block patterning system, where each brick is discretized with two Hexa8 elements, enabling a precise representation of masonry units. The pattern follows a systematically defined sequence across both layers. To distinguish adjacent blocks, and thus recognizing interfaces, within the UBLA code, color-coded unique identifiers are assigned to bricks, basing on specific properties such as orientation and dimensions. As illustrated in Figure 4 different naming conventions, using characters A, B, C, and D, are used to ensure a clear differentiation:

- Bottom: (ABCC×4 + AB) ×2
- Top: (ABDD×4 + AB) ×2

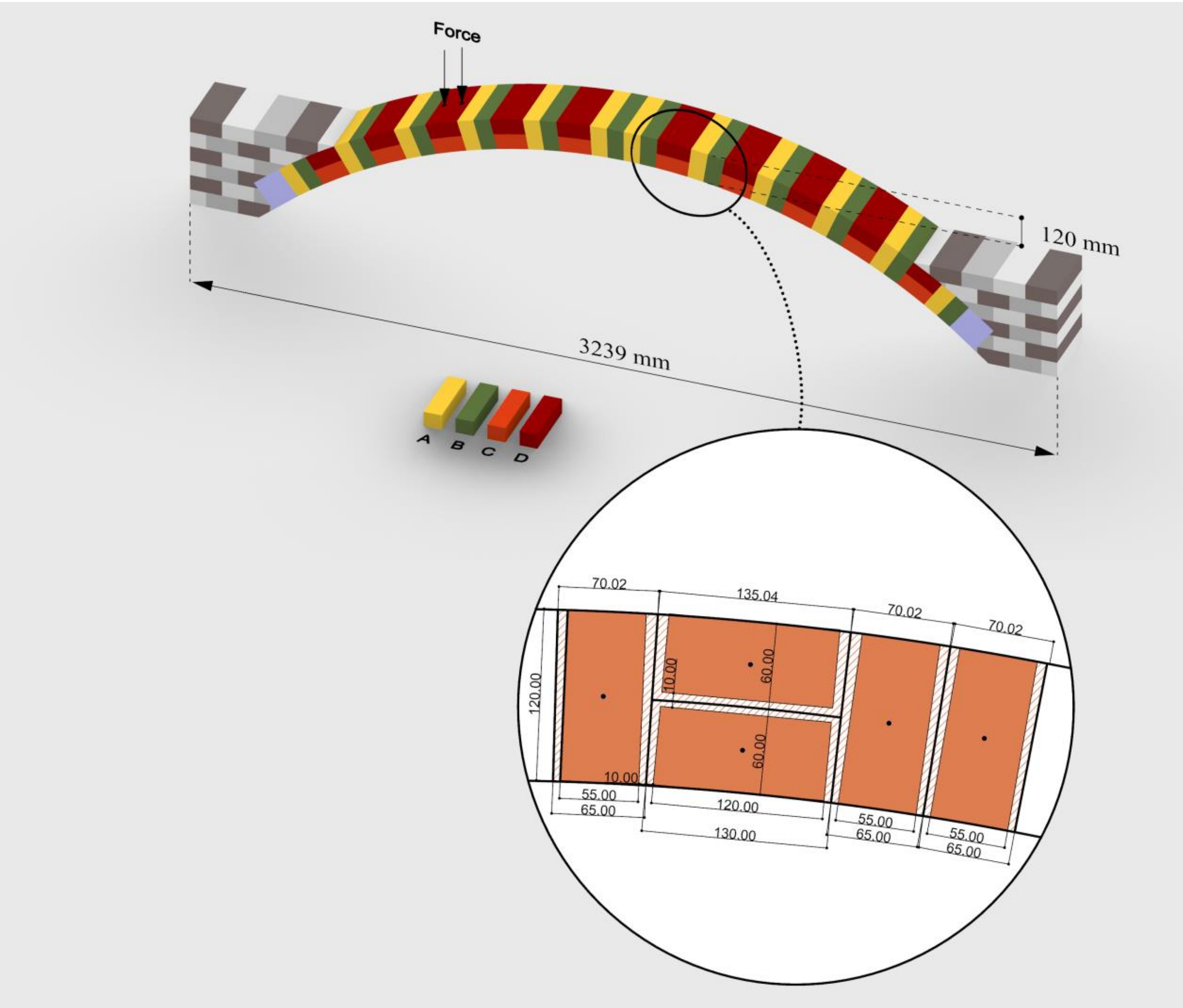

Figure 4: The arch patterning process

This quadruple-unit patterning system ensures proper interlocking of masonry units while maintaining the structural logic of traditional stereotomy.

The mesh processing pipeline employs the following advanced computational geometry techniques:

1. Vertex unification: a dictionary-based vertex indexing system ensures topological consistency while eliminating duplicate vertices through welding operations;

2. Mesh combination: individual bricks are aggregated into a unified mesh structure while preserving element connectivity;

3. Element classification: automatic detection of hexahedral (Hex8) and prismatic (Wedge6) elements accommodate geometric variations.

All the aforementioned information is collected in a text file which can be directly imported into the UBLA code. It contains, as illustrated in Figure 5, the following information: (i) node coordinates, (ii) element topology and (iii) interface definitions.

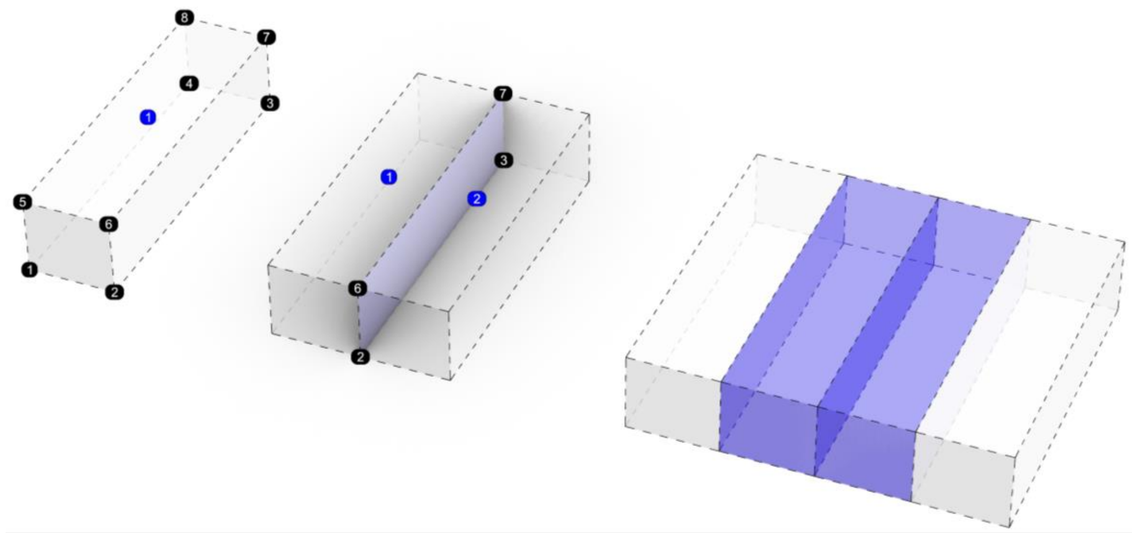

Figure 5: Information needed – in .txt format - by UBLA code: node coordinates, element topology with branch indexing, and interface definitions for adjacent elements.

The geometry generation process is outlined in Figure 6.

Algorithm 1 Mesh Stereotomy Generation for UBLA Code

Input:
Brick dimensions: $55mm \times 120mm \times 250mm$
Arch thickness: $120mm$
Arch rise: $830mm$
Arch radius: $1995mm$
Number of bricks in x direction: 36
Number of brick layers in y direction: 2 (3 levels)
Number of layers in z direction: 1 (out-of-plane)
Block pattern: Only 2 patterns used (A and B)
Step 1: Create Arch Geometry
Generate the arch based on input dimensions (thickness, rise, radius, brick count).
Set up grid positions for bricks in x, y, and z directions.
Step 2: Apply Brick Pattern
Define the brick pattern as:
Lower brick layer: $(ABCC \times 4 + AB) \times 2$
Upper brick layer: $(ABDD \times 4 + AB) \times 2$
Apply the pattern on the arch geometry.
Step 3: Add Support Blocks
Add 3 support blocks at beginning and end of each string.
Ensure support width equals 3 brick dimensions ($3 \times 250mm = 750mm$).
Align support blocks with main structure.
Step 4: Prepare Mesh for Export
Create a list of ordered vertices.
Use a dictionary to store vertex indices for fast lookup.
Combine all meshes and weld vertices to create a unified mesh.
Loop through the data tree of meshes and add each mesh's vertices.
Step 5: Export Mesh to File for MATLAB
Create a text file for limit analysis export.
For each mesh, write the node coordinates to the file.
For each brick element, write its type and vertex indices.
For interface topology, write the shared vertices between meshes.
Save the file with the mesh information.
Output:
Exported file with ordered vertices, brick elements, and interface topology for UBLA.

Figure 6: Geometry generation process

The resulting text file is then imported into the UBLA code, implemented in MATLAB. The numerical analysis is performed under the no-tension material assumption, with a cohesion sufficiently high to preclude sliding (*0.07 MPa*) -in agreement with Heyman's hypothesis-, compressive strength and friction angle set equal to *2.5 MPa* and *30°*, respectively.

The failure mechanism found is plotted in Figure 7.

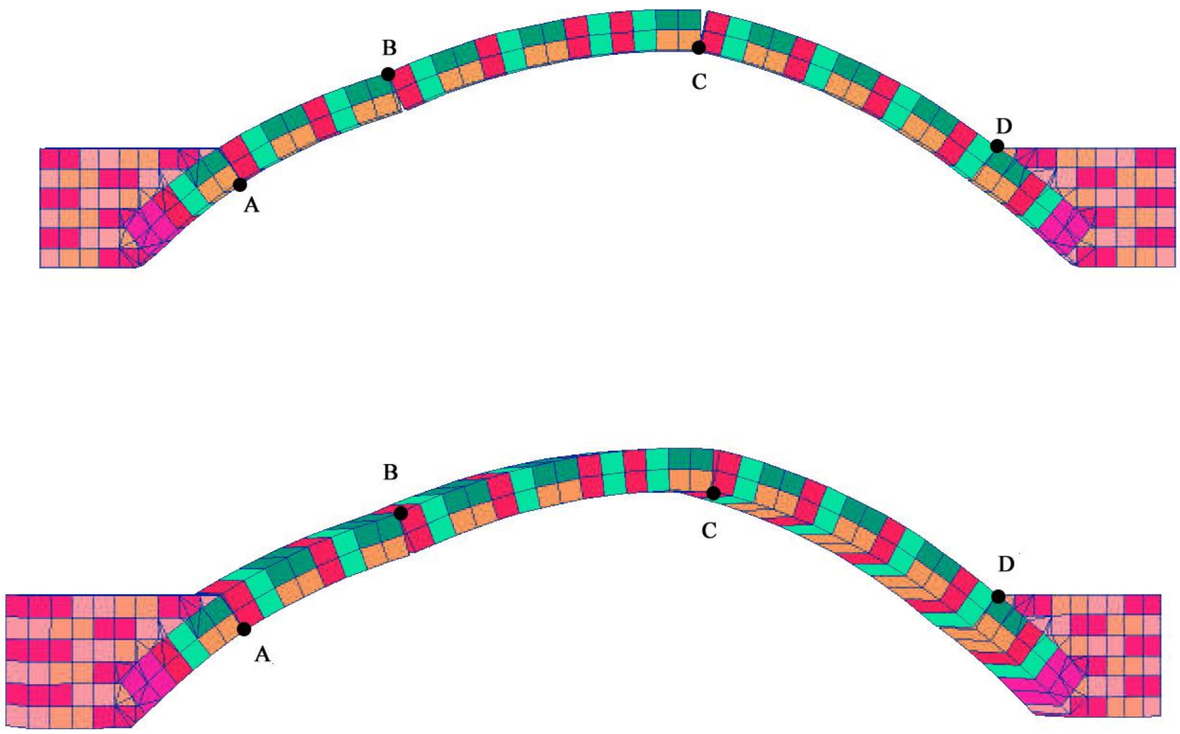

Figure 7: Failure mechanism found with PoliBrick combined with UBLA

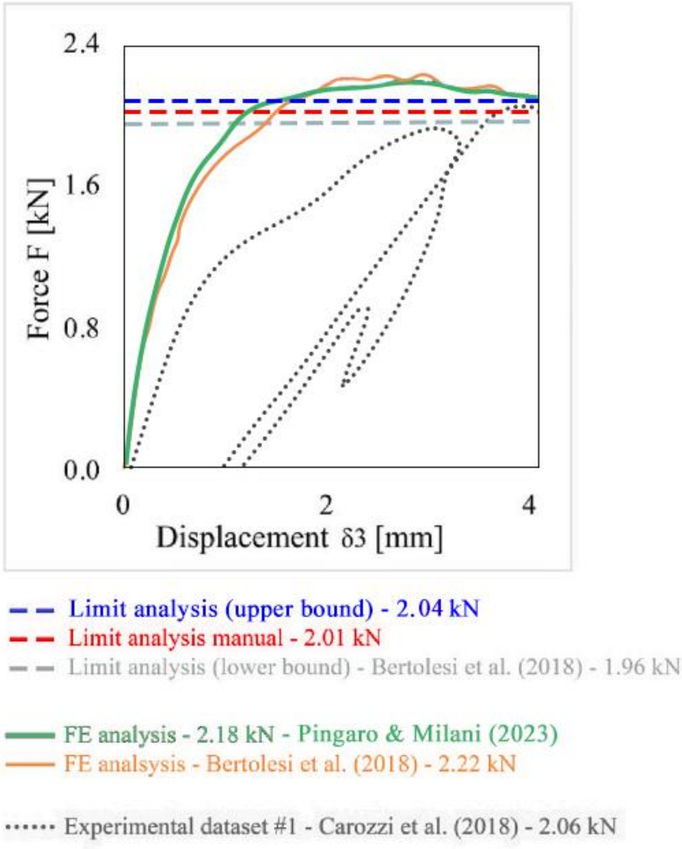

Figure 8: Collapse load obtained with the present analysis compared with results available in the literature [12]-[14]

Finally, the collapse load obtained using the approach proposed by PoliBrick in combination with UBLA is calculated as *2.04 kN*. This result is compared with those available in the literature (see Figure 8), finding a good agreement with previously proposed alternative numerical approaches [12]-[14].

4 CONCLUSIONS

This study has presented a novel framework for the analysis of single and double curvature masonry structures using the PoliBrick plugin integrated with Finite Element Limit Analysis (FELA). By combining parametric modeling with a computerized Limit Analysis based on the Upper Bound theorem, the proposed methodology has allowed for an efficient and accurate prediction of both collapse load and failure mechanism exhibited by a masonry arch subjected to a vertical concentrated force. The integration of stereotomy into the modeling process through PoliBrick will offer a unique advantage in this research field, as it (i) significantly reduces the computational burden and (ii) can provide a more accurate prediction of the actual response of the structure under various loading conditions.

The validation, carried out for a masonry arch, has been obtained comparing the results obtained with available experimental and numerical data. It has demonstrated that the numerical methodology discussed is both robust and reliable in estimating the ultimate load-bearing capacity of curved masonry structures. The results have proved the efficiency of the PoliBrick plugin in facilitating the generation of complex geometries and interfacing them with limit analysis solvers.

In conclusion, this work is meant to give a contribution in the field of masonry preservation by providing a practical and rapid computational tool that can support engineers and conservationists in assessing the stability of historical structures, particularly in contexts where experimental testing may be difficult or costly. Future developments include further refinement of the plugin capabilities to handle more complex case-studies in terms of geometry, disposition of bricks and external loads, e.g. seismic actions, as well as an extension for the utilization in presence of other types of masonry structures.

ACKNOWLEDGMENT

The study was partially developed within the research activities carried out in the frame of 2024-2026 ReLUIS Project – WP10 masonry structures (Coordinator – Prof. Guido Magenes), specifically by means of the research carried out in WP10.1.1 and WP10.2 Masonry Structures (Task Coordinators – respectively Prof. Serena Cattari and Prof. Sergio Lagomarsino). The Italian Department of Civil Protection has funded such a project.

Finally, the study was partially developed with the financial support of the Italian Ministry of Scientific Research MUR within the research project PRIN-2022 (https://www.dabc.polimi.it/en/progetto/advanced-mechanical-models-and-computational-methods-for-large-scale-3d-printing-of-innovative-concrete-structures/) titled "Advanced mechanical models and computational methods for large-scale 3D printing of innovative con crete structures (COM^3D-CREATE)" (National PI: Prof. Andrea Chiozzi, Local PI: Prof. Gabriele Milani). Finanziato dall'Unione europea- Next Generation EU, Missione 4 Componente 1 CUP D53D23004070006.

REFERENCES

[1] N. Pingaro, M. Buzzetti, G. Milani, Advanced FE nonlinear numerical modeling to predict historical masonry vaults failure: Assessment of risk collapse for a long span cloister vault heavily loaded at the crown by means of a general-purpose numerical protocol. *Engineering Failure Analysis*, **167**, 109070, 2025. https://doi.org/10.1016/j.engfailanal.2024.109070

[2] M. Alforno, F. Venuti, A. Monaco, C. Calderini, Seismic behaviour of cross vaults with different brick pattern. *Bulletin of Earthquake Engineering*, **20(8)**, 3921–39, 2022. https://doi.org/10.1007/s10518-022-01347-6.

[3] J. Heyman, The stone skeleton. *International Journal of Solids and Structures*, **2(2)**, 249–279, 1966. https://doi.org/10.1016/0020-7683(66)90018-7

[4] A. Chiozzi, G. Milani, A. Tralli, A Genetic Algorithm NURBS-based new approach for fast kinematic limit analysis of masonry vaults. *Computers & Structures*, **182**, 187–204, 2016. https://doi.org/10.1016/j.compstruc.2016.11.003

[5] E. Milani, G. Milani, A. Tralli, Limit analysis of masonry vaults by means of curved shell Finite Elements and homogenization. *International Journal of Solids and Structures*, **45**, 5258–5288, 2008. https://doi.org/10.1016/j.ijsolstr.2008.05.019

[6] M. Gilbert, C. Casapulla, H. Ahmed, Limit analysis of masonry block structures with non-associative frictional joints using linear programming. *Computers & Structures*, **84(13–14)**, 873–887, 2006. https://doi.org/10.1016/j.compstruc.2006.02.005

[7] G. Milani, P. B. Lourenço, A. Tralli, 3D homogenized limit analysis of masonry buildings under horizontal loads. *Engineering Structures*, **29(11)**, 3134–3148, 2007. https://doi.org/10.1016/j.engstruct.2007.03.003

[8] G. Milani, 3D upper bound limit analysis of multi-leaf masonry walls. *International Journal of Mechanical Sciences*, **50(4)**, 817–836, 2007. https://doi.org/10.1016/j.ijmecsci.2007.11.003

[9] M. Buzzetti, N. Pingaro, G. Milani, Automatic detection of local collapse mechanisms in historical masonry buildings: Fast and robust FE upper bound limit analysis. *Engineering Failure Analysis*, **170**, 109310, 2025. https://doi.org/10.1016/j.engfailanal.2025.109310

[10] P. Wang, G. Milani, Seismic vulnerability prediction of masonry aggregates: Iterative Finite element Upper Bound limit analysis approximating no tensile resistance. Engineering Structures, **293**, 116595, 2023. https://doi.org/10.1016/j.engstruct.2023.116595

[11] M. Pourfouladi, N. Pingaro, M. Valente, PoliBrick plugin as a parametric tool for digital stereotomy modelling. *Computers & Structures*, **311**, 107722, 2025. https://doi.org/10.1016/j.compstruc.2025.107722

[12] F. G. Carozzi, C. Poggi, E. Bertolesi, G. Milani, Ancient masonry arches and vaults strengthened with TRM, SRG and FRP composites: Experimental evaluation. *Composite Structures*, **187**, 466–480, 2017. https://doi.org/10.1016/j.compstruct.2017.12.075

[13] E. Bertolesi, G. Milani, F. G. Carozzi, C. Poggi, Ancient masonry arches and vaults strengthened with TRM, SRG and FRP composites: Numerical analyses. *Composite Structures*, **187**, 385–402, 2017. https://doi.org/10.1016/j.compstruct.2017.12.021

[14] N. Pingaro, G. Milani, Simple non-linear numerical modelling of masonry arches reinforced with SRG using elasto-fragile and elasto-ductile truss finite elements. *Engineering Structures*, **293**, 116637, 2023. https://doi.org/10.1016/j.engstruct.2023.116637