ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

ON NUMERICAL METHODS FOR SOLVING RUN-UP PROBLEMS. COMPARATIVE ANALYSIS OF NUMERICAL ALGORITHMS AND NUMERICAL RESULTS

Leonid B. Chubarov[1,2], Alexandr D. Rychkov[1], Gayaz S. Khakimzyanov[1,2], and Yurii I. Shokin[1]

[1]Institute of Computational Technologies SB RAS
6, Lavrentiev Avenue, 630090, Novosibirsk, Russia
e-mail: (chubarov, rych, khak, shokin)@ict.nsc.ru

[2] Novosibirsk State University
2, Pirogova Str., 630090, Novosibirsk, Russia
e-mail: dec@mmf.nsu.ru

Keywords: Surface Waves, Shallow Water, Run-up, Numerical Simulation.

Abstract. *The numerical simulation of the run-up of long surface waves on a plane slope is presented. Using a method based on the combination of the TVD scheme and the SPH method the shallow water approximation is applied to the solution of the well known model problem of a run-up of a wave approaching from an area of constant depth towards a plane slope. The numerical method has proved to be reliable and effective not only in the range of small amplitudes, but also outside of the theoretical limits of applicability of the shallow water theory, such as for the modelling of breaking waves. The qualitative and partially quantitative comparison with the results of numerical calculations of other authors are presented. The differences in the results caused by the differences in the numerical algorithms are highlighted.*

1 INTRODUCTION

The interaction of waves with coasts and coastal installations is one of the most interesting and complex phenomena studied in the framework of the mathematical models of wave hydrodynamics. The investigation of the final phase of tsunami waves' existence – their run-up on coast and run-down – belongs to this class of problems. It is necessary to define the extreme values of run-up, distances of wave propagation to a coast, drying areas, inundation depths, and duration of land's being under water with necessary precision and well ahead of time.

The development of reliable algorithms for the solution of the considered class of problems requires not only the theoretical analysis of the mathematical models and the numerical methods used for their implementation, but also the validation on the test problems. One of the "canonical" problems of this type is the problem on the definition of the parameters of a long wave run-up on a plane slope with the given slope angle, where a wave runs up a slope from the side of the bottom part with a constant depth [1]. It is customary to assume that such problem reflects the main physical aspects of the considered phenomenon with a sufficient completeness.

The current paper is devoted to the application of our combined TVD+SPH method [2] to the solution of the above described "canonical" problem under the conditions, which reconstruct the run-ups of long waves with various configurations and amplitudes on the slopes with different slope angles. Our task is to identify the basic characteristics of the phenomena for weakly nonlinear (non-breaking) as well as for highly nonlinear (breaking) waves. The stability (robustness) of the algorithm has also to be estimated for a wide range of parameter variation.

The first part of the paper briefly describes the problem formulation; the second part presents the results of the numerical modelling of the wave processes, which were investigated experimentally in the Large Wave Flume of the University of Hannover [3]. The obtained results are compared with those of the numerical modelling by the research group of Prof. E.N. Pelinovsky [4].

2 PROBLEM FORMULATION AND NUMERICAL ALGORITHMS

One-dimensional problem of wave run-up on a plane slope adjacent to a horizontal bottom with a constant depth h_0 is considered in the framework of the shallow water model. The Cartesian coordinate system Oxz is used with the vertical axis Oz directed upward and with the coordinate line $z = 0$ coinciding with the unperturbed free surface of an ideal fluid layer. A fluid layer is bounded from below by a fixed bottom $z = -h(x)$, and from above – by a moving free boundary $z = \eta(x, t)$, where t is the time. In this case the system of shallow water equations is written in the form of the conservation laws:

$$\mathbf{q}_t + \mathbf{F}_x = \mathbf{G}, \qquad t > 0, \tag{1}$$

where

$$\mathbf{q} = \begin{pmatrix} H \\ Hu \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Hu \\ Hu^2 + gH^2/2 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} 0 \\ gHh_x \end{pmatrix},$$

g is the gravity acceleration. The sought values are the full depth of a fluid layer $H(x,t) = \eta(x,t) + h(x) \geq 0$ and $u(x,t)$ – its velocity, which is vertically averaged from bottom to free surface.

In [2] the channel was considered, where the plane slope was positioned to the left from the bottom part with a constant depth. However, in the present work the plane slope is placed to the right from the horizontal segment, making the comparison to the results of other authors more convinient. In this case the solution domain is limited from the left by a impermeable wall at the point $x = 0$, and from the right – by a moving boundary $x_0\,(t)$, which separates water from land (waterfront point). A moving boundary is unknown, therefore, the number of sought functions increases.

The problem for the equation (1) is closed by the initial and boundary conditions. Under the assumptions that the initial position $x_0\,(0)$ of the waterfront point is known and the wave moves from left to right, the boundary conditions take the following form:

$$H\,(x_0(t), t) = 0, \quad u\,(0, t) = 0, \quad t \geq 0; \tag{2}$$

$$\eta\,(x, 0) = \eta_0\,(x)\,, \quad u\,(x, 0) = u_0\,(x)\,, \quad 0 \leq x \leq x_0\,(0)\,. \tag{3}$$

The relief of bottom and adjacent land is given by the function

$$z = -h(x) = \begin{cases} -h_0, & 0 \leq x \leq x_s, \\ -h_0 + (x - x_s)\tan\,\beta, & x_s \leq x \leq L_x, \end{cases} \tag{4}$$

where $\beta > 0$ is the plane slope angle, $x_s > 0$ is the given abscissa of the transition point between the inclined and the horizontal bottom segments, $L_x = x_s + (z_0 + h_0)\cot\beta$, $z_0 > 0$ is the land height at the point $x = L_x$. The value z_0 is chosen so that the maximal vertical run-up of a wave on a coast is less than z_0. Therefore, the inequality $x_0\,(t) < L_x$ is satisfied for all $t > 0$.

In our investigations presented in this paper, we have used two numerical methods of the second accuracy order for modelling of wave run-up on a coast. The first method is based on the approach suggested in [5] and represents an original combination of the SPH (smoothed-particle hydrodynamics) method and the finite-difference scheme with TVD properties (named below as the TVD+SPH method). We supplemented this method with the algorithm for computing a moving waterfront point. The comparative analysis of the methods for modelling of long surface wave run-up in the framework of the shallow water theory [2] has shown that the TVD+SPH method appears to be the most advanced for this class of problems.

The second method, conventionally named as "exact", is presented in [6, 7]. It uses a moving grid, and the analytical solution of the problem in a small vicinity of a waterfront point is applied to compute its position and velocity.

3 NUMERICAL RESULTS

Let us consider some results of the numerical solution of the test problem, where the characteristics of run-ups of positive and negative pulses (positive and negative polarity correspondingly) on a slope are defined for a wide range of initial wave amplitudes. We compare our numerical results with the results of other authors. In particular, qualitative and quantitative comparison is done with the results of the research group of Prof. E.N. Pelinovsky [4, 8, 9], which include the analytical solutions (for non-breaking waves) and the data of laboratory and numerical investigations. The experiments were performed in the Large Wave Flume of the University of Hannover, using a wave flume consisting of a segment with a constant depth $h_0 = 3.5$ m and length $x_s = 250$ m, adjacent to a plane slope $1 : 6$ which was positioned near the right boundary of the flume [3]. The numerical computation in [4] were done using the software package CLAWPACK [10].

The initial data were given by the relations

$$\eta\,(x, 0) = A\cosh^{-2}\Big[(x - x_w)/L\Big], \quad u\,(x, 0) = 2\left[\sqrt{g\,(h_0 + \eta\,(x, 0))} - \sqrt{gh_0}\right]. \tag{5}$$

As it is shown in [11, 12], these relations correspond to the exact solution (Riemann wave) of the shallow water equations

$$H\,(x,t) = H_0\,[x - V\,(H)\,t]\,, \quad V\,(H) = 3\sqrt{gH} - 2\sqrt{gh_0}, \quad H_0\,(x) = h_0 + \eta\,(x,0)\,.$$

The authors of [4] interpret the value L as the half of the length of the initial elevation and take $L = 11$ m in their computations. The extremal values of surface displacements (wave amplitudes) were initially positioned near the point $x_w = 50$ m, and they varied in the range from 0.05 to 3.5 m for the waves with positive polarity, and in the range from -0.05 m to -3.49 m – for the waves with negative polarity.

In comparison to [4], we have additionally modelled the waves with smaller initial amplitudes, namely 0.001 m, 0.0025 m, 0.005 m, 0.01 m. Let us note that the applicability limits for the shallow water theory, which are defined by the possibility of using the analytical formula of Synolakis, are bounded in our computations by the interval of initial amplitude variation $0.008064 \leq A < 0.228976576$. However, the results for the amplitudes which fall outside the limits of the given interval, can be used for testing new algorithms for wave run-up modelling in the framework of the shallow water model. At the least, these results explicitly point out that the used numerical methods can work even outside the theoretical limits of applicability of the approximated mathematical model.

We have chosen the display format for the results according to the examples provided in [4]. Unfortunately, some peculiarities of the formulations of numerical modelling problems [4] do not allow direct quantitative comparison of the results, but qualitative comparison is possible.

The first series of graphs (Fig. 1) shows the profiles of free surface computed for the same set of amplitudes as in [4]. Each graph presents the waves at the initial time moment (lines (*1*)); near the transition point between the inclined and the horizontal bottom segments (lines (*2*), which correspond to the time moment $t = 30$ s for the amplitudes $A = 0.1$ m and $A = 0.5$ m, and $t = 20$ s – for $A = 1.5$ m and $A = 3.5$ m); at the moment of nearly vertical run-up (lines (*3*), which correspond to the time moment $t = 40$ s for the amplitudes $A = 0.1$ m and $A = 0.5$ m, and $t = 30$ s – for $A = 1.5$ m and $A = 3.5$ m); and at the moment of approach of the wave, reflected by the slope, to the opposite boundary of the computational domain (lines (*4*), which correspond to the time moment $t = 70$ s for the amplitudes $A = 0.1$ m and $A = 0.5$ m, and $t = 60$ s – for $A = 1.5$ m and $A = 3.5$ m). Let us note that for small amplitudes our results are not only in qualitative agreement with the results from [4], but in quantitative agreement as well; whereas the differences in free surface profiles, which evolve after the interaction of waves with slope, significantly increase with the increase of initial amplitudes.

The influence of the non-linearity can be already seen for the wave with the smallest initial amplitude. The leading edge of the wave gradually becomes steeper, but wave breaking (gradient catastrophe) does not happen (Fig. 1 (*a*)), the amplitude of the wave, which propagates above the horizontal bottom segment, does not essentially decline. After the reflection from the slope the wave form changes significantly and in fact changes into the N-wave, which contains the fragment of depression lower than zero level.

With the increase of an initial wave amplitude the influence of non-linear effects on wave characteristics is also enhanced. Under the impact on non-linearity the leading edge of waves become significantly steeper, whereas trailing edges flatten out so that the wave lengths increase and the amplitudes decline. As a result the already "breaking" waves approach a plane slope and run up to it. These considerations are proved by the Figure 2, which presents the characteristics of the computed incident and reflected waves above the flume segment with a constant

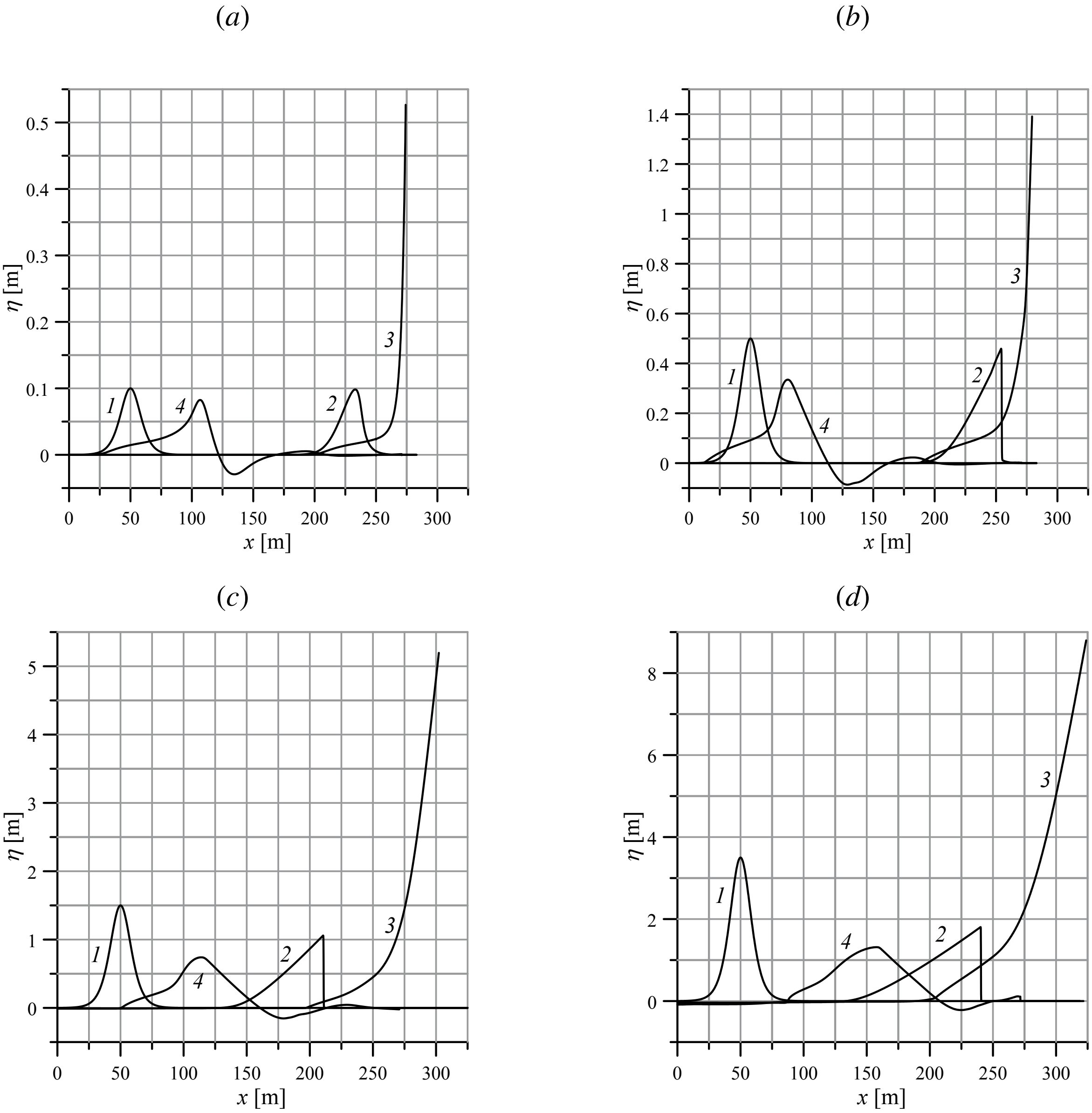

Figure 1: The free surface profiles for run-up on a plane slope for the waves with positive polarity and the amplitudes $A = 0.1$ m (*a*), 0.5 m (*b*), 1.5 m (*c*), 3.5 m (*d*) at the initial time moment, near the transition point between the inclined and the horizontal bottom segments, the moment of nearly vertical run-up, the moment of approach of the wave, reflected by the slope, to the opposite boundary of the computational domain. The values of time moments are given in the text

depth (Fig. 2 (*a*)) and above the transition point between the inclined and the horizontal bottom segments (Fig. 2 (*b*)). Fig. 2 (*a*) shows that incident and reflected waves are time-spaced, the incident waves have vertical leading edges, whereas the reflected waves have a smooth form. Above the transition point between the inclined and the horizontal bottom segments the waves are not time-spaced, here an incident wave meets a reflected wave before completely passing this point $x = x_s$.

The comparison of the results from the Fig. 3 (*a*) with the results from [4] demonstrates not only quantitative but also significant qualitative differences. The run-up heights computed with the TVD+SPH method are much larger, whereas the duration of run-up is much shorter. Besides, in contrast to the results of [4], the dependencies of maximal (in absolute magnitude) run-

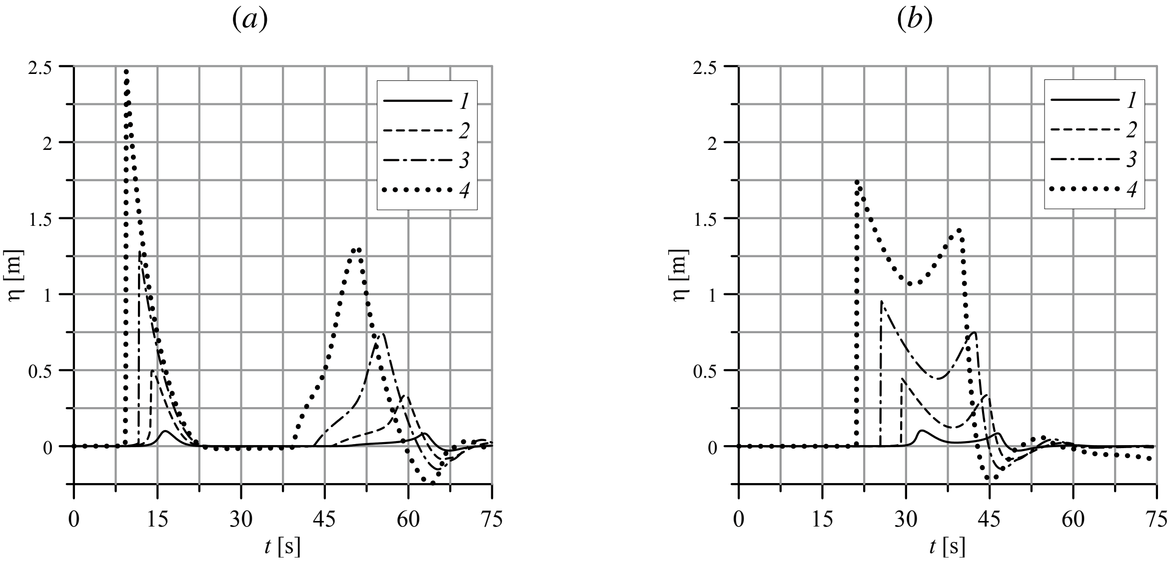

Figure 2: The mareograms, computed above the segment with constant depth of the virtual wave flume at the point $x = 150$ m (*a*), the transition point $x = 250$ m between the inclined and the horizontal bottom segments (*b*), for the run up of the waves with initial amplitudes $A = 0.1$ m (*1*), 0.5 m (*2*), 1.5 m (*3*), 3.5 m (*4*)

downs on initial amplitudes are monotone, and the dependencies of vertical run-ups are smooth for all amplitudes. Finally, in [4] the absolute values of run-downs change non-monotonically and decrease with tending to zero (with that being in poor agreement with the ideas of the physics of modelled processes) for the largest initial amplitudes starting from $A = 2$ m. In the results presented in 3 (*a*) these values increase slowly, but monotonically. There are also differences in velocities: for the same conditions the run-up and run-down processes happen significantly slower in [4].

Some of the data obtained by the solution of the considered problem is presented in Table 1.

The next results were obtained for the modelling of run-up of the waves with negative polarity ($A < 0$). The initial positions coincided with those of the waves with positive polarity. The

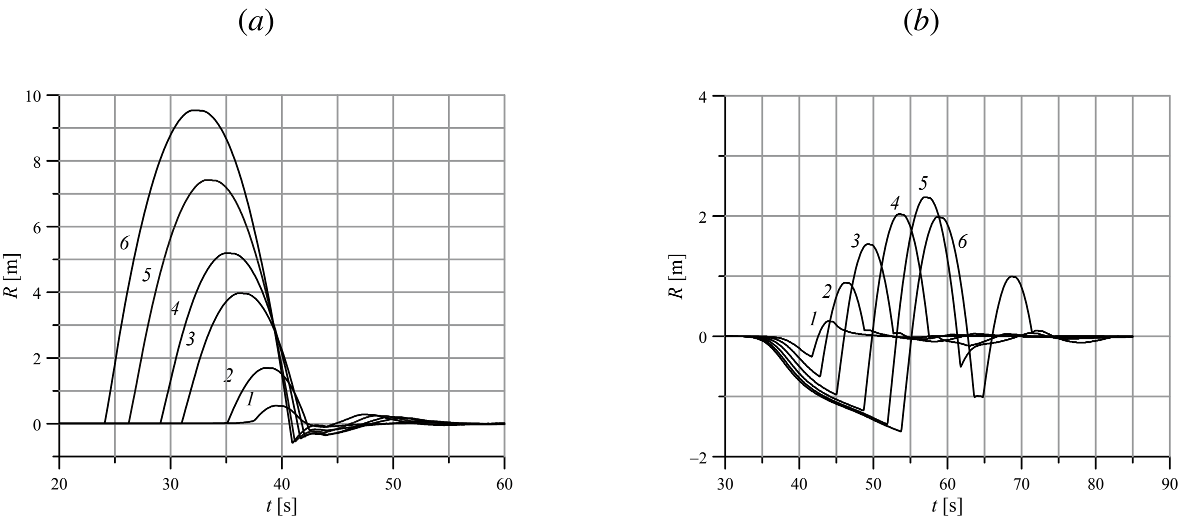

Figure 3: The dynamics of the vertical run-up on a plane slope of the waves with positive (*a*, $A > 0$) and negative (*b*, $A < 0$) polarity with the intial amplitudes $A = \pm 0.1$ m (*1*), ± 0.3 m (*2*), ± 0.7 m (*3*), ± 1.5 m (*4*), ± 2.5 m (*5*), $A = 3.5$ m (*a*) and $A = -3.49$ m (*b*) – (*6*)

	Ratio of maximal run-up to initial amplitude		Ratio of maximal run-down to initial amplitude		Ratio of maximal run-down to maximal run-up	
A, m	TVD+SPH	[4]	TVD+SPH	[4]	TVD+SPH	[4]
0.05	4.868	4.7354	1.104	1.0053	0.2268	0.2365
0.1	5.499	4.6779	1.024	1.0015	0.1862	0.2211
0.3	5.658	3.5827	0.682	0.7363	0.1205	0.2055
0.5	5.0096	2.9408	0.5778	0.5733	0.1153	0.1962
0.7	4.503	2.5674	0.4829	0.4672	0.1072	0.1820
1	3.9676	2.2246	0.396	0.3653	0.0998	0.1642
1.5	3.4581	1.9056	0.2983	0.1877	0.0863	0.0983
2	3.1606	1.7105	0.2547	0.0899	0.0806	0.0526
2.5	2.9644	1.5826	0.2144	0.0229	0.0723	0.0144
3	2.8293	1.4976	0.189	0.0065	0.0668	0.0044
3.5	2.7244	1.3807	0.1663	0.0009	0.0611	0.0007

Table 1: Main characteristics of the run-ups of the waves with positive polarity on a plane slope, computed using two numerical algorithms

initial form of the waves coincided as well with an accuracy up to the plane reflection. As in the case of the positive pulses, the results obtained by the TVD+SPH method are close to the results from [4] for small amplitudes, but significantly differ for $|A| > 1$ m. The primary analysis of the computed profiles shows that the for large amplitudes the times of arrivals of the extremal values to the same points do not coincide. Thus, the profiles in Fig. 4 correspond to the time moments other than those in [4]. Each graph shows the waves at the initial time moment (lines (*1*)); near the transition point between the inclined and the horizontal bottom segments (lines (*2*), which correspond to the time moment $t = 20$ s); at the moment of nearly vertical run-up (lines (*3*), which correspond to the time moment $t = 41$ s for the amplitude $A = -0.1$ m, $t = 43$ s – for $A = -0.5$ m, $t = 46.5$ s – for $A = -1$ m, $t = 53.86$ s – for $A = -3.49$ m); and at the moment of approach of the wave, reflected by the slope, to the opposite boundary of the computational domain (lines (*4*), which correspond to the time moment $t = 75$ s for the amplitudes $A = -0.1$ m, -0.5 m and -1 m, and $t = 85$ s – for $A = -3.49$ m).

Thus, for $A = 1.0$ m some advance is seen in the positions of run-up and run-down profiles in comparison to the results of the work [4], where as for $A = -3.49$ m – the delay is present. The numerical experiment with such amplitude of the wave with negative polarity can be hardly considered to be reasonable because of the obvious inapplicability of the shallow water theory. However, the possibility of performing the computations of this kind testify to the high working efficiency of the algorithm. The corresponding fragment of the figure (Fig. 4, *d*) demonstrates the significant differences for all time moments with the meaningfully close figure from the work [4], which demonstrate themselves in much more complicated forms of incident and reflected waves.

The general idea about the waterfront dynamics during the run-up of waves with negative polarity on a plane slope can be given by the results (Fig. 3 (*b*)), obtained by the TVD+SPH on the grid with 6000 nodes. However, these results appeared to be practically identical to those obtained on the grid with 4000. The graphs on this figure demonstrate the monotonic increase of the values of both primary and secondary run-downs, yet the value of run-up increases up to the initial amplitude -2.5 m and then decreases for the amplitudes $A = -3$ m and $A = -3.499$ m.

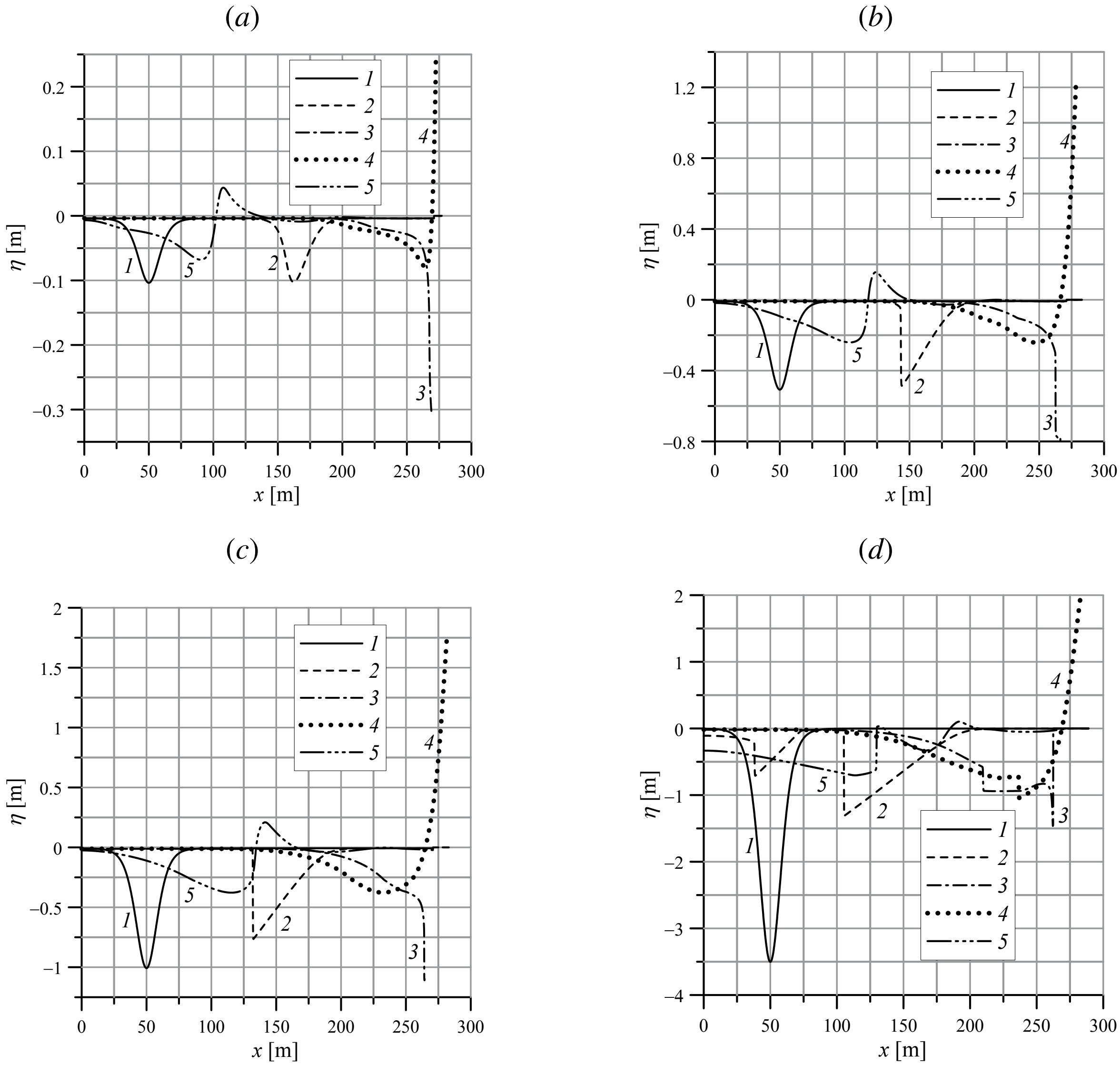

Figure 4: The free surface profiles for the run-up on a plane slope of the waves with negative polarity with initial amplitudes $A = -0.1$ m (*a*), -0.5 m (*b*), -1 m (*c*), -3.49 m (*d*) at the initial time moment, and at the moment of approach of the wave to the transition point between the inclined and the horizontal bottom segments, at the moment of nearly vertical run-up, and at the moment of approach of the wave, reflected by the slope, to the opposite boundary of the computational domain. The values of time moments are given in the text

Further we compare our results with those of the work [4]. Our results were obtained by the TVD+SPH method and the algorithm, based of the definition of the waterfront point using the analytical solutions of the shallow water equations for the formulation of the difference boundary conditions on a moving waterfront [6]. The values from [4] were obtained partially from its text and partially by the digitization of the graphs presented there.

The first graphs (Fig. 5) demonstrate the dependency of the maximal values of R/A on the initial amplitude in the phase of run-up of the waves with positive polarity. These values, obtained using the TVD+SPH method, are significantly larger than the results from the work [4] for practically all considered amplitudes. Yet, they are close to the results obtained using the methodology from the work [6], which, however, happen to be somewhat smaller for the largest amplitudes. Let us note, that if for the absolute values of the run-up values R the monotonic increase takes place, then the corresponding relative values R/A increase first (up to the ampli-

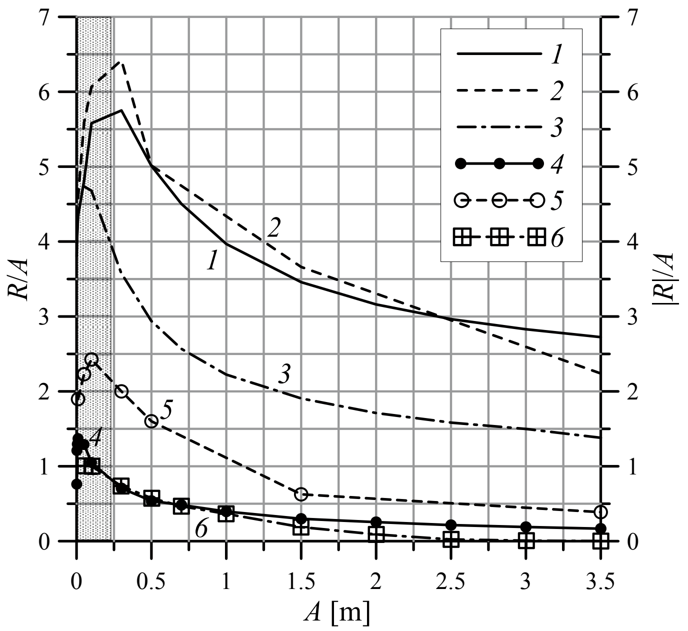

Figure 5: Relative characteristics of the run-ups of the wave with positive polarity with respect to their initial amplitudes: maximal values of R/A during run-up phases (*1–3*) and $|R|/A$ – during run-down phases (*4–6*), computed with the TVD+SPH method (*1*, *4*), the methods from [6] (*2*, *5*) and [4] (*3*, *6*); the domain of applicability of the analytical solution [13] is marked by grey color.

tude 0.25 m), and only after that they begin to decrease monotonically. This can be explained by the strengthening of the non-linear effects' influence, which lead to the increase of the leading edge steepness, decrease of the heights and increase of lengths for the waves approaching a slope. In the results [4] the extremum is not found because of the absence of the data for small amplitudes. The significantly smaller values from [4] can be explained by the fact that the implicit algorithm is implemented in the software package CLAWPACK used by the authors. The absolute stability of the algorithm allows computing with a rather large time step, leading to the significant growth of the numerical dissipation and a subsequent decrease of the results quality.

Considering the characteristics in the run-down phases, some changes can be noted. For small amplitudes (nearly up to $A = 1$ m) the values, obtained by the TVD+SPH method, increase monotonically and practically coincides with the results from [4], which starts to decrease sharply and tend to zero for the largest amplitude. On the contrary, the results, obtained by the TVD+SPH algorithm, continue the stable monotonic increase and stay smaller than the results from [6] for the whole range of amplitude variation. The analysis of the graphs with the maximal values $|R|/A$ in the run-down phase points in the first place to the preserving anomalous tendency of the results from [4] to zero for $A > 1$ m and their closeness to the results, obtained by the TVD+SPH algorithm, up to the amplitude $A = 1$ m. The behavior of the TVD+SPH results are qualitatively close to the distribution computed by the methodology from [6], which however is positioned significantly higher. The results obtained for the smaller initial amplitudes, as in the run-up phase, demonstrate the presence of the extremum, which is slightly shifted to the direction of the smallest values of A. Unfortunately, the results, computed in the discussed range of the argument variation and needed for the comparison, are absent on the work [4].

The relations of the maximal values of $|R|$ in the run-down phase to the maximal values of R in the run-up phases decrease with the increase of initial amplitude of the wave approaching a slope. As before, the results from [4] tend to zero; the results from [6] have the largest values; and the results obtained by the TVD+SPH algorithm demonstrate the presence of extremum in the domain of the very small amplitudes.

The analogues of the above considered characteristics of the run-up of waves with negative polarity show that, as in the case of the positive pulses, the results obtained by the TVD+SPH algorithm are larger than the results from [4]. This is expressed to a greater degree in the run-up characteristics, whereas the corresponding run-down characteristics appear to be very close. Both sets of relative values for the run-up phase (Fig. 6) possess the maximums in the range of small initial amplitudes. After reaching these maximums the corresponding distributions decrease monotonically. The relative characteristics of the run-down phase do not have extremes and decrease monotonically with the increase of initial amplitude. Let us note some peculiarity of the value R variation, computed using the TVD+SPH algorithm. This peculiarity consists in the fact that near the initial amplitude $A = 2.5$ m the monotonous increase of the maximal vertical position of the moving waterfront point is changed to equally monotonous decrease. We can assume that this is caused by the specificity of the non-linear effects' simulation by the TVD+SPH algorithm, which leads to an early breaking of the waves with large amplitudes, approaching a slope, with the simultaneous decrease of their positive fragments' heights. This conclusion finds its proof on the profiles of the corresponding free surfaces (Fig. 4) as well as on the graphs of the dynamics of the moving waterfront point (Fig. 3 (*b*)). The distributions of the relations of the maximal values of $|R|$ in the run-down phase to the maximal values of $|R|$ in the run-up phase are essentially close qualitatively. At first, they decrease rather steeply and take the minimal value for the initial amplitude $A = -1$ m, and then they increase slowly. Here the distributions from the work [4] appear to be higher.

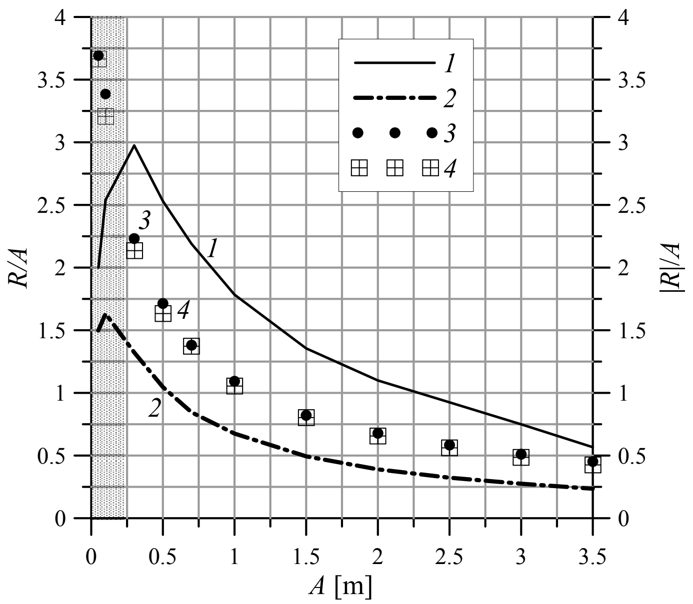

Figure 6: The dependence of the relative characteristics of the run-ups of the waves with negative polarity on their initial amplitudes: maximal values of R/A in the run-up phases (*1*, *2*) and $|R|/A$ – in the run-down phases (*3*, *4*), computed with the TVD+SPH method (*1*, *3*) and [4] (*2*, *4*); the domain of applicability of the analytical solution [13] is marked by grey color

The analysis of the peculiarities of the run-ups of waves with different polarities (positive and negative pulses) has shown that the waves with positive polarity lead to the larger values of the vertical run-down R. However, the velocities of the waterfront point for the run-up of waves with negative polarity are significantly larger. This leads to the steeper trailing edges of waves, being in the qualitative agreement with the results of the work [8]. Let us also note that for the run-up of waves with positive polarity the vertical displacements of the waterfront points in the run-up phase exceed significantly the displacements in the run-down phase, whereas for the waves with negative polarity these values appear to be much closer. The non-monotonicity of free surface profiles above the breakpoint of the bottom relief (Fig. 2), which is absent in the results from [8], is the consequence of the interaction of the approaching wave and the wave reflected from the slope.

Summarizing the comparison with the results from the work [4], which are connected to some extent with the conditions of the physical experiments in the wave flume in the University of Hannover, let us note that even the application of the well-known computer code, created by highly professional specialists, requires the detailed investigation of the underlying algorithm, understanding of its peculiarities, and correct parameter settings. Otherwise it can happen that the numerical effects, which are intrinsic for the considered algorithm, can corrupt significantly the values of the computed variables, complicate the adequate interpretation of the obtained results, and lead to the physically wrong conclusions.

Let us also note that the explicit algorithm, which is used in the TVD+SPH method, guarantees the necessary computational accuracy and proves the correctness of the choice of numerical schemes for the modelling of fine effects arising in wave run-ups even on simple model slopes.

4 CONCLUSIONS

The presented results have demonstrated the possibilities of the combined TVD+SPH numerical method in the definition of the main characteristics of run-ups of the waves with various configurations in the “canonical” problem on long wave run-up on a plane slope. This method has proved to be functional and effective not only in the range of small amplitudes where the shallow water approximation os theoretically allowed, but also beyond its theoretical applicability in the case of breaking waves.

The analysis of the run-ups of solitary waves with various polarity (positive and negative pulses) has shown that the positive pulses have lead to the larger values of vertical run-up in comparison to the negative pulses. This conclusion is in the qualitative agreement with the results of the work [4]. It was also shown that the interaction of the incident and reflected waves above the breakpoint of the bottom relief has lead to the non-monotonicity of free surface profiles, which has influenced the run-up characteristics.

We consider the development and careful analysis of the version of the TVD+SPH algorithm for the modelling of run-ups of catastrophic waves of various configurations in the spatial formulations and with taking into account the real properties of water areas and adjacent coasts to be a promising research direction.

Funding: The study was supported by the Russian Scientific Foundation (project No. 14-17-00219).

REFERENCES

[1] S. Tadepalli, C.E. Synolakis, The Run-up of N-waves on sloping beaches. *Proceedings of Royal Society of London. A*, **445**, 99–112, 1994.

[2] Yu.I. Shokin, A.D. Rychkov, G.S. Khakimzyanov, L.B. Chubarov, On numerical methods for solving run-up problems. I. Comparative analysis of numerical algorithms for one-dimensional problems. *Computational Technologies*, **20** (5), 214–232, 2015 (In Russian).

[3] P. Denissenko, I. Didenkulova, A. Rodin, M. Listak, E. Pelinovsky, Experimental statistics of long wave runup on a plane beach. *Journal of Coastal Research*, **65** (Special issue), 195–200, 2013.

[4] A. Rodin, I. Didenkulova, E. Pelinovsky, Numerical study for run-up of breaking waves of different polarities on a sloping beach. E. Pelinovsky, C. Kharif eds. *Extreme Ocean Waves*, Springer International Publishing, 155–172, 2016.

[5] S.S. Hrapov, A.V. Khoperskov, N.M. Kuzmin, A.V. Pisarev, I.A. Kobelev, Numerical scheme for simulating the dynamics of surface water on the basis of the combined SPH-TVD approach. *Computational Methods and Programming Techniques*, **12**, 282–297, 2011 (In Russian).

[6] S.P. Bautin, S.L. Deryabin, A.F. Sommer, G.S. Khakimzyanov, N.Yu. Shokina, Use of analytic solutions in the statement of difference boundary conditions on a movable shoreline. *Russian Journal of Numerical Analysis and Mathematical Modelling*, **26** (4), 353–377, 2011.

[7] G. Khakimzyanov, N.Yu. Shokina, D. Dutykh, D. Mitsotakis, A new run-up algorithm based on local high-order analytic expansions. *Journal of Computational and Applied Mathematics*, **298**, 82–96, 2016.

[8] I.I. Didenkulova, E.N. Pelinovsky, O.I. Didenkulov, Run-up of long solitary waves of different polarities on a plane beach. *Izvestiya, Atmospheric and Oceanic Physics*, **50** (5), 532–538, 2014.

[9] A.A. Rodin, Numerical calculations of run-up of solitary breaking waves on a plane beach. *Transactions of Nizhni Novgorod State Technical University named by R.E. Alekseev*, **1** (98), 36–43, 2013 (In Russian).

[10] Clawpack-5 home page // http://www.clawpack.org/

[11] E.N. Pelinovsky, A.A. Rodin, Nonlinear deformation of a large-amplitude wave on shallow water. *Doklady Physics*, **56** (5), 305–308, 2011.

[12] E.N. Pelinovsky, A.A. Rodin, Transformation of a strongly nonlinear wave in a shallow-water basin. *Izvestiya, Atmospheric and Oceanic Physics*, **48** (3), 383–390, 2012.

[13] C.E. Synolakis, The runup of solitary waves. *Journal of Fluid Mechanics*, **185** (6), 523–545, 1987.