

AN ANALYTICAL MODEL FOR THE DYNAMIC RESPONSE OF AN ELASTIC SDOF SYSTEM FIXED ON TOP OF A ROCKING SINGLE-STORY FRAME STRUCTURE: EXPERIMENTAL VALIDATION

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Abstract. *It has been shown that assemblies of large rocking bodies have remarkable seismic stability and minimal residual displacement. Such dynamic response characteristics make freely rocking systems desirable for their superior seismic performance, but challenging to design. In Russia and the former Soviet Union structures have been constructed with an intentionally “soft story” that comprises rocking columns able to take the seismic displacement. The uplift of the columns works as a mechanical fuse and limits the forces transmitted to rest of the structure. This paper describes an experimental investigation of the dynamic response of specimens comprising an elastic single-degree-of-freedom system fixed to a rigid beam that is rocking on rigid columns. The goal of the tests is to verify and validate the dynamic rocking model of a two-degree-of-freedom system of such rocking podium structures. The time history responses computed using the numerical model are in good agreement with the results obtained in the experimental tests. Furthermore, the model predicts both the rocking column rotations and the superstructure deformation relatively well. The prediction is better for smaller tilt angles.*

1 INTRODUCTION

Base isolation has been used for decades. It decreases the design forces of the superstructure and takes most of the displacement demand through a soft and specially designed layer between the structure and the ground. Usually this soft layer comprises either rubber or (concave) sliding bearings. Simplified models, experimental validation of them, and code provisions have resulted in the increasing use of seismic isolation.

In Russia and the former Soviet Union states another method of seismic isolation has been used. The soft layer does not comprise bearings placed under the base slab, but the entire bottom story intentionally designed as “soft”. Its concrete columns are designed to uplift and sustain rocking motion during an earthquake. Thus, the design forces of the superstructure are controlled by the uplift force of the bottom story. The critical design parameters for such podium building structure are the geometric properties of the columns. Column ends are protected by steel plates to avoid concrete crushing when they uplift (Figure 1). Unlike for concave or lead-rubber bearings (which have a hysteretic form of damping that results in residual displacements) energy dissipation due to uplift and rocking is instantaneous and happens at every impact. Therefore the rocking podium system has minimal (if any) residual deformation and has a resilient behavior. Added dampers can be used to diminish the magnitude of the rocking motion. Interestingly, full-scale dynamic tests of rocking podium structures have been performed on real structures. The structures were excited using a hydraulic jack to push the structure to an initial displacement and then release it [1, 2]. It should be mentioned that this system does not rely on the size of the rocking elements for its stability, as the rocking isolation techniques proposed for solitary columns or rocking assemblies do [3–60].

The force-displacement response of concave friction-pendulum and lead-rubber bearings can be approximated by a bilinear envelope curve with positive stiffnesses. This allows for a rough approximation of the base isolated structure response using a secant stiffness linear model with viscous damping [61, 62]. On the contrary, a rocking podium structure has negative post-uplift stiffness (the restoring force decreases as the lateral displacement increases). It has been proven that these kind of systems cannot be approximated by a SDOF elastic systems and, hence, the widely used elastic response spectra are not applicable [12].

This paper presents a simplified model to describe the behavior of an elastic structure sitting on a rocking podium. The model was validated using the results from small-scale experiments performed in the Swiss Federal Institute of Technology (ETH) in Zurich, in the context of the Masters Semester project of the 2nd and 3rd author.



Figure 1: Left: Basement with rocking columns. Right: Close up of a column out of reinforced concrete. [1]

2 DYNAMIC MODEL

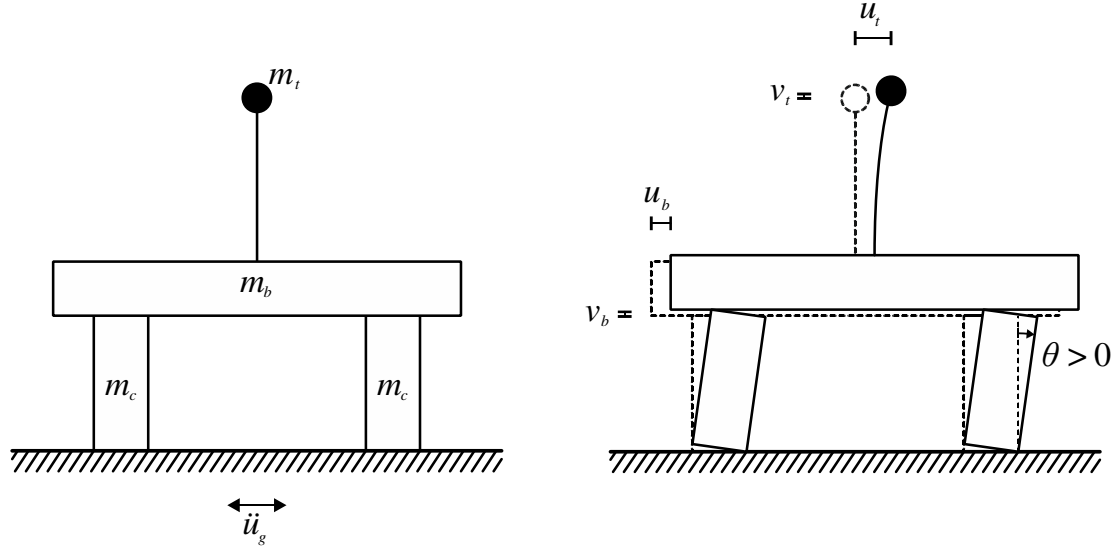


Figure 2: Dynamic model of a rocking podium structure. Left: Initial position. Right: Rocking position.

In order to gain insight into the behavior of rocking podium structure, an elementary planar analysis can be performed using a simple 2-DOF model. The parameters of the model are shown in Figure 2. It comprises two rocking columns ($N = 2$), one cap beam, and a SDOF system fixed to the beam. The elastic behavior of a more complex superstructure is described using an equivalent SDOF elastic system, thus neglecting any higher modes effects. The columns and the slab are assumed to behave rigidly. Two dimensionless parameters γ and η are introduced.

$$\begin{aligned}\gamma &= \frac{m_b}{Nm_c} = \frac{m_b}{2m_c} \\ \eta &= \frac{m_t}{Nm_c} = \frac{m_t}{2m_c}\end{aligned}\tag{1}$$

The SDOF system on top with the mass m_t has a fixed-base natural frequency ω_s and a viscous damping ratio ζ . Using a Lagrangian formulation, the following equations of motion are obtained:

$$\ddot{u}_t + \omega_s^2 (u_t - u_b) + \ddot{u}_g + 2\zeta\omega_s (\dot{u}_t - \dot{u}_b) = 0\tag{2}$$

$$\begin{aligned}\ddot{\theta} \left[\gamma + \frac{I_c}{4R^2 m_c} + \frac{1}{4} + \eta \sin(\pm\alpha - \theta) \right] \frac{2R}{g} - \dot{\theta}^2 \eta [\cos(\pm\alpha - q) \sin(\pm\alpha - q)] \frac{2R}{g} + \\ + \left(\gamma + \frac{1}{2} + \eta \right) \sin(\pm\alpha - \theta) + \frac{\ddot{u}_g}{g} \left(\gamma + \frac{1}{2} \right) \cos(\pm\alpha - \theta) + \\ - \frac{\omega_s^2 \eta}{g} (u_t - u_b) \cos(\pm\alpha - \theta) - \frac{2\zeta\omega_s \eta}{g} \cos(\pm\alpha - \theta) (\dot{u}_t - \dot{u}_b) = 0\end{aligned}\tag{3}$$

where I_c is the rotational moment of inertia of one column around its center of mass. The slenderness α of the column (Figure 3a) controls the magnitude of the forces transferred to the

superstructure. The ground acceleration that leads to the uplift of a single rigid column, and therefore initiates rocking motion, is:

$$\begin{aligned}\ddot{u}_{g,uplift} &= g \tan(\alpha) \\ \tan(\alpha) &= \frac{2B}{2H} = \frac{B}{H}\end{aligned}\quad (4)$$

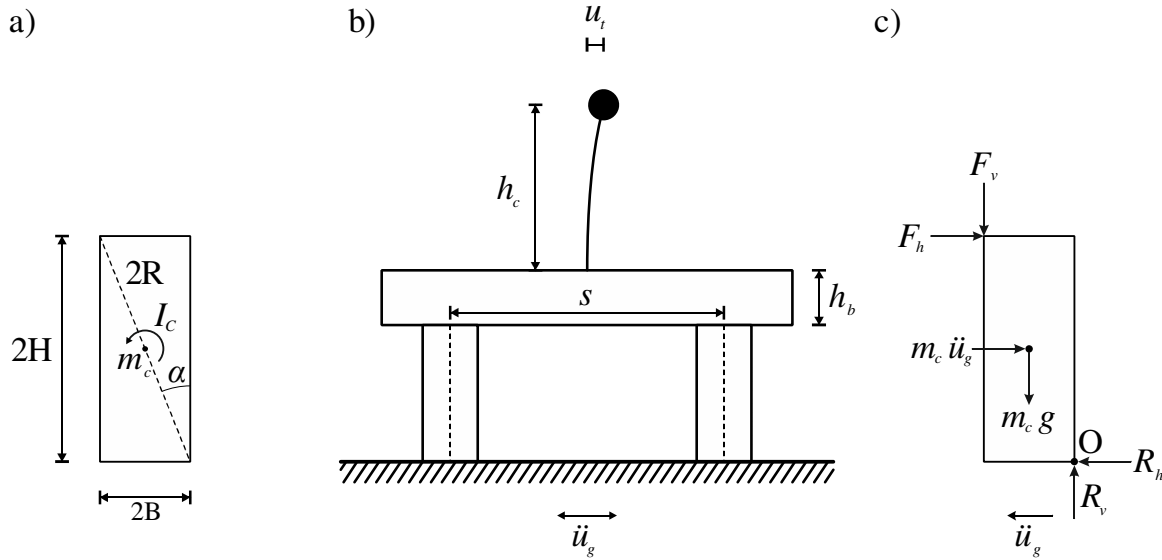


Figure 3: a) Column. b) State of the structure at incipient uplift. c) Forces at incipient uplift.

The contact forces acting on a single column right at the verge of uplift, rotating around point O, are shown in Figure 3c and given in equation (5).

$$F_h = \frac{m_t (\omega_s^2 u_t + 2\zeta \omega_s \dot{u}_t)}{2} - \frac{m_b \ddot{u}_g}{2}, \quad F_v = \frac{(m_t + m_b)g}{2} \pm \frac{m_t (\omega_s^2 u_t + 2\zeta \omega_s \dot{u}_t)(h_c + h_b)}{s} \quad (5)$$

For the model under consideration, the principle of virtual work at incipient uplift gives:

$$\delta\theta (Hm_c + 2Hm_b) \ddot{u}_g - \delta\theta (\omega_s^2 m_t u_t + 2\zeta \omega_s m_t \dot{u}_t) 2H - \delta\theta [Bm_c + 2Bm_b + 2Bm_t] g = 0 \quad (6)$$

The SDOF superstructure will always start to oscillate before the columns start to rock because rocking starts only when the ground acceleration reaches the uplift limit of the podium structure. This limit not only depends on the ground acceleration $\ddot{u}_g$ but also on the actual displacement u_t of the top mass (Figure 3b). Equation (6) leads to the two following thresholds for the podium structure uplift acceleration $\ddot{u}_g$:

$$\begin{aligned}\ddot{u}_g &\geq g \tan(\alpha) \left[\frac{1+2\gamma+2\eta}{1+2\gamma} \right] + \omega_s^2 u_t \left[\frac{2\eta}{1+2\gamma} \right] + 2\zeta \omega_s \left[\frac{2\eta}{1+2\gamma} \right] \\ \ddot{u}_g &\leq -g \tan(\alpha) \left[\frac{1+2\gamma+2\eta}{1+2\gamma} \right] + \omega_s^2 u_t \left[\frac{2\eta}{1+2\gamma} \right] + 2\zeta \omega_s \left[\frac{2\eta}{1+2\gamma} \right]\end{aligned}\quad (7)$$

Hence, the uplift acceleration for the podium structure can be higher or lower than $g \tan(\alpha)$, the uplift acceleration for a single rocking column (equation (4)).

As soon one of the thresholds in equation (7) is reached, the rocking motion of the podium structure starts. When the structure rocks back and hits the ground it is assumed to have

stopped rocking until equation (7) is fulfilled again – this could happen instantly, or not at all. There are limitations for this assumption: if the top mass is light a structure might not start rocking again in the numerical model although it could do so in reality. This, however, was not the case in the conducted experiments since only relatively top-heavy superstructures were used. A different approach is needed to model the response of rocking podium structures with light superstructures.

The model dissipates input energy through structural damping of the SDOF model of the superstructure and through rocking impacts. Since the podium structure model has two degrees of freedom, two equations are necessary to formulate the energy loss. The model proposed by Housner [63] assumes that impact happens instantaneously and that the contact forces are concentrated at the new pivot point. For a solitary column this means that the angular momentum about the new pivot point is conserved. Applying Housner's assumptions to the 2-DOF model led to the first equation. The second equation was derived by assuming that the horizontal velocity of the top mass m_t stays the same before and after impact [13, 43]. Thus, the coefficient of restitution for the 2-DOF system was derived:

$$c = \frac{\dot{\theta}_{after}^2}{\dot{\theta}_{before}^2} = \left[1 - \frac{\sin^2(\alpha) \left(2\gamma + 2\eta + \frac{1}{2} \right)}{\frac{1}{4} + \frac{I_c}{4m_c R^2} + \gamma + \eta} \right]^2 \quad (8)$$

Equation (8) can be compared to the expressions for coefficient of restitution of other, simpler, systems. With $\eta = \gamma = 0$ it yields exactly what Housner proposed for his rigid block, and with $\eta = 0$ it yields the solution of Makris and Vassiliou [39] for an array of free-standing columns capped with a free-standing rigid beam.

Uplift of the podium structure affects the vibration properties of the SDOF superstructure [7, 27, 58]. An eigenfrequency analysis reveals that there are two distinct mode shapes of the podium structure: one is overturning of the rocking frame structure with a natural frequency of 0 Hz, and the other is the vibration of the SDOF system when the rocking frame structure is uplifted. In this state the natural frequency of the SDOF $f_{s,u}$ is amplified compared to its fixed-base counterpart, f_s , and is given by equation (9):

$$f_{s,u} = \sqrt{\frac{3\gamma + 3\eta + 1}{3\gamma + 3\eta \sin^2(\alpha) + 1}} f_s \quad (9)$$

For top-heavy structures ($\eta \rightarrow \infty$) the amplification factor simplifies to $\sin^{-1}(\alpha)$.

3 SPECIMEN AND TEST SETUP

The 2-DOF model presented in the previous section was instantiated in the lab using two frames on top of each other, the first representing the rocking podium, and the second one representing the elastic SDOF system (Figure 4). In order to allow for replacements or improvements during the testing campaign, the specimen was screwed together. The base plate was fixed to the shake table. The material used for the specimens and the setup, apart from the steel masses on top, was aluminum (EN AW-6060).

The rocking podium structure (i.e. the first story) has a total height of 500 mm and a column slenderness α of 0.1385 rad, determined by the column width of 69.7 mm. The two 480 mm tall columns are identical, and stiff enough to be modelled as rigid. To achieve a high stiffness, each column is built out of two hollow 60 mm x 60 mm x 4 mm sections. Rectangular plates (5 mm x 50 mm x 400 mm), two at the bottom and two at the top of the columns are

used to connect the columns into a rigid frame. When rocking, the frame is assumed to rotate around the outer edges of these plates.

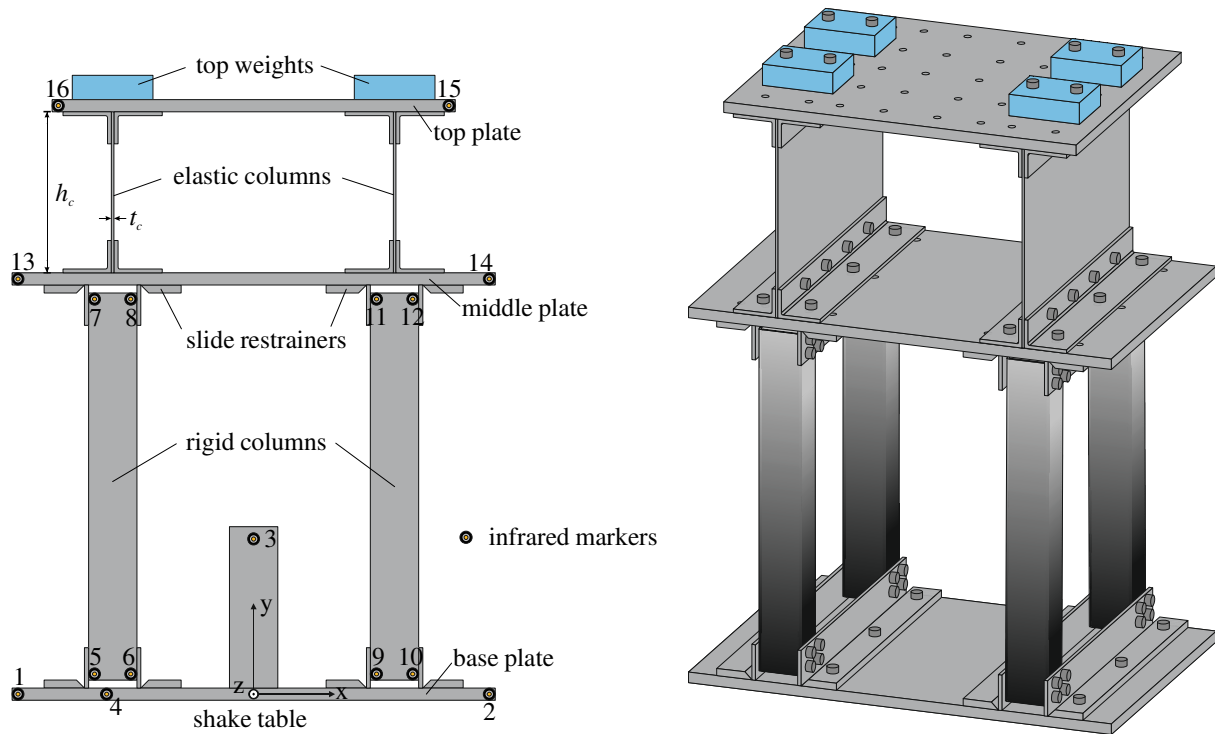


Figure 4: Left: front view of the specimen with locations of the infrared markers. Right: 3D view of the specimen.

The elastic SDOF superstructure was fixed to the aluminum plate positioned on top of the rocking columns. It comprises two elastic columns, the top plate, the top weights, and the connecting L-sections. The height of an aluminum sheet representing the elastic column is 441 mm, the thickness is 3 mm and the depth is 400 mm. The connecting extruded L sections have a length of 400 mm and a cross section of 60 mm x 40 mm x 5 mm.

Part	Mass [kg]	Quantity [-]	Ground [kg]	Columns [kg]	Beam [kg]	Top [kg]
base plate	10.96	1	10.96			
slide restrainers	0.54	8	2.16		2.16	
1 rigid column setup	3.94	2		7.88		
middle plate	11.09	1			11.09	
L section	0.56	8			2.24	2.24
1 elastic column	1.56	2			1.56	1.56
top plate	7.88	1				7.88
top weights	7.92	2				15.84
Total	65.57		13.12	7.88	17.05	27.52

Table 1: Masses of the different parts.

Table 1 shows the weights of the different parts of the experimental setup, including screws and other small parts. The mass of the elastic columns of the superstructure is evenly distributed to the top and bottom mass.

Then the mass ratios are:

$$\begin{aligned}\gamma &= \frac{m_b}{Nm_c} = \frac{17.05\text{kg}}{7.88\text{kg}} = 2.16 \\ \eta &= \frac{m_t}{Nm_c} = \frac{27.52\text{kg}}{7.88\text{kg}} = 3.49\end{aligned}\tag{10}$$

3.1 Similitude Analysis

The above mentioned model dimensions were chosen so that it reproduces, as closely as possible, a typical full size prototype podium structure. The main laboratory constraint was the size of the model, which resulted in a scaling down of the rocking column length dimension by a factor of 6 (given that a typical floor would have a height of 3.0 m). The other properties of the model were chosen so that similitude is maintained. The similitude analysis between the Prototype full-scale podium structure and the Model is shown in Table 2.

Parameter	Prototype	Model
Story height	3.0 m	500 mm
Excitation frequency	f_p	$\sqrt{6} f_p$
Superstructure period	0.544 s	0.222 s
Mass ratio γ	2.16	2.16
Mass ratio η	3.49	3.49

Table 2: Properties of the Prototype and Model podium structures.

The fundamental vibration of the prototype superstructure was targeted to be in the range between 0.5 s and 0.6 s, chosen as typical for a 5-story masonry building. The fundamental vibration period of the fixed-base superstructure model was 0.222 s, corresponding to a fundamental period of 0.544 s for the prototype. The mass ratio $\gamma = 2.16$ (Table 1) is larger than what may be expected (as the slab weight would be much larger than the total column weight) but lower values were not possible in the lab, given that the model column had to be practically undeformable under its loads. The uplifted period was measured at 0.145 s which is 1.53 times smaller than the fixed-base natural period of 0.222 s, confirming the value that equation (9) predicts, namely 1.53.

3.2 Experimental Setup

The experimental setup is shown in Figure 5. To avoid sliding, restrainers were added to the baseplate. The green steel columns and plywood plates were used for safety reasons and to prevent the specimen from collapsing.

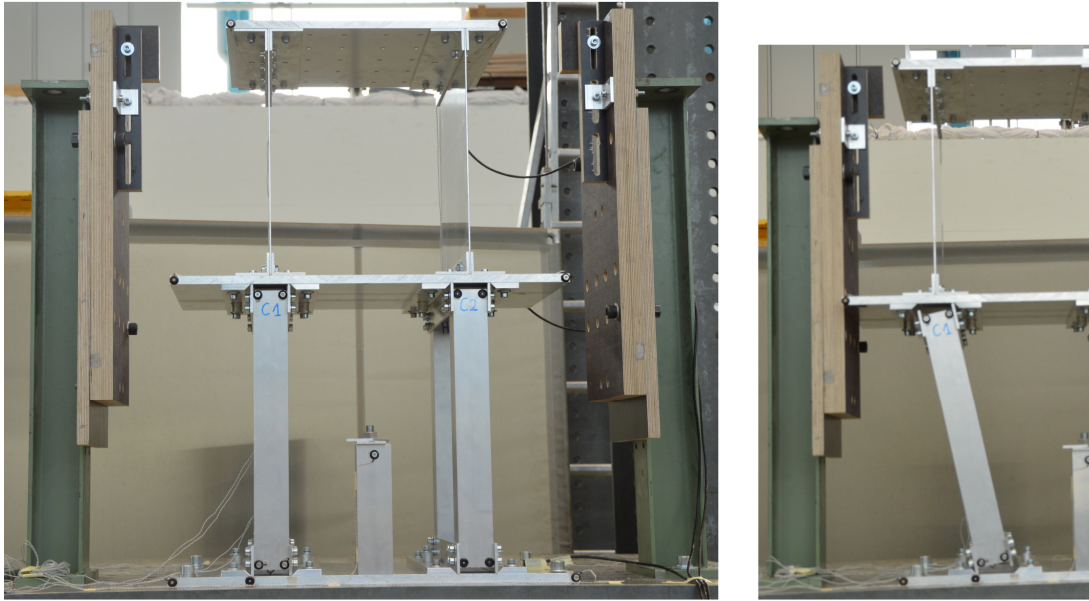


Figure 5: Left: Front view of the specimen. Right: Overturned specimen at the end of the test.

3.3 Excitation

The ETH 1-D shaking table was used to apply a base motion to the specimen. It is supported by roller bearings and actuated using servo-hydraulic actuators to move only in one horizontal direction. The stroke of the table is 250 mm, and the maximum velocity is 225 mm/s. Each specimen was tested using 12 different symmetric Ricker wavelets and 12 different anti-symmetric Ricker wavelets.

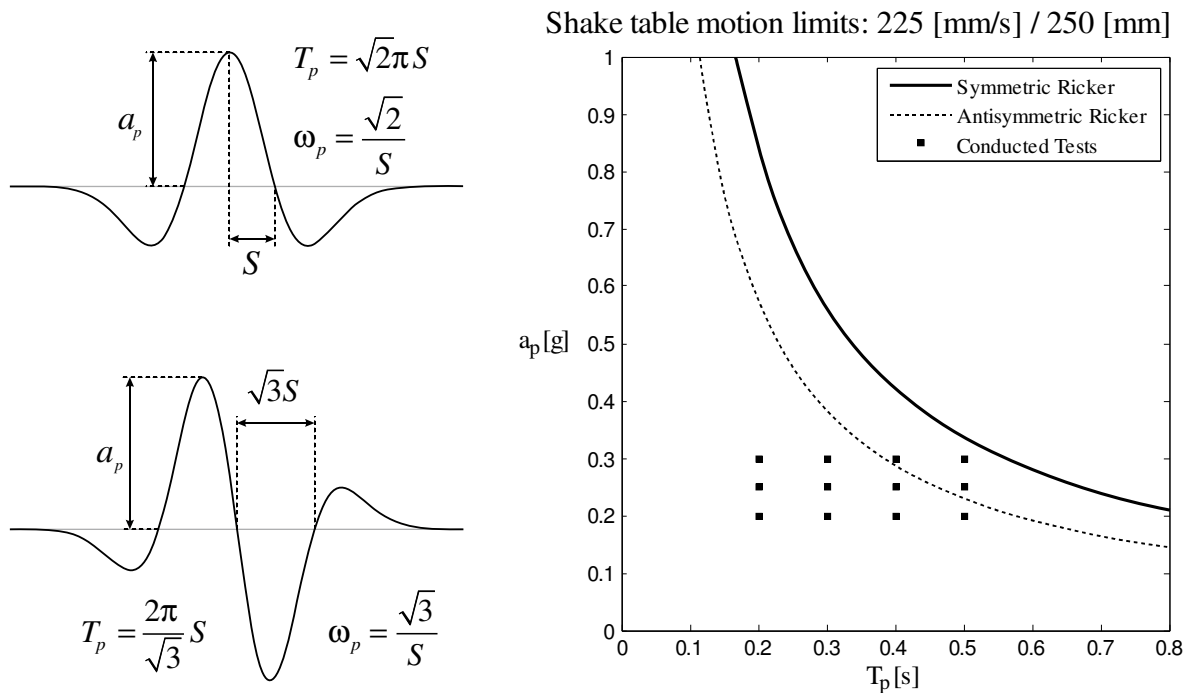


Figure 6: Left: Symmetric and antisymmetric Ricker wavelets. Right: Shake table motion limits

A Ricker wavelet approximates the main pulse of pulse-type ground motions [64]. Typical symmetric and antisymmetric Ricker wavelets are shown in Figure 6. The acceleration amplitude, a_p , and period, T_p , of a Ricker wavelet are discussed further in [33, 65]. The Ricker wavelets used in this study had acceleration amplitudes of $a_p = \{0.20, 0.25, 0.30\}g$ and the pulse periods of $T_p = \{0.20, 0.30, 0.40, 0.50\}s$. At the Prototype scale these values correspond to periods of $\{0.49, 0.74, 0.98, 1.23\}s$.

For 3 out of the 24 different Ricker wavelets the shake table would reach its motion limits (Figure 6), namely the velocity limit of 225 mm/s. For our study this did not pose any problems. The acceleration and displacement output of the shake table was measured in all tests and subsequently used as input for the numerical model for comparison. Unfortunately, the shaking table reproduced the command motions with different degrees of accuracy during the test campaign, depending on the characteristics of the motion and numerous factors associated with the state of the shaking table.

3.4 Data Acquisition System

An Optotrak Certus System, manufactured by Northern Digital Inc., was used to track the position of the specimens during the tests. This system uses active infrared-emitting diodes as markers and a trinocular camera system to determine the position of the markers (Figure 4). Three markers (1, 2, 3) were placed on the shake table and parts of the specimen rigidly connected to the shake table. These markers were used to define a rigid plane; the displacements measured during the tests were relative to marker 1. Eight other markers (5-8 & 13-16) were placed on the moving parts of the specimen to measure their position. The built-in tools of the Optotrak Certus System software were able to measure angles, either in 2D or 3D, between lines defined by two markers. Lines defined by markers 1-2 and 5-7 were used to identify the rocking angle in real time during the experiments. Other output parameters were defined to show the out-of-plane behavior of the specimen, to measure the differential rotation of the middle and top plate compared to the base plate. The position sampling frequency was 250 Hz.

The accuracy of the system has been determined in previous experiments [56]. In the x- and y-directions in the plane where rocking motion takes place, the accuracy is about 0.02 mm. In the z-direction, perpendicular to the plane of motion, the accuracy is about 0.10 mm.

3.5 Experiment Outcomes and Observations

Table 3 lists the 24 tests with Ricker wavelets, numbered for reference.

No.	Excitation type	a_p [g]	T_p [s]	Exp.	Num.
1	Antisymmetric Ricker	0.20	0.20		
2	Antisymmetric Ricker	0.20	0.30		
3	Antisymmetric Ricker	0.20	0.40		
4	Antisymmetric Ricker	0.20	0.50	overturn	overturn
5	Antisymmetric Ricker	0.25	0.20		
6	Antisymmetric Ricker	0.25	0.30		
7	Antisymmetric Ricker	0.25	0.40		
8	Antisymmetric Ricker	0.25	0.50	overturn	overturn
9	Antisymmetric Ricker	0.30	0.20		
10	Antisymmetric Ricker	0.30	0.30		

No.	Excitation type	a_p [g]	T_p [s]	Exp.	Num.
11	Antisymmetric Ricker	0.30	0.40		
12	Antisymmetric Ricker	0.30	0.50		overturn
13	Symmetric Ricker	0.20	0.20		
14	Symmetric Ricker	0.20	0.30		
15	Symmetric Ricker	0.20	0.40		
16	Symmetric Ricker	0.20	0.50		
17	Symmetric Ricker	0.25	0.20		
18	Symmetric Ricker	0.25	0.30		
19	Symmetric Ricker	0.25	0.40		
20	Symmetric Ricker	0.25	0.50		
21	Symmetric Ricker	0.30	0.20		
22	Symmetric Ricker	0.30	0.30		
23	Symmetric Ricker	0.30	0.40		
24	Symmetric Ricker	0.30	0.50	overturn	

Table 3: List of tests with Ricker wavelet excitation

Typical test response is one where the frame uplifts and rocks without overturning, without out-of-plane motion, or stepping or sliding on the rocking surface, and where the superstructure SDOF vibrates in its uplifted frequency. The following deviations in specimen response were observed:

- **Sliding:**
Two different types of sliding were observed. One type was sliding occurring at every impact during rocking and the other type was sliding occurring when there was no rocking. The amount of sliding during the rocking motion was small, and very difficult to detect in measured data (the transition from one edge of the rocking column to the other was usually fast and clear). Sliding while the columns were not rocking was significantly larger and could be detected in measurements. Such sliding occurs because the still-vibrating SDOF superstructure excites the podium frame, causing the relatively light columns to overcome friction at their bases and slide. Restrainers were placed near the column bases to keep such sliding to a minimum (< 0.3 mm) for small and light specimens tested in this study.
- **Out-of-plane movement:**
The large number of rocking interfaces (2 rocking columns with 4 rocking edges each) accompanied by unnoticeable but still present imperfections and asymmetry caused small out-of-plane movements during a few tests, accumulating to 2 mm in the worst case.

4 COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL RESULTS

The objective of the experimental campaign was to validate the analytical model of a rocking podium structure presented in the Section 2. The specimen was tested against 12 different symmetric and antisymmetric Ricker pulses. Figures 9-32 compare the numerical and experimental time histories of the normalized tilt angle (θ/α), the absolute and relative top mass displacement (u_T and $u_T - u_B$), and the accelerations of the ground, beam, and the top mass (a_{ground} , a_{beam} , and a_{top}). For the numerical solution, the measured shaking table (i.e. ground) acceleration was used as input.

Even though some tilt angle time histories were captured very well by the numerical solution, it is generally difficult to predict the entire time history correctly. Unlike elastic systems, the “period” of rocking oscillators strongly depends on their amplitude of vibration. In turn, the amplitude depends on the energy dissipated at each impact. Therefore, any error introduced grows larger, since the solution goes out of phase with the experiment. This confirms the observations of many researchers and suggests that a stochastic (rather than a deterministic) treatment of the rocking problem should be employed.

Nevertheless, for analytical pulses, where the maximum tilt angle occurs in the beginning of the time history, the matching in terms of tilt angle maxima is generally good, even though the shake table controller was not able to enforce a clear pulse motion. Figures 7 and 8 demonstrate this match by comparing the maximum tilt angles. For small absolute tilt angles ($\theta/\alpha < 0.5$) the results compare better. As the tilt angles increase, the negative post-uplift stiffness of the rocking structure affects the highly nonlinear behavior more and more, leading to larger discrepancies between the computed and measured maximum tilt angles.

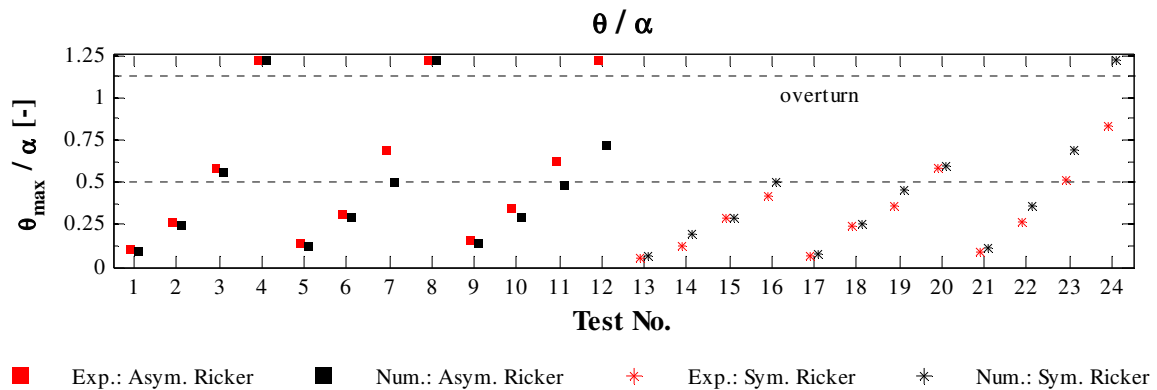


Figure 7: Comparison of computed and measured maximum normalized tilt angle θ/α

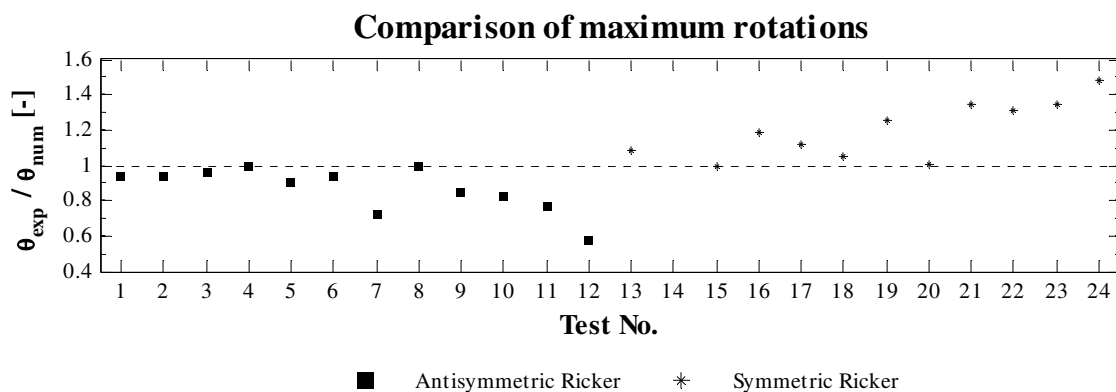


Figure 8: Comparison of computed and measured maximum tilt angle θ

The maximum superstructure deformation ($u_T - u_B$) is also captured quite well, with an average error of 8% (Table 4). The tests highlighted in gray were not taken into account because in these tests either the numerical and/or the experimental model overturned.

Antisymmetric Ricker			Symmetric Ricker		
No.	Exp.	Num.	No.	Exp.	Num.
[-]	[mm]	[mm]	[-]	[mm]	[mm]
1	5.03	6.09	13	4.61	4.39
2	6.48	6.20	14	5.92	6.53
3	8.27	7.49	15	8.14	7.26
4	5.83	4.47	16	8.51	7.60
5	5.93	5.82	17	5.54	4.53
6	8.79	7.77	18	6.57	6.14
7	8.63	7.51	19	8.34	7.10
8	4.54	4.50	20	7.74	7.15
9	6.46	6.16	21	6.69	5.30
10	7.83	6.49	22	8.35	6.82
11	8.55	8.40	23	8.02	7.40
12	8.44	3.74	24	6.63	7.43

Table 4: Comparison of computed and measured superstructure deformation $u_T - u_B$ [mm]. Overturning occurred in tests highlighted in gray.

The recorded motions of the top mass and have a pronounced high frequency component. This component is attributed to the rocking impacts. It cannot be captured by the analytical model, since the impacts excite higher modes of vibration which are not taken into account in the analytical model.

5 CONCLUSIONS

A small-scale model of a rocking podium structure was constructed and tested. A 2-DOF analytical model of a rocking podium structure was derived and verified and validated against the test results. The computed responses compare well to the test results in terms of the maximum tilt angle and peak superstructure deformation, as well as in terms of the eigenfrequency of the superstructure in the uplifted state. The computed and measured response time histories generally do not match equally well because rocking motion is very sensitive to imperfections at the rocking surfaces, imperfections in the specimens and errors in applying the excitation. The above observation urges for a probabilistic treatment of the rocking problem.

6 TIME HISTORIES

Test No. 1

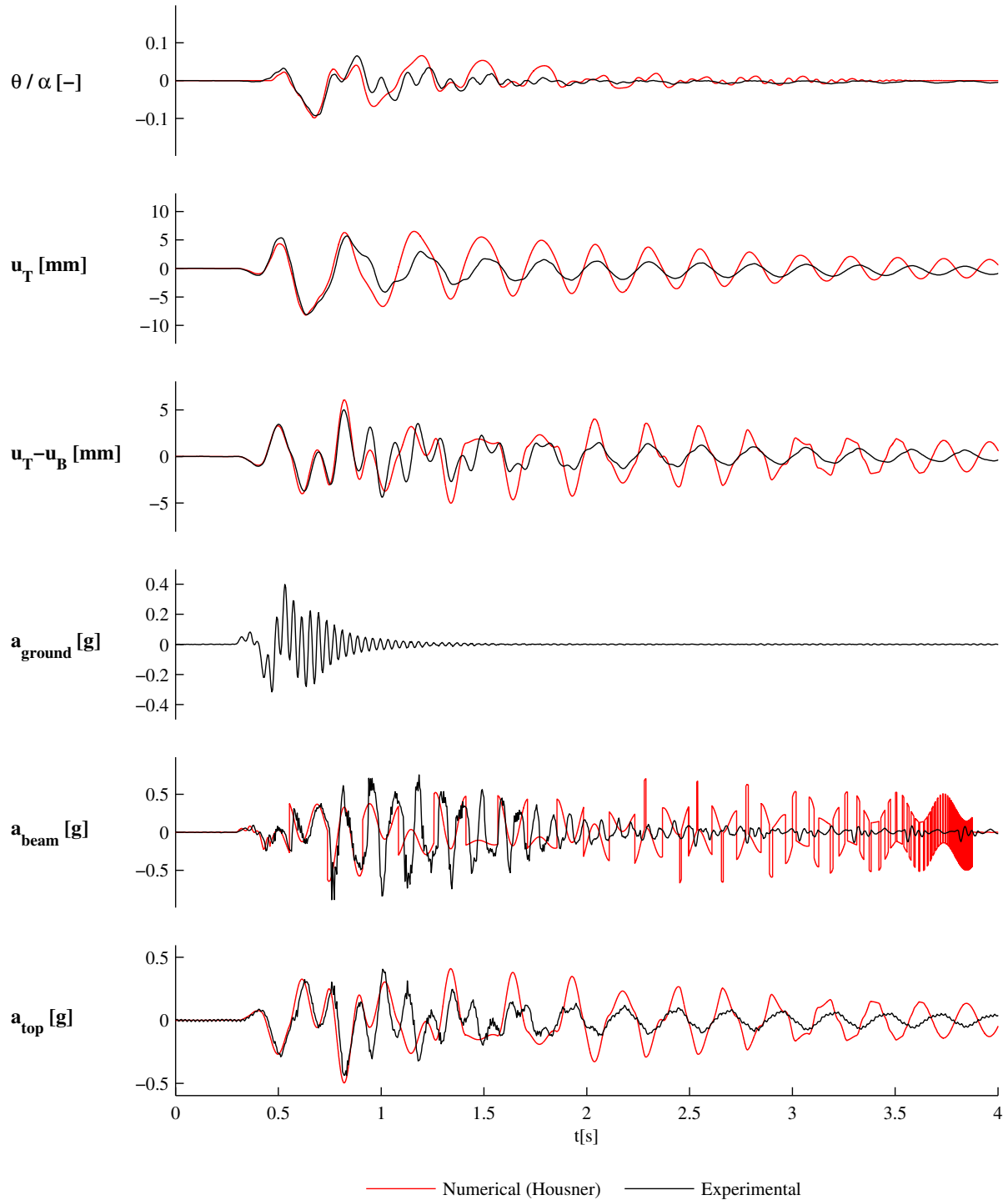


Figure 9: Test No. 1: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

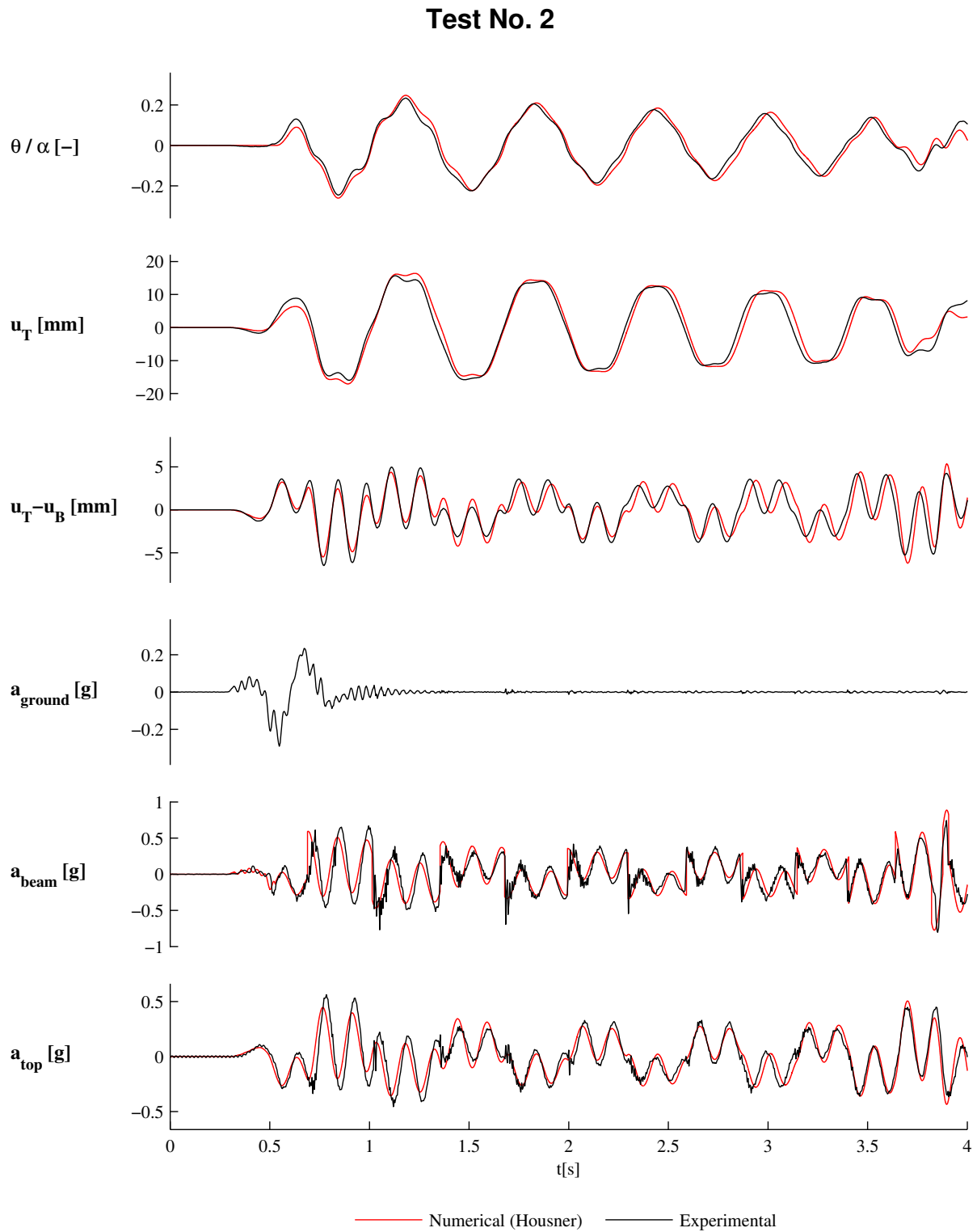


Figure 10: Test No. 2: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

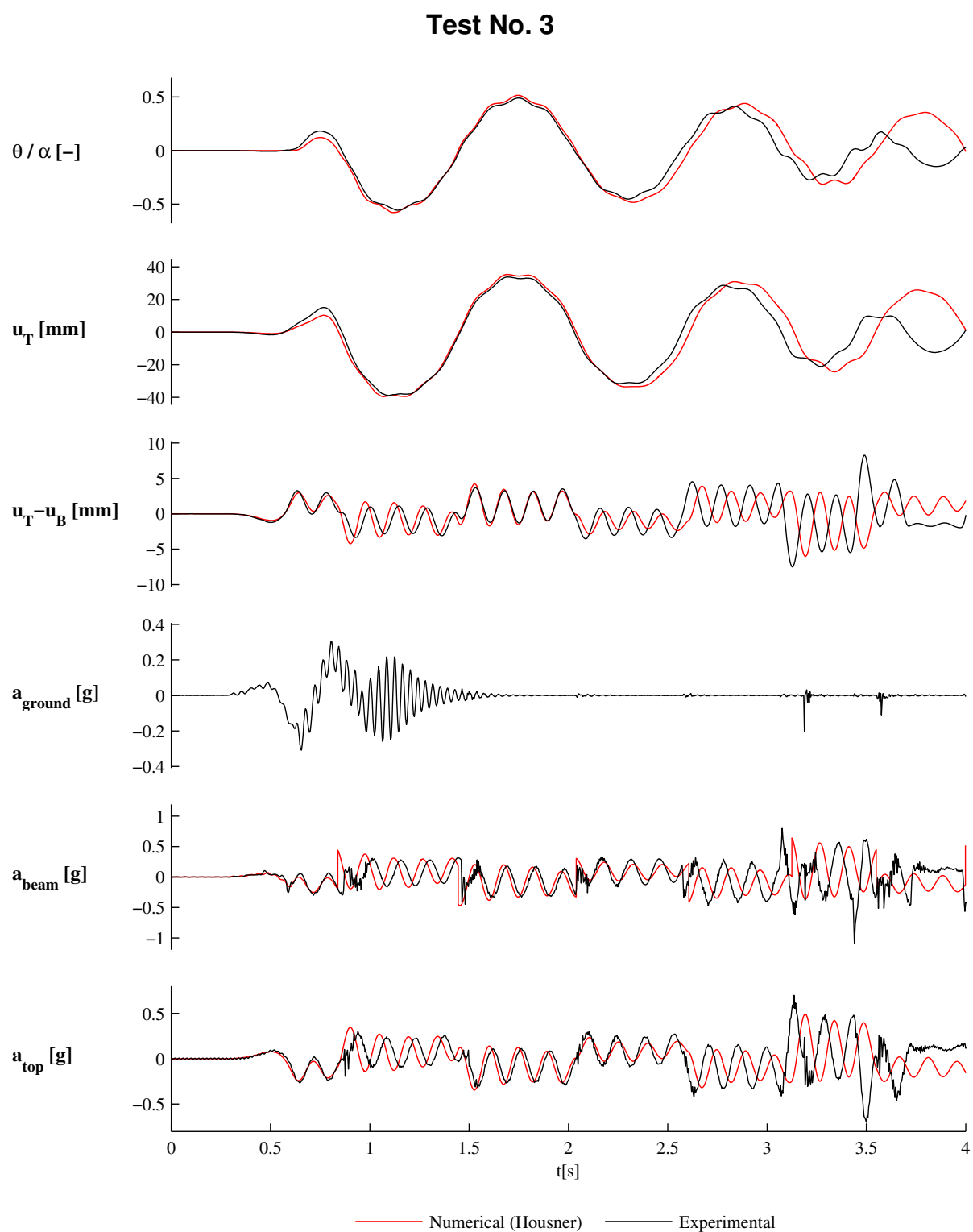


Figure 11: Test No. 3: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement u_T-u_B , ground acceleration, beam acceleration, top mass acceleration

Test No. 4

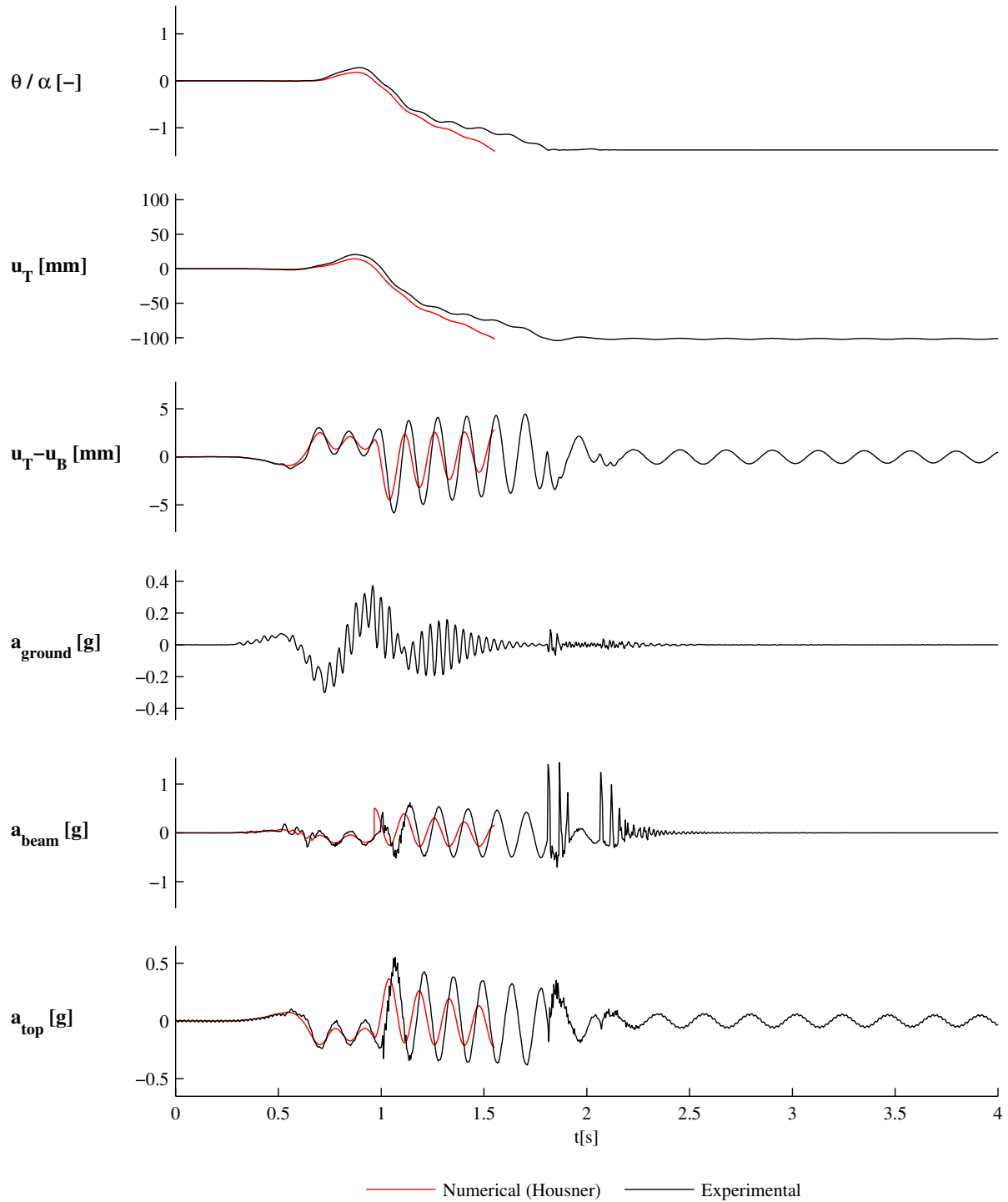


Figure 12: Test No. 4: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 5

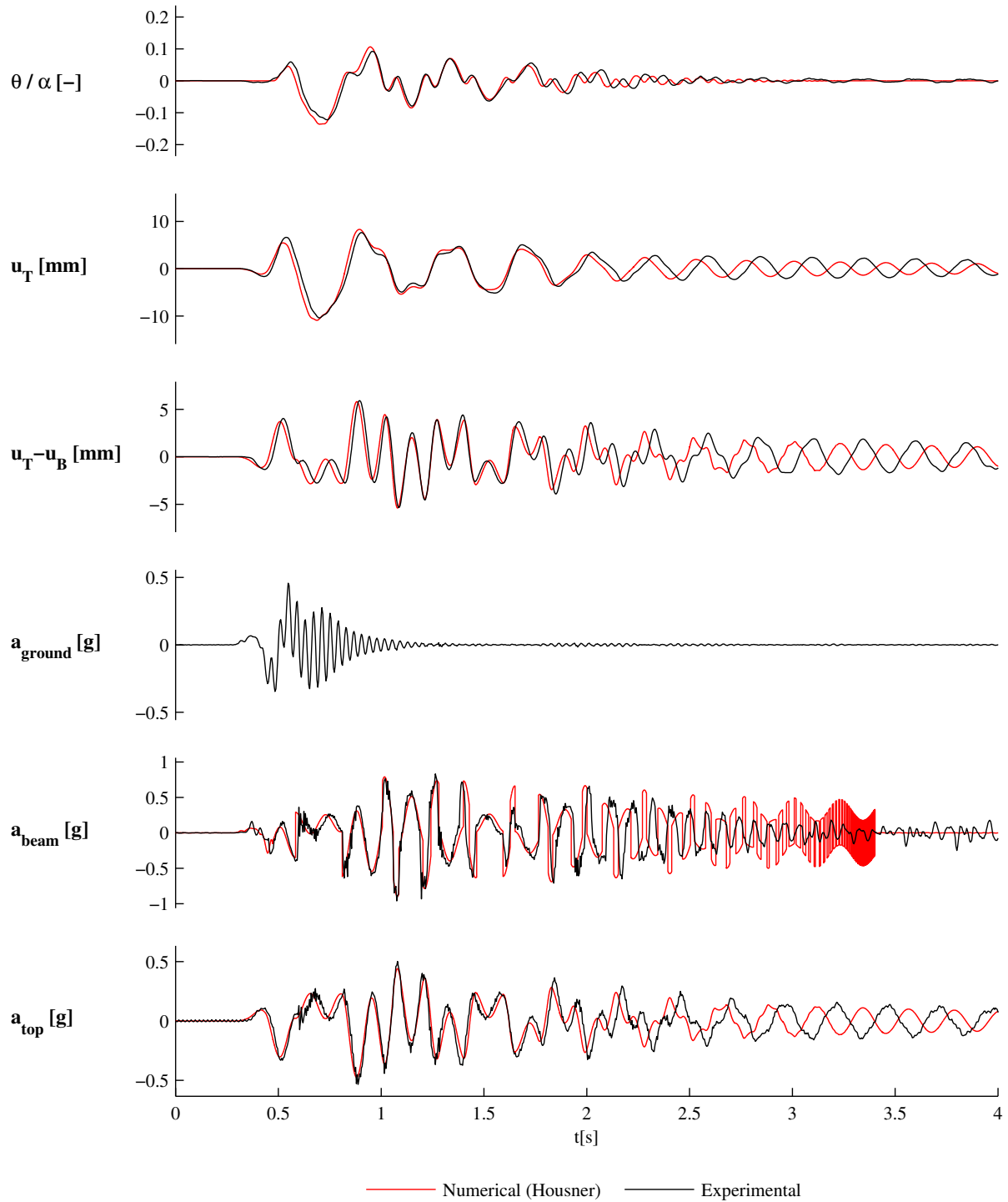


Figure 13: Test No. 5: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

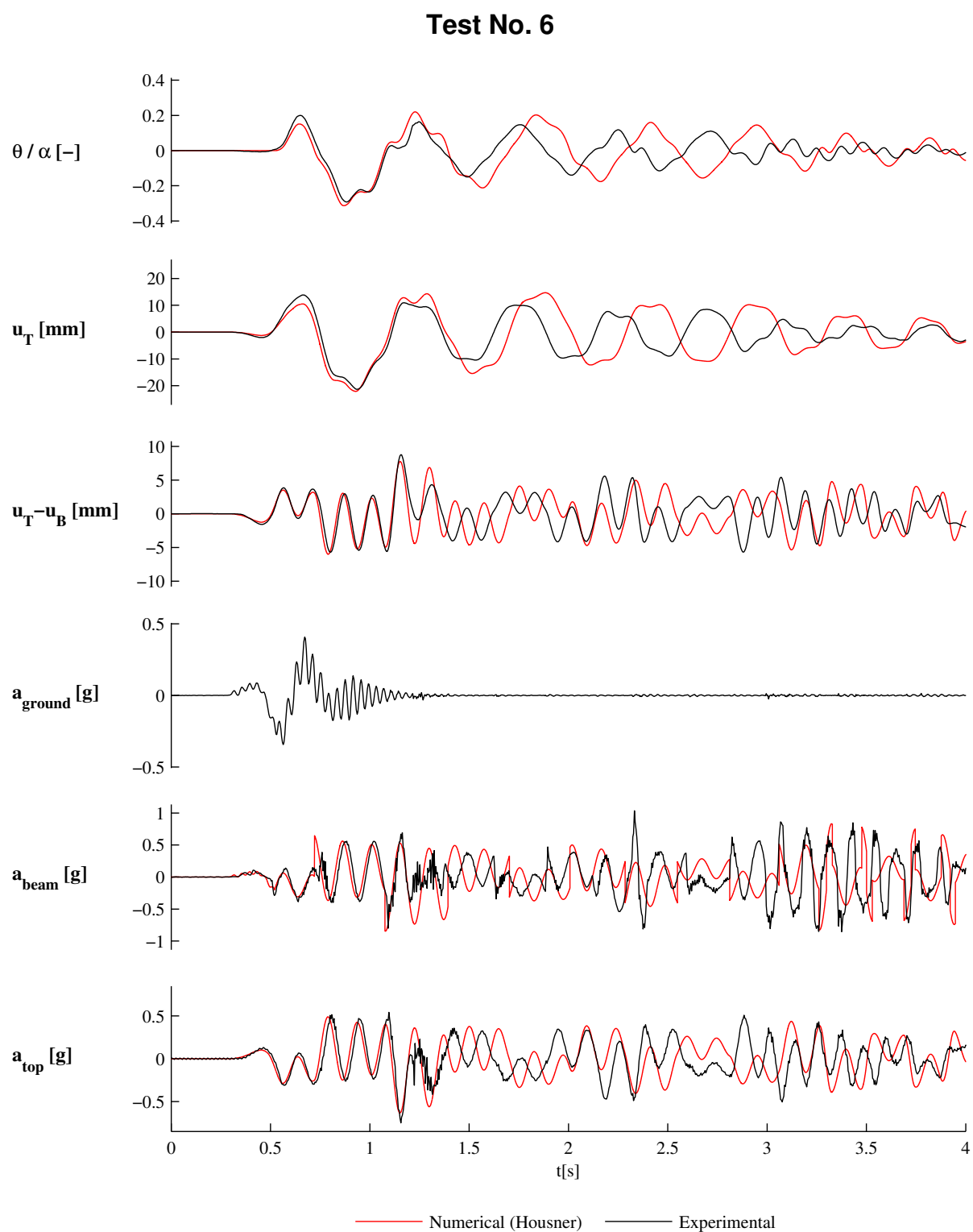


Figure 14: Test No. 6: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

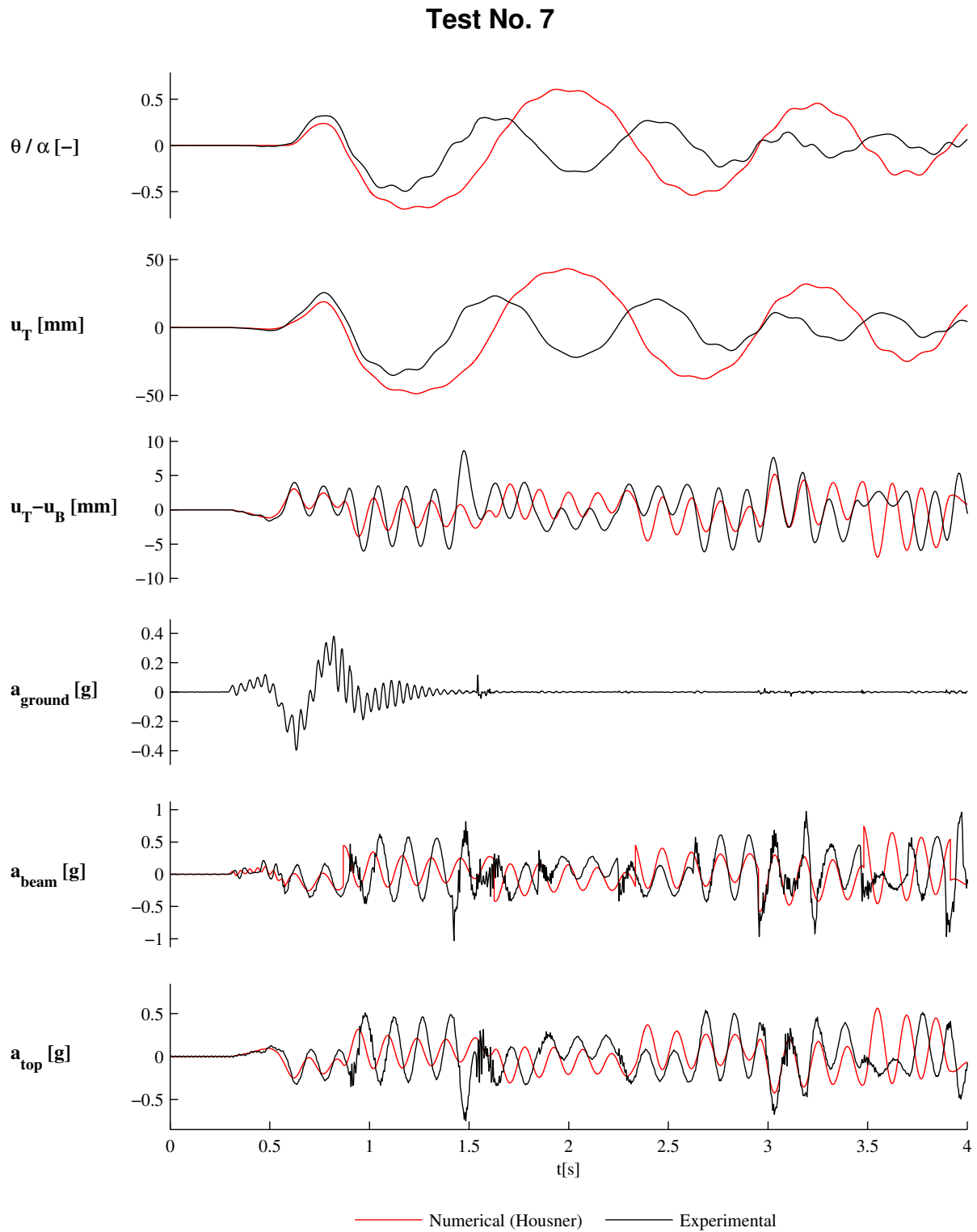


Figure 15: Test No. 7: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 8

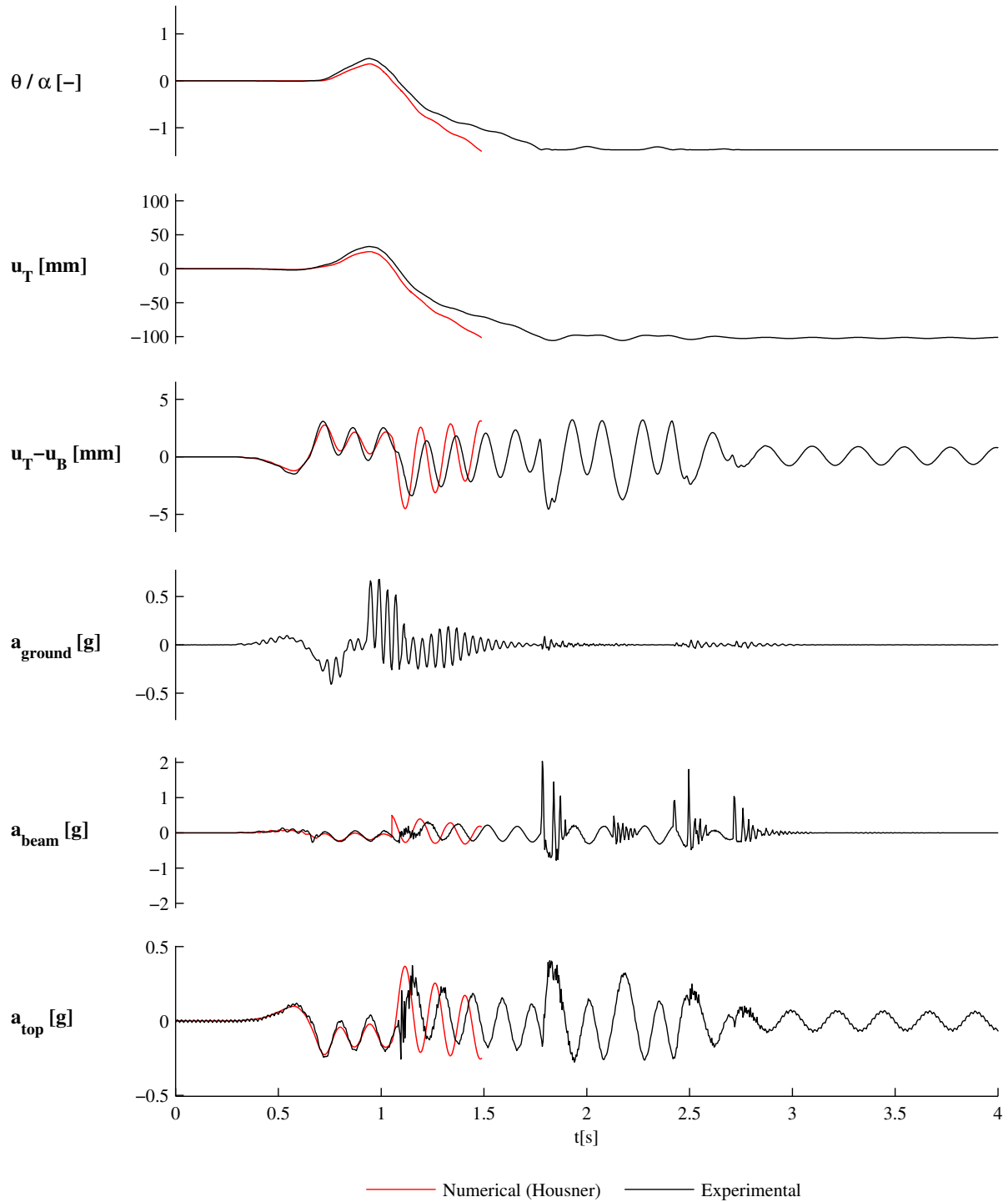


Figure 16: Test No. 8: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 9

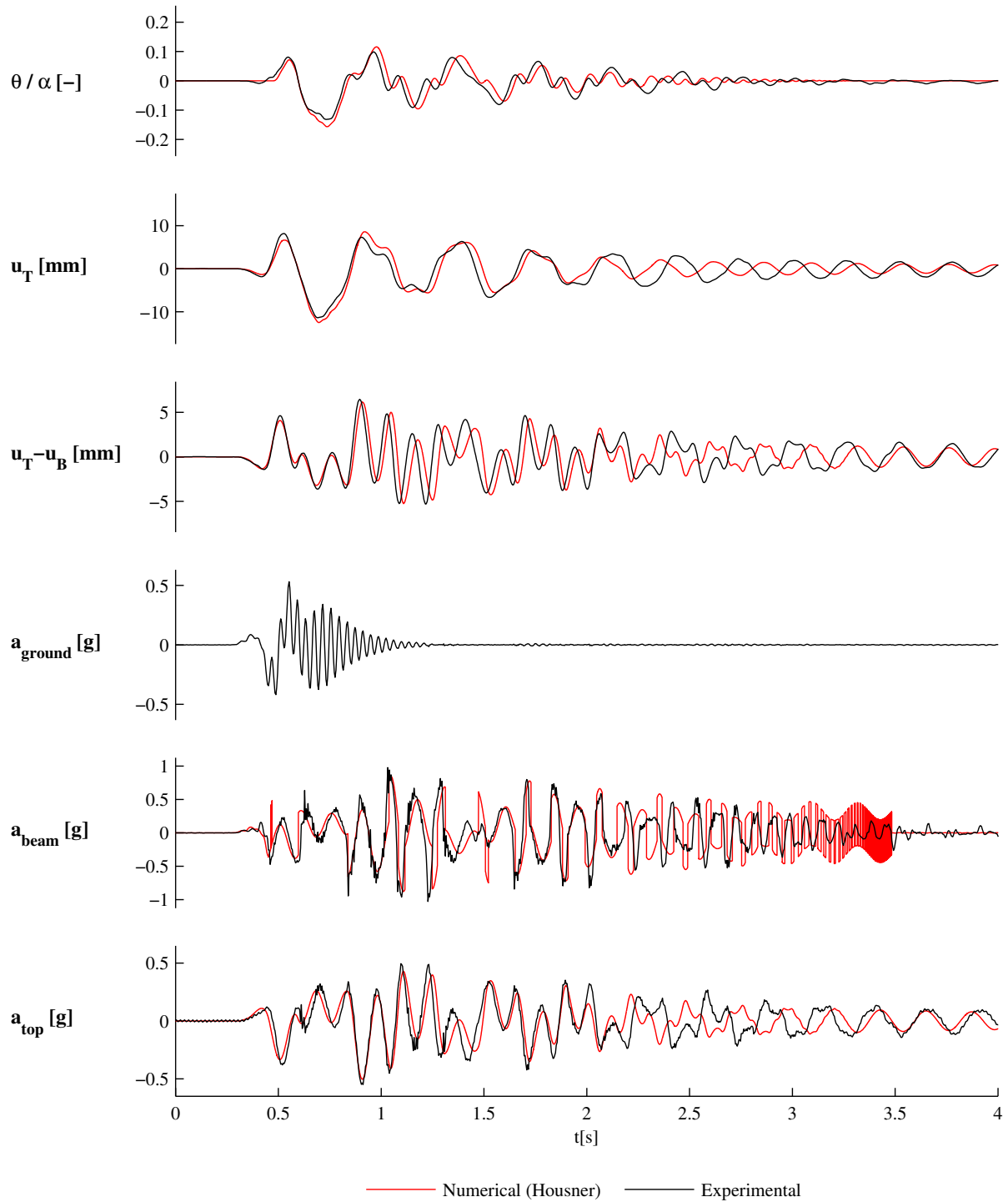


Figure 17: Test No. 9: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

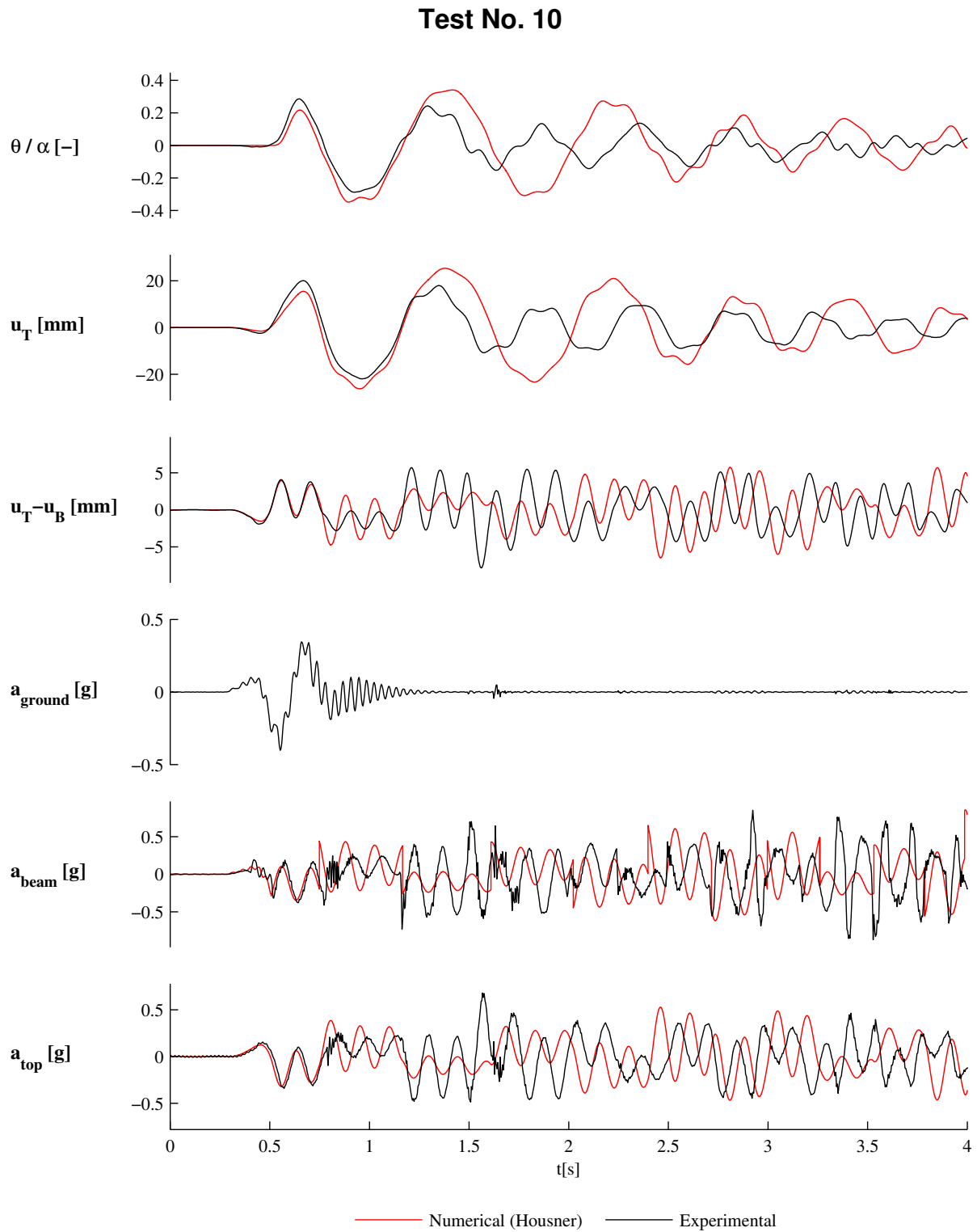


Figure 18: Test No. 10: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

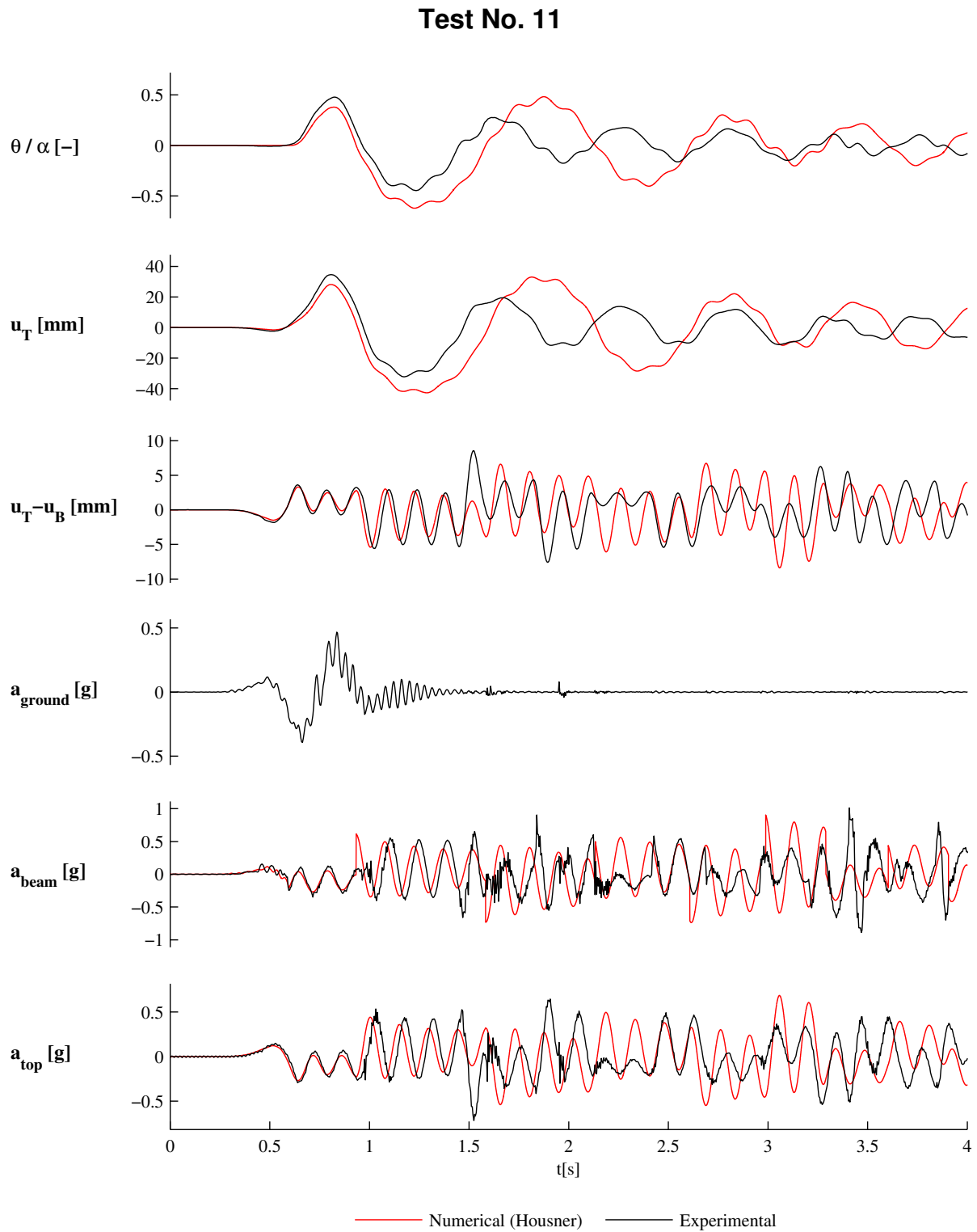


Figure 19: Test No. 11: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 12

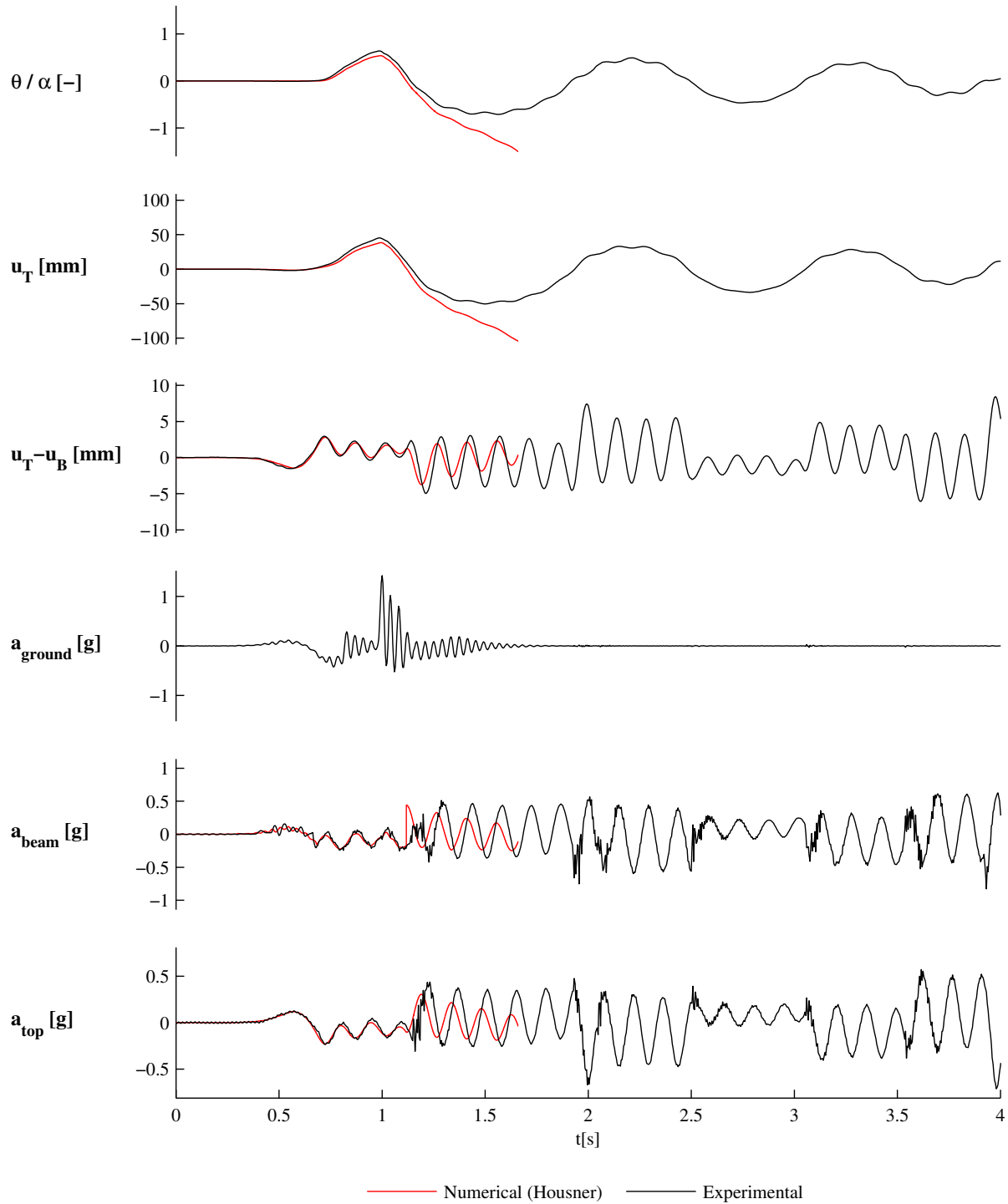


Figure 20: Test No. 12: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 13

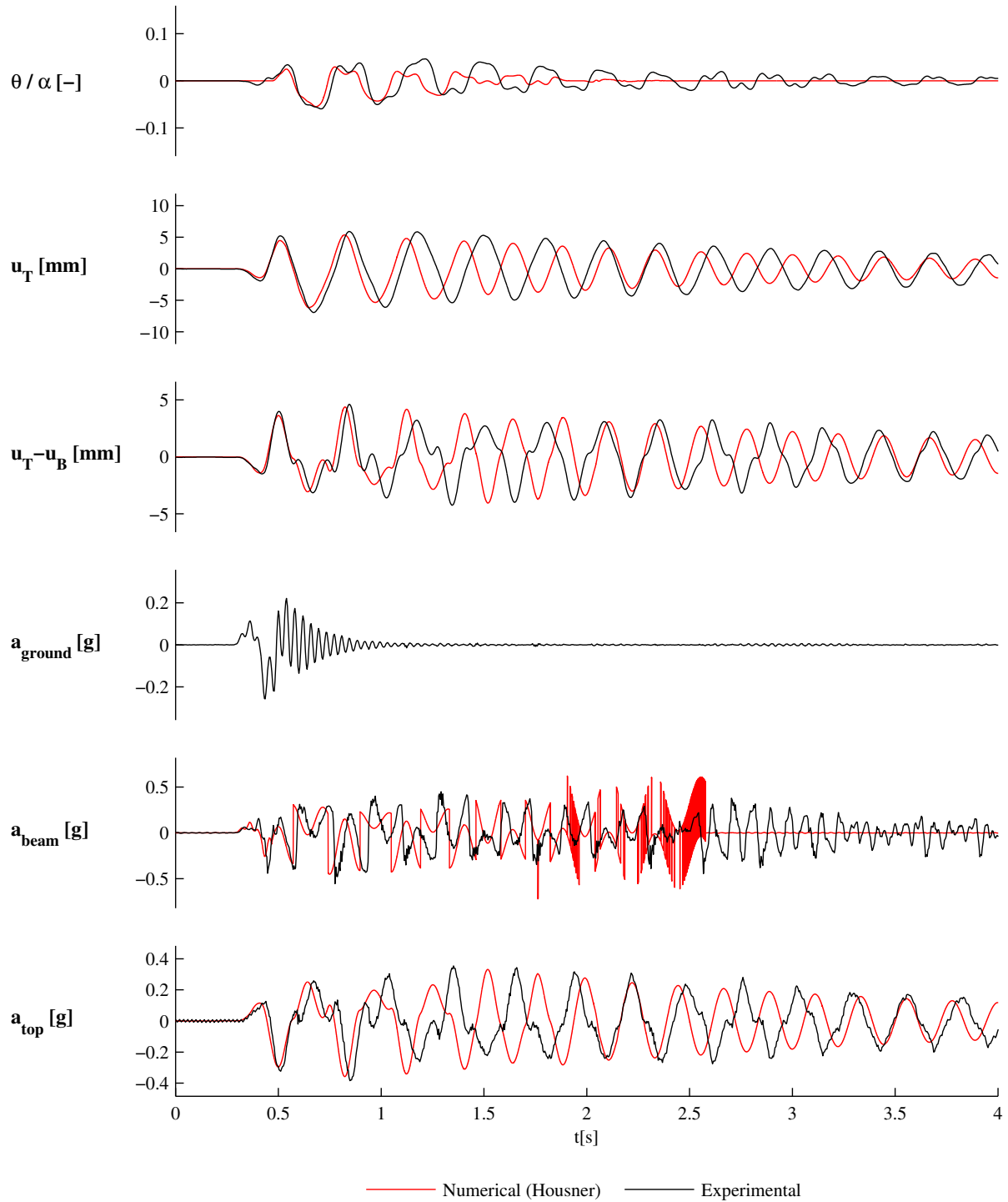


Figure 21: Test No. 13: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 14

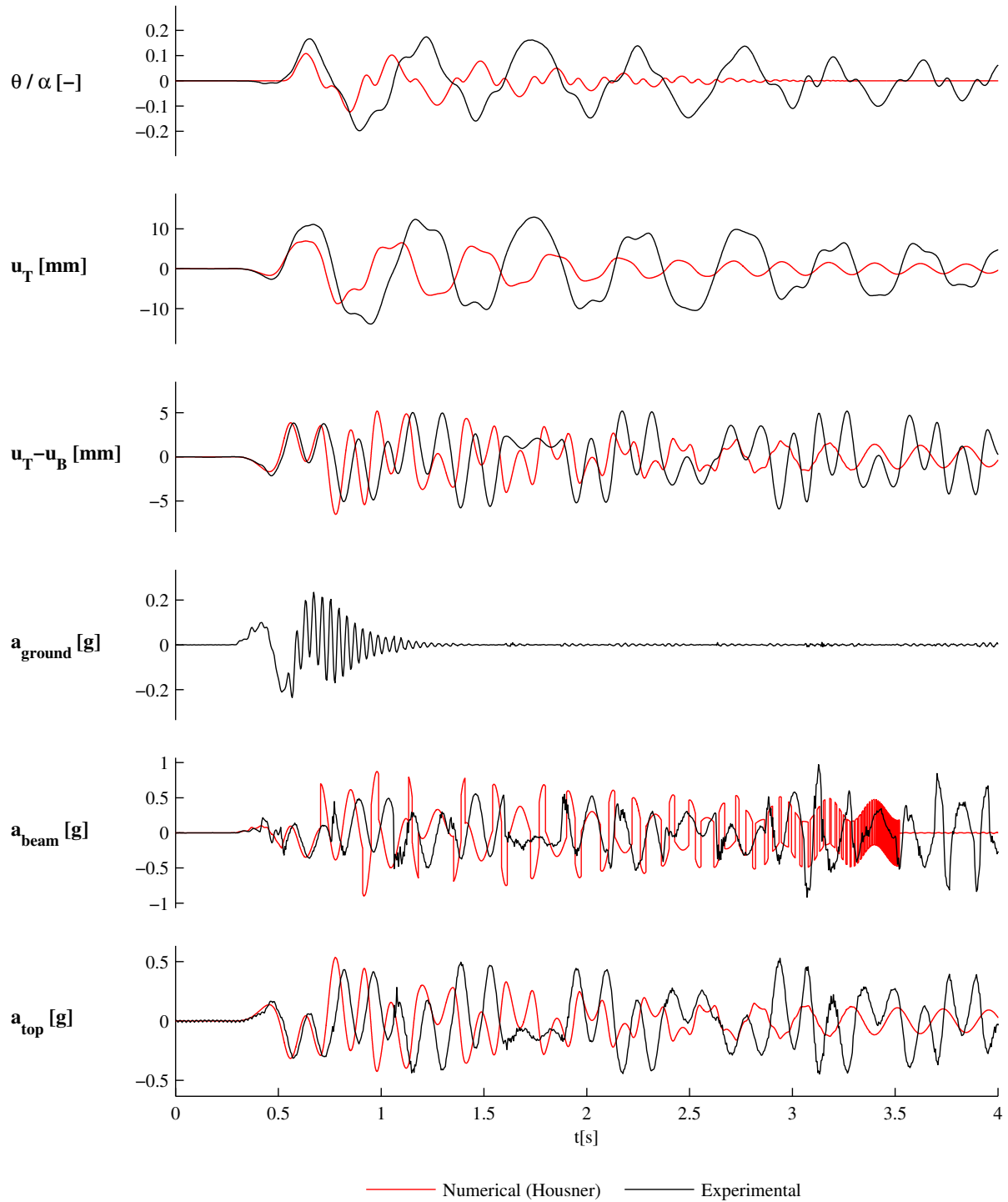


Figure 22: Test No. 14: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

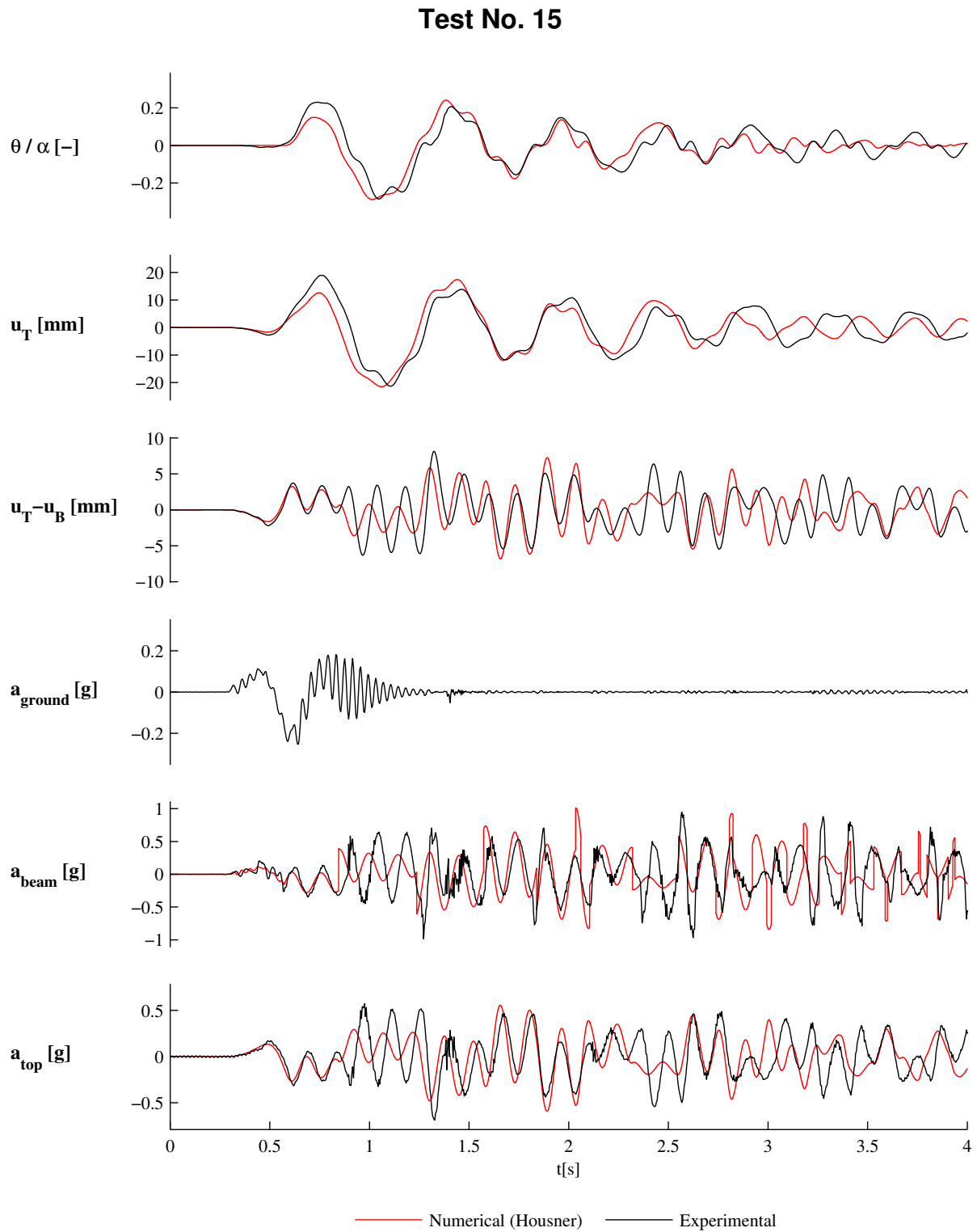


Figure 23: Test No. 15: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

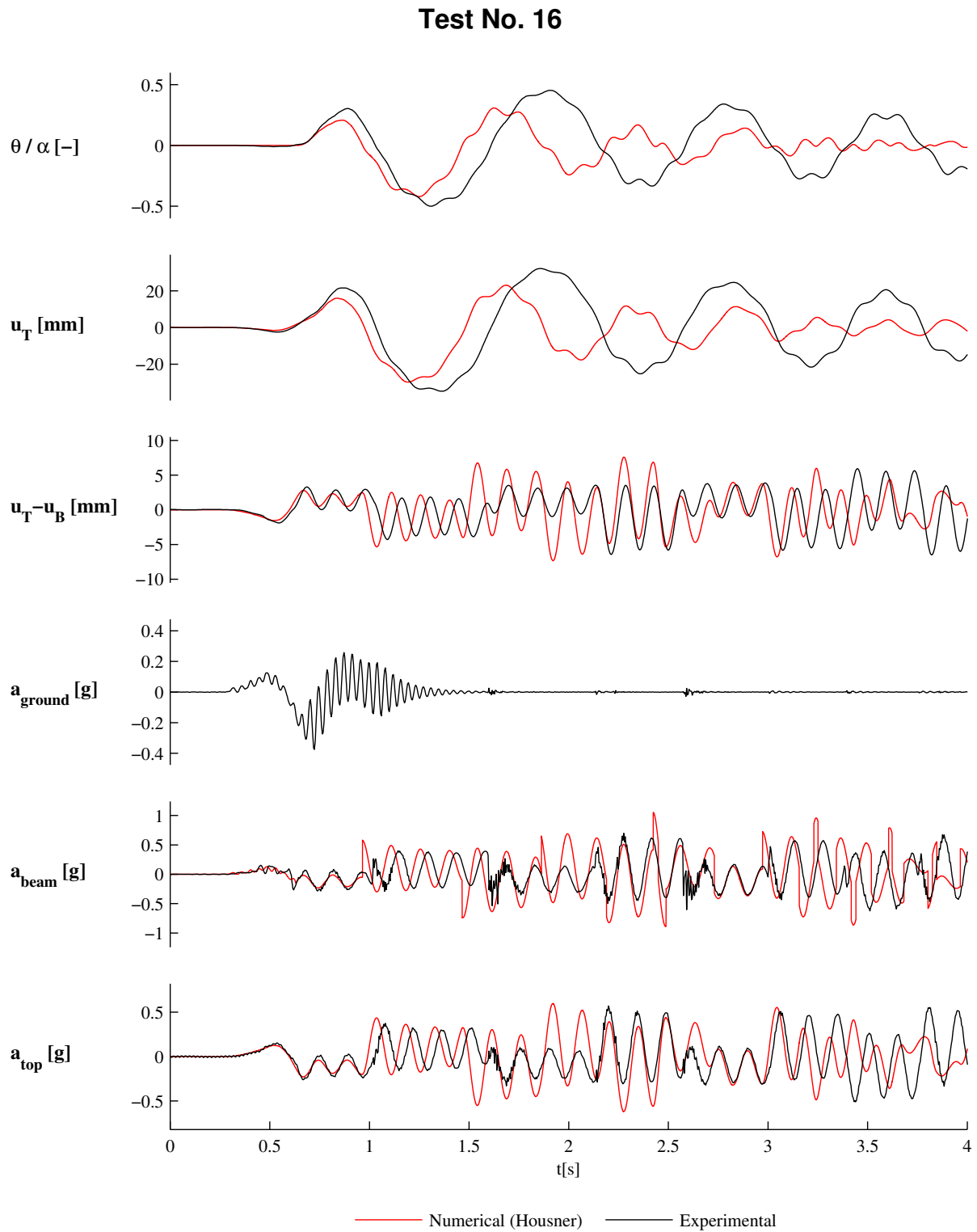


Figure 24: Test No. 16: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 17

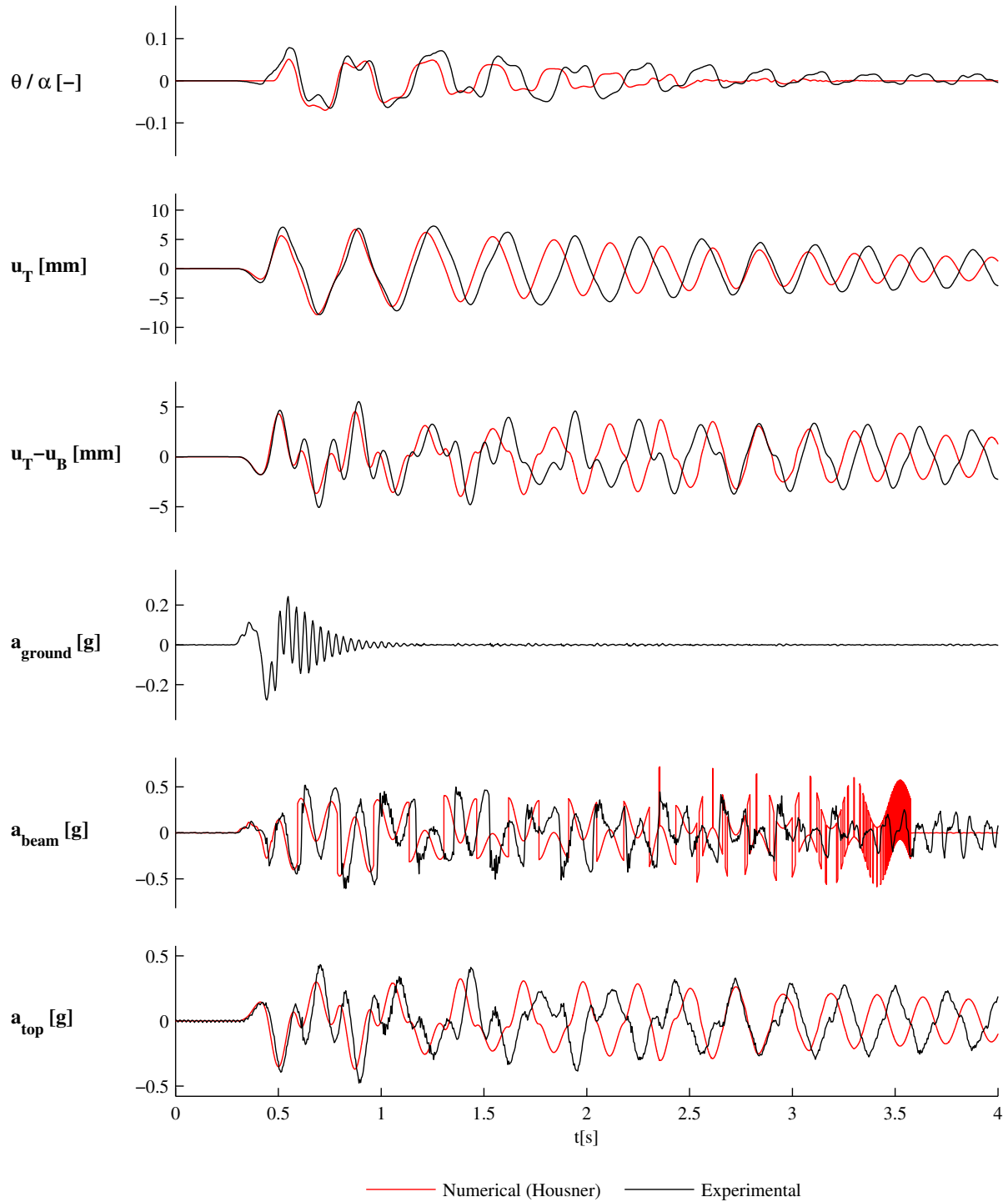


Figure 25: Test No. 17: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 18

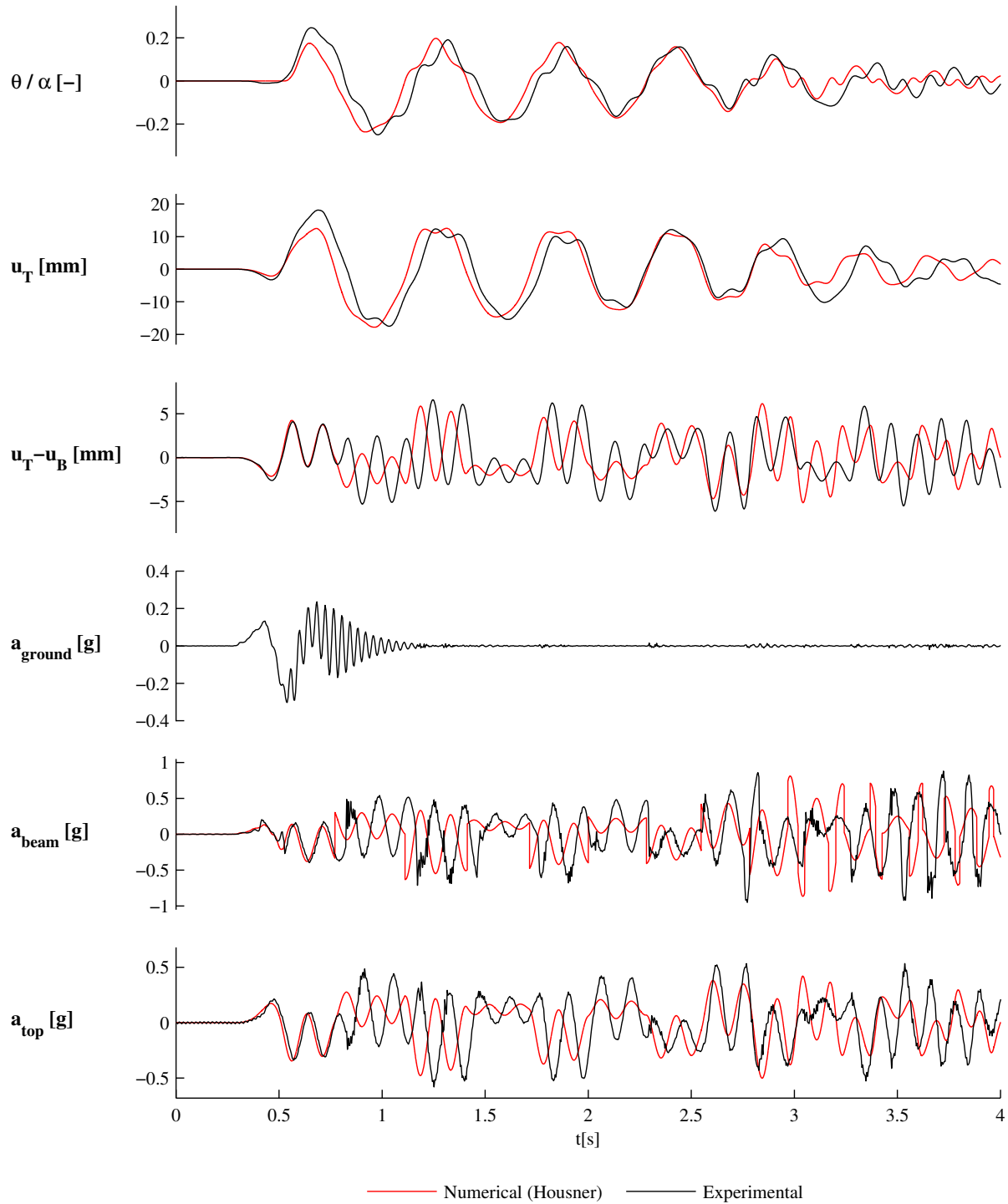


Figure 26: Test No. 18: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

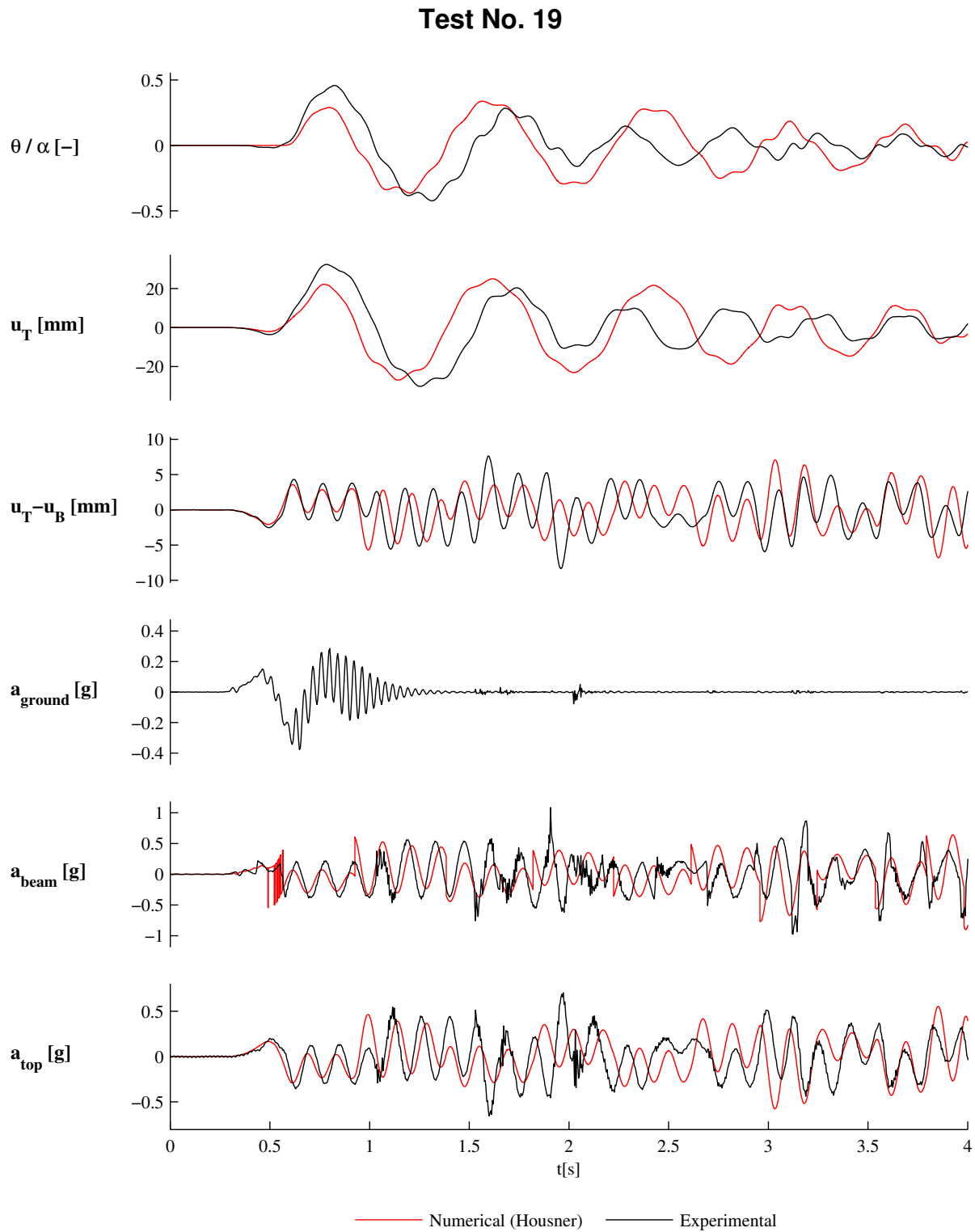


Figure 27: Test No. 19: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

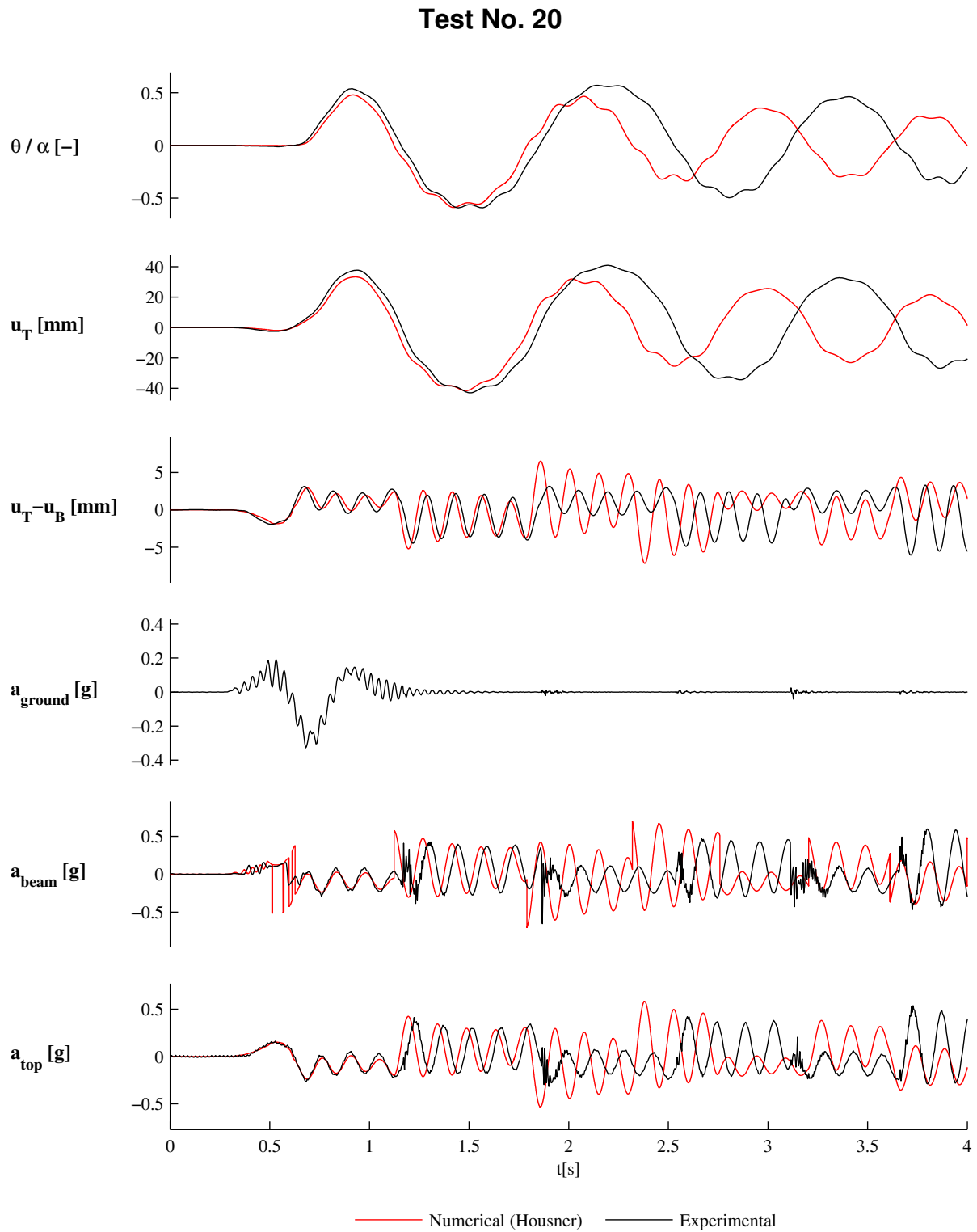


Figure 28: Test No. 20: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

Test No. 21

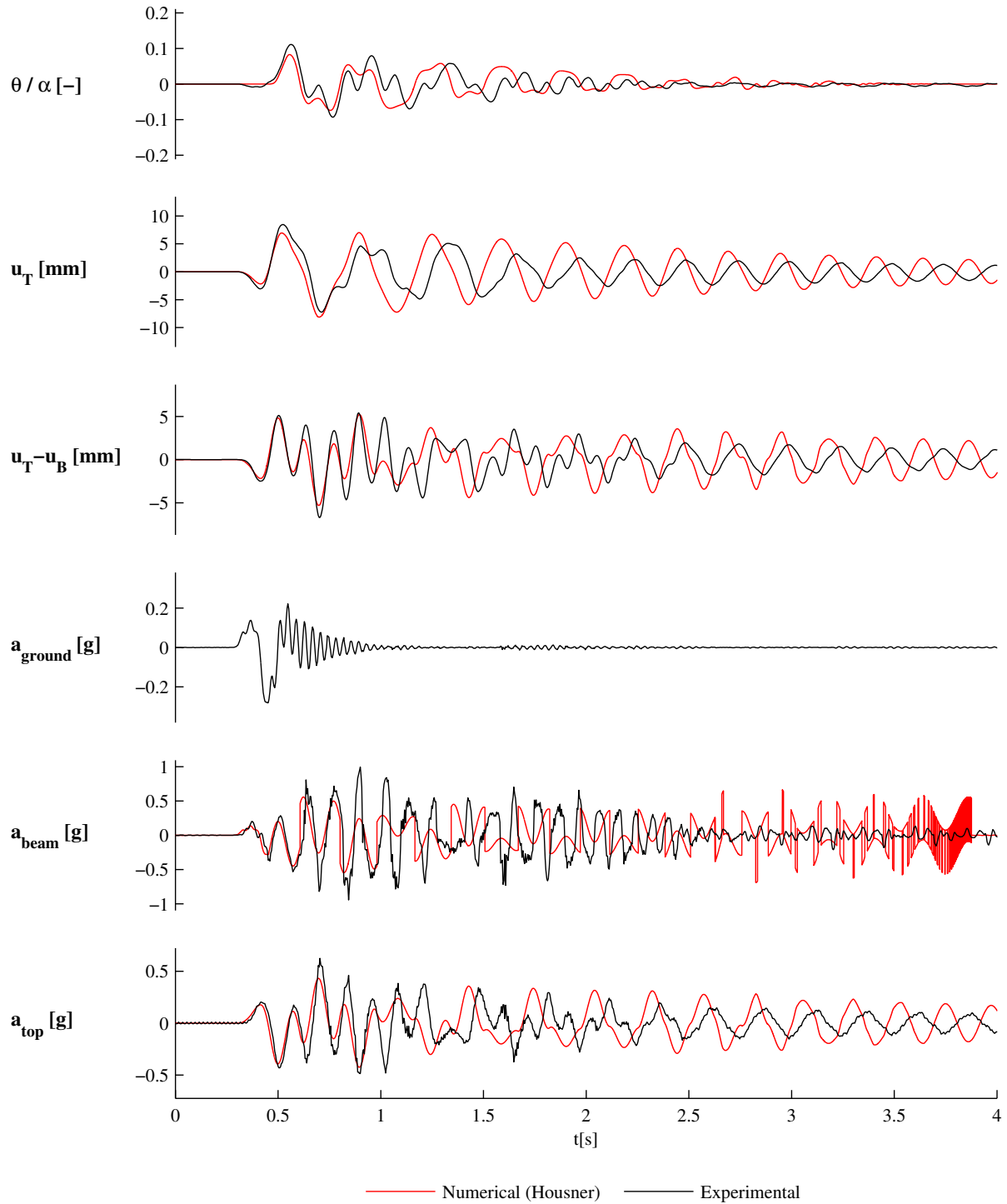


Figure 29: Test No. 21: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

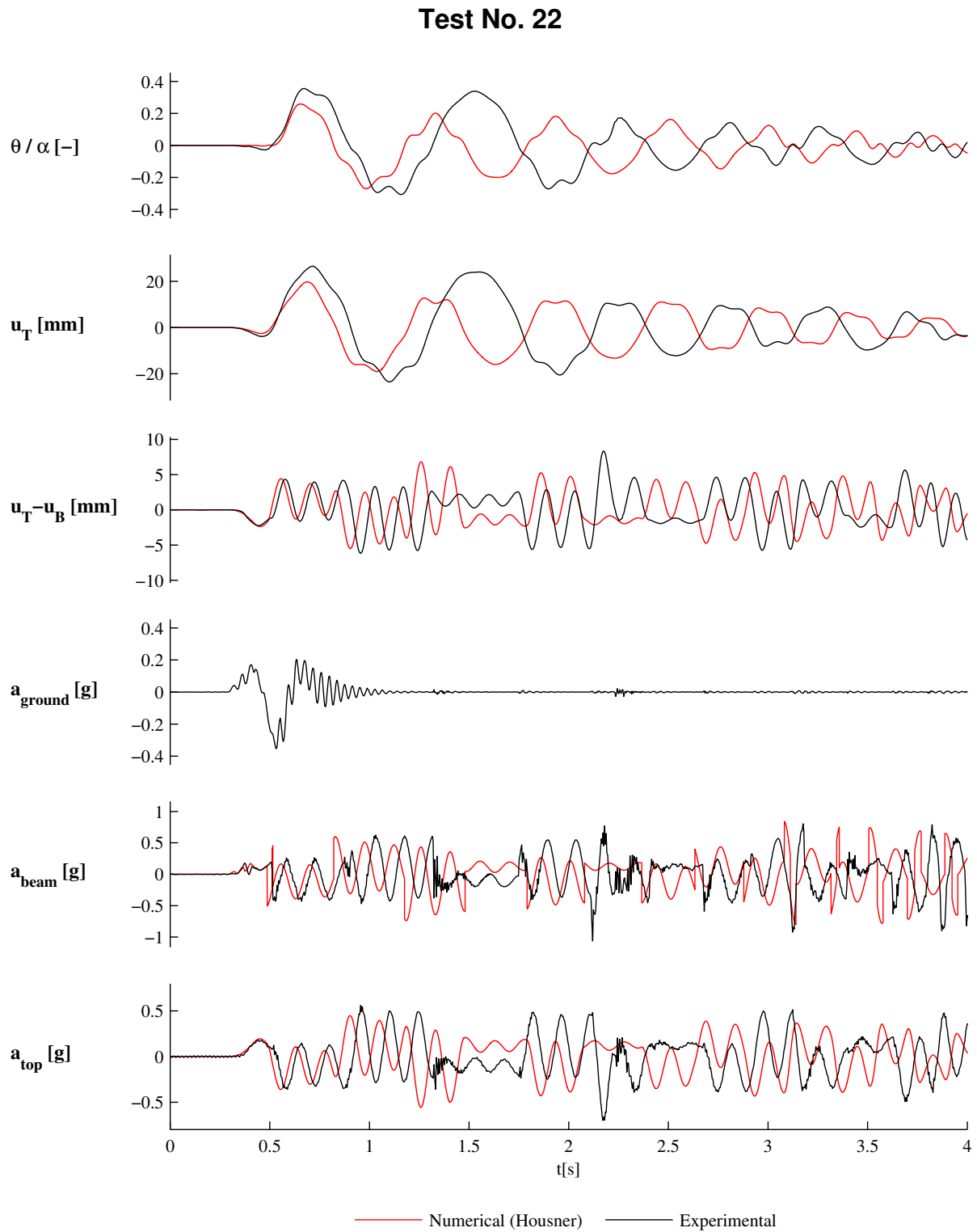


Figure 30: Test No. 22: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

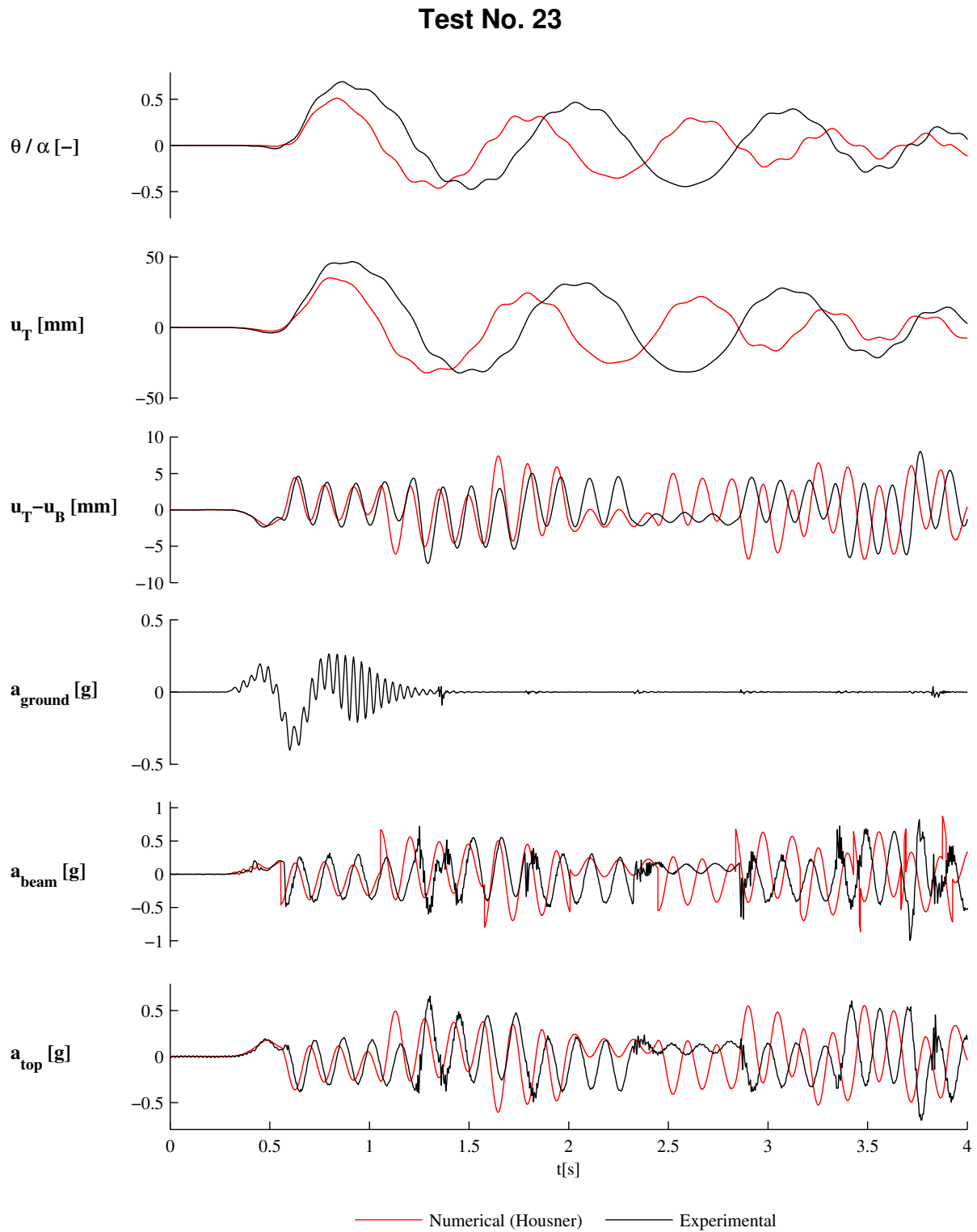


Figure 31: Test No. 23: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

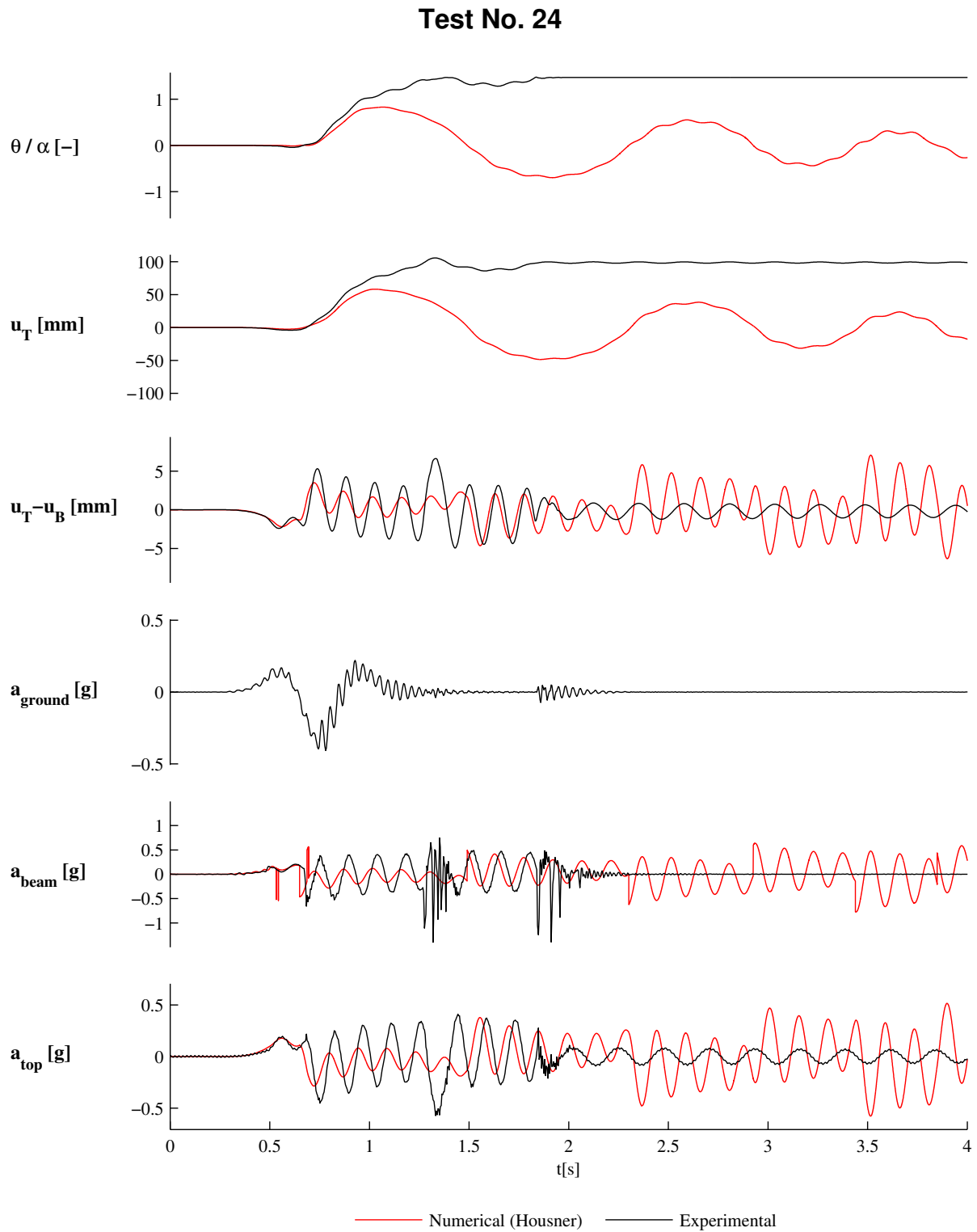


Figure 32: Test No. 24: Time histories for: tilt angle θ/α , top displacement u_T , relative top displacement $u_T - u_B$, ground acceleration, beam acceleration, top mass acceleration

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