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BLOOD FLOW SIMULATION USING SMOOTHED PARTICLE HYDRODYNAMICS

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ABSTRACT: *To understand the characteristics of blood flow, it is important to identify key parameters that influence the flow of blood. The characterisation of blood flow will also enable us to understand the flow parameters associated with physiological conditions such as atherosclerosis. Thrombosis plays a crucial role in atherosclerosis and it also helps to stop bleeding when a blood vessel is injured. This article focuses on using a meshless particle-based Lagrangian numerical technique, named the smoothed particles hydrodynamic (SPH) method, to study the flow behaviour of blood and to explore the flow conditions that induce the formation of thrombus in a blood vessel. Due to its simplicity and effectiveness, the SPH method is employed here to simulate the process of thrombogenesis for various blood flow parameters. In the present SPH simulation, blood is modelled by particles that have the characteristics of plasma and of platelets. To simulate the coagulation of platelets which forms thrombus, the adhesion and aggregation processes of the platelets are modelled by an effective inter-particle attraction force model. With these models, the motion of platelets in flowing blood, and their adhesion and aggregation are effectively coupled with viscous blood flow. In this study, the adhesion and aggregation of blood particles are analysed on a (straight vessel) under various velocities of blood scenarios. The results are compared with experimental results, and a good agreement is found between the simulated and experimental results.*

1 INTRODUCTION

A thrombus is considered to be one of the most important causes of many diseases in human body. On the other hand, A blood clot anchored to a damaged vascular wall can stop bleeding or it can prevent atherosclerosis in arteries. The danger is that a thrombus can affect the blood flow in the vessels and this can cause potentially deadly accidents, such as cardiac infarction (or heart attack) or ischemic stroke when the damage occurs in the coronary or the carotid arteries, respectively[1, 2]. The formation of a thrombus depends on platelet flow; for example, the transport to denuded subendothelium, formation of membrane tethers, adhesion to the subendothelium, and aggregation[3]. Many experimental studies have provided information on the biochemical effects of fluid forces on thrombogenesis. In recent years, due to the availability of vast computational power, research on computer simulation of thrombosis has become a field of deep interest. Although fluids can be simulated in either the Eulerian or Lagrangian method, the Lagrangian method is considered to be more suitable for this type of simulation due to their obvious advantages in tracking movement of particles similar to platelets[4]. The purpose of this study is to analyse flow parameters that influence the formation of thrombosis inside arteries. A Lagrangian smoothed particles hydrodynamics (SPH) is used for numerical simulations of the blood flow consisting plasma and platelets.

2 NUMERICAL METHODOLOGY

The governing equations for solving incompressible or weakly compressible isothermal fluid flow using SPH are mass and momentum conservation equations given by,

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla.\mathbf{v} = 0 \tag{1}$$

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla \mathrm{p} + v\nabla^2\mathbf{v} + \mathbf{F} \tag{2}$$

where ρ, t, v, $\mathbf{v}$, and p represent the density, time, kinematic viscosity, velocity and pressure of the fluid particles and, **F** represents the external force acting on fluid particles. The fluid pressure for weakly compressible SPH formulation is obtained by an equation of state as presented in [5]**.** The numerical procedure to calculate fluid velocity is derived from the momentum equation (2) as,

$$\mathbf{v}^{n+1} = \mathbf{v}^n + \left(-\frac{1}{\rho}\nabla P + v\nabla^2\mathbf{v}^n + \mathbf{F}\right)\Delta t \tag{3}$$

Where superscript n and $n+1$ refer to current and next time steps, respectively, and Δt is the numerical time step. The position and density of the fluid can be updated respectively at every time step by,

$$\mathbf{x}^{n+1} = \mathbf{x} + \mathbf{v}^{n+1}\Delta t, \tag{4}$$

And (from the continuity equation (1)),

$$\rho^{n+1} = \rho^n - \rho^n(\nabla.\mathbf{v}^{n+1})\Delta t. \tag{5}$$

The pressure is then estimated from the updated density.

3 MODELLING PLATELET MOTION

The platelets tend to adhere and aggregate when the blood vessel is damaged. This can lead to formation of a primary thrombus. Inside the primary thrombus the neighbour platelets link together, which are then bound by vWF fibrinogen in plasma and collagen in the sub-endothelial tissue [6]. This process takes place by making a link between neighbouring platelets and bound by vWF fibrinogen in plasma and collagen in the sub-endothelial tissue. To numerically model such platelet motion, an algorithm based on a penalty or spring force mechanism [7] is adopted. This model dictates the interactions between platelets and plasma inside the blood vessel. When the platelets are within a distance d_{ad} from the damaged area, the platelets are attracted towards the damaged wall by an adhesive force given by eq. (6). The platelets adhering to the wall are then activated and attract other platelets which are within a distance of d_{ag} from them. This attractive force is called an aggregation force which is given by eq. (7). The aggregation force takes the same form as that of the adhesive force but has a different spring constant.

$$\boldsymbol{F}_{ad} = \begin{cases} K_{ad}(|\boldsymbol{r}_{ij}| - r_o)\boldsymbol{n}_{ij} & (|\boldsymbol{r}_{ij}| \le d_{ad}) \\ 0 & (|\boldsymbol{r}_{ij}| > d_{ad}) \end{cases} \tag{6}$$

$$\boldsymbol{F}_{ag} = \begin{cases} K_{ag}(|\boldsymbol{r}_{ij}| - r_o)\boldsymbol{n}_{ij} & (|\boldsymbol{r}_{ij}| \le d_{ag}) \\ 0 & (|\boldsymbol{r}_{ij}| > d_{ag}) \end{cases} \tag{7}$$

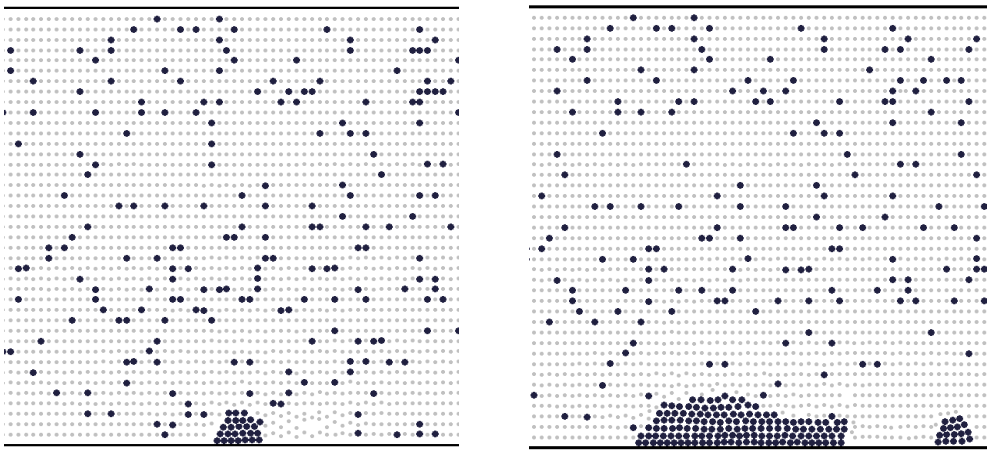
In the above equations $\boldsymbol{F}_{ad}, \boldsymbol{F}_{ag}$ are the adhesive and aggregate forces and K_{ad}, K_{ag} are the corresponding spring constants. The $\boldsymbol{r}_{ij}$ here is distance between activated platelet and vessel wall (or other non-activated platelets), r_o is the original or natural length of the spring and $\boldsymbol{n}_{ij}$ is a unit vector linking platelet and damaged wall (or linking activated platelet and other surrounding platelets). These two forces are introduced in equation (2) for platelet particles which are influenced by adhesion and aggregation.

4 BLOOD FLOW MODEL

In this work, the blood flow simulations were performed inside a straight blood vessel with velocity range of blood flow between100-700 $\mu m/s$ defined at the inlet of vessel. The total length of the vessel (L) and the width between two walls (D) are respectively 130μm and 40 μm. The dimensions of the damaged wall (L_i) is 30μm (refer to the length of the wall damage) and the distance from the inlet to the damaged wall (L_o) is 40μm (see Fig.1). The total number of particles used in the simulation was 5371. Four layers of boundary dummy particles were also used. The initial distance between particles is 1.0 μm. The density ρ and kinematic viscosity v of the plasma and platelets, were set as $\rho = 1\text{x}10^3 \text{kg/m}^3$ and $v = 1\text{x}10^{-6}\text{m}^2/\text{s}$. The boundary conditions were; a uniform velocity at the inlet, zero pressure at the outlet and, non-slip condition at the walls enforced by dummy boundary particles. The amount of the platelet particles used in the simulation is approximately 8.8% of the plasma to resemble normal physiological condition. The time step was set to $5\text{x}10^{-7}\text{s}$ to ensure the stability of numerical integration scheme. In the reported numerical simulations, the spring constants K_{ad} and the K_{ag} are $9.0\text{x}10^9\, N/\text{m}$ and $4.5\text{x}10^9\, N/m$ respectively, while d_{ad}= 3.0μm = d_{ag}, and r_o = 2.0μm.

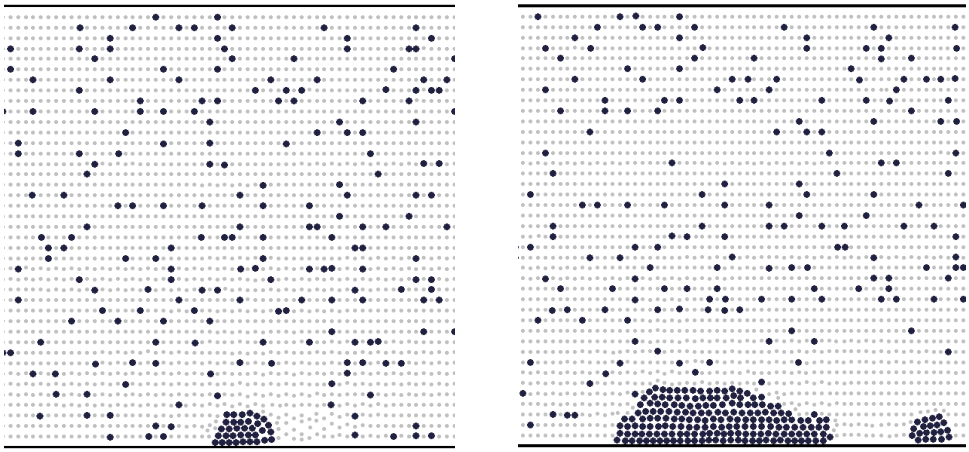
5 NUMERICAL RESULTS

The purpose of this study is to demonstrate the formation of thrombus and to investigate the applicability of SPH in modelling such process. The corrected SPH is used to improve the accuracy [8] of the simulation. Normally, a thrombus is formed by adhesion and aggregation of platelets which are transported by the blood flow in different geometries of arteries or vessels, where the growth rate of thrombus formation varies with the stenosis and the flow rate of blood. Figure 1 illustrates the formation of thrombus at two different stages of the flow. In these figures, for clarity, plasma and platelet particles are shown by two different the plasma and platelets are denoted by light and the dark grey respectively. The platelets are activated when they are within d_{ad} distance from the damaged region and form a primary thrombus. During the course of time, a primary thrombus is developed to cover the whole damage area by forming several layers of platelets. When thrombus grows to a certain volume, part of the thrombus is separated and transported downstream by the blood flow. Figures 1, 2 and 3 depict the growth of thrombus at different times for velocities 100 ,500 and 700 $\mu m/s$ of the blood flow. From the figures below, various stages of thrombus growth on the damaged area of the wall are clearly evident. It can be noted from Fig. 1(b), 2(b) and 3(b) that, part of the thrombus is separated from primary thrombus once the primary thrombus grows to a substantial volume. It is interesting to observe that the volume of the primary thrombus and the time at which separation of the thrombus takes place are affected by the flow rate. From these figures it can also be noted that, when the flow rate was 700 $\mu m/s$ the thrombus growth was thinner compare to the cases where the blood flow rate was 100 and 500$\mu m/s$. Further, it was observed that with higher flow rates the separation of thrombus takes place quicker. In Figure 4, the growth rate of thrombus against various blood flow velocities are plotted. It transpires from Fig. 4 that the growth rate of thrombus gradually increases with blood velocity until approximately 500 $\mu m/s$. Beyond 500 $\mu m/s$, the thrombus growth rate drops to a lower level. The results illustrated in Figure 4 qualitatively agree with experimental observation made in [9]. As results reported here are from 2-dimensional simulations, direct comparisons could not be made at this point. It is evident from all the results listed below that the blood flow rate plays a crucial role in the build-up and separation of thrombus. The results show that the growth rate of the thrombus, its thickness, and formation/separation vary according to the blood flow rate and these results are consistent with experimental observations reported in [9].



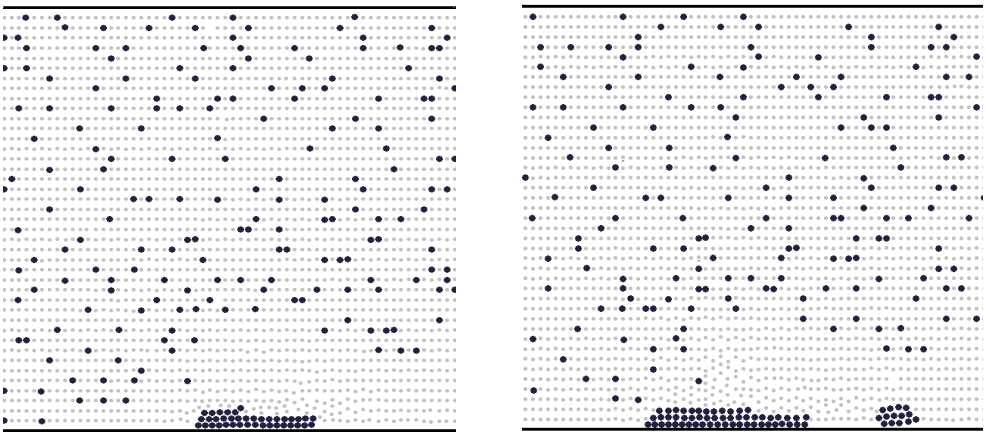
(a) (b)

Figure1: The platelet aggregation in the velocity=100 $\mu m/s$ at (a) t=0.2s; (b) t=0.6s



(a) (b)

Figure2: The platelet aggregation in the velocity=500 μm/s at (a) t=0.2s; (b) t=0.6s



(a) (b)

Figure 3: The platelet aggregation in the velocity=700 μm/s at (a) t=0.2s; (b) t=0.6s

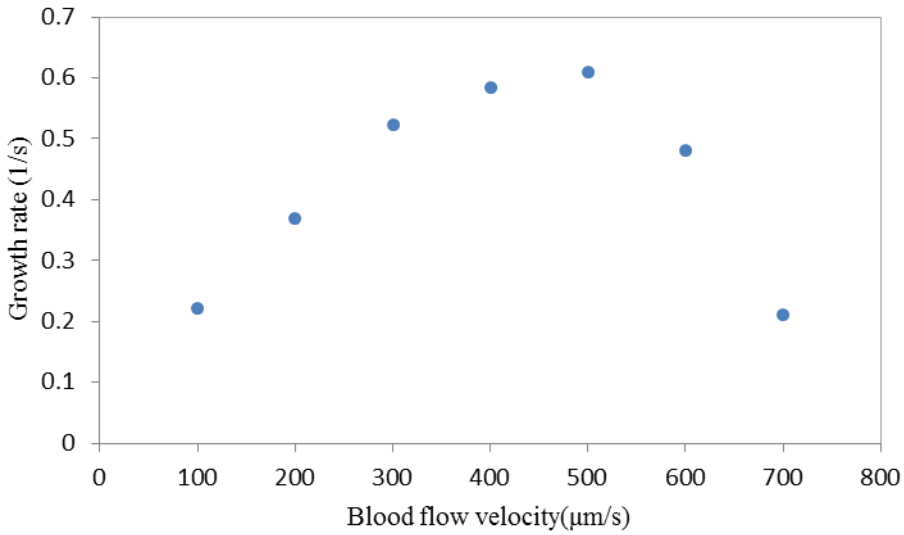


Figure 4: Effect of blood flow velocity with thrombus growth rate

7 CONCLUSIONS

This work has focused on the simulation of the thrombogenesis process using the SPH method by considering platelet adhesion/aggregation and the influence of blood flow rates on thrombus growth. In the numerical simulations, blood inside a straight vessel is discretised by particles which are assumed to have the characteristics of blood constituents, such as plasma and platelets. The platelet adhesion and aggregation process during the blood flow is modelled by adopting an inter-particle penalty force method. The aforementioned model proved efficient in simulating adhesion and aggregation process without rigorous computational efforts. The potential of SPH method to simulate thrombogenisis process is demonstrated via numerical examples. The numerical simulations were able to indicate how blood flow velocity influenced thrombus growth rate in a straight vessel. Further, the numerical results also qualitatively agree with experimental observations reported in literature. This study also demonstrates the ability and accuracy of the SPH method in modelling blood flow with low Reynolds numbers.

It is essential to further investigate the accuracy of the methods developed here in a 3-dimensional context. The 3-dimensional simulations will enable the results to be directly and quantitatively compared with the experimental observations reported in [9]. In addition, the assumed penalty values (or spring constants) used in the analysis discussed above can be more accurately estimated or calibrated by 3-dimensional numerical models.

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