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ON STABILITY LOSS OF A THIN-WALLED SPHERICAL SHELL SUBJECTED TO EXTERNAL PRESSURE AND INTERNAL HOMOGENEOUS CORROSION

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Abstract. *Stability loss of a thin-walled elastic closed spherical shell, subjected to external pressure and internal corrosion, is studied. It is shown that critical time of stability loss of the shell can be established if the upper critical load value for static stability loss of the shell without corrosion and the corrosion rate law are known. Numerical results are obtained for carbon steel shells with different wall thickness at different temperatures.*

1 INTRODUCTION

Thin-walled elastic vessels, having the form of a closed sphere, are widely used in the industry, for example, in chemical engineering, as high-pressure reservoirs. In this relation the problem of stability loss of a thin closed spherical shell, subjected to a uniform external pressure, is of special importance. It was studied in a number of papers, particularly, in [1-4], where stability loss of thin-walled elastic shells, subjected to various loads, including external pressure, was investigated.

However, existing models don't take into account the fact that in the process of structure operation the shell interaction with an active environment is possible. Such interactions can change, for instance, the thickness of the shell's wall - it is known [5] that corrosion leads to sufficiently fast thinning of different elements of metal constructions. Experimental data indicate that corrosion rate depends also on the magnitude of mechanical stresses in the material. This effect must be taken into account when the service life of the structural elements operating in the presence of general corrosion is estimated [5].

The onset of breaking is related usually to approaching of certain limiting stress in the shell. Nevertheless, in certain cases of practical importance, the stability loss can occur in the shell before the breaking stress is reached. In such cases the structure life time will be completely determined by the criterion of the stability loss. For instance, the method of the life time estimation for elastic thin-walled closed circular cylindrical shell, subjected to simultaneous actions of uniform external pressure or longitudinal compressive forces, and homogeneous corrosion, was developed in [6, 7].

In the present paper this method is extended for the case of stability loss of a thin-walled elastic closed spherical shell, subjected to simultaneous action of a uniform external pressure and internal homogeneous (general) corrosion.

2 PROBLEM DESCRIPTION

The corrosion is considered here as a uniform dissolution of the internal surface of the shell (general corrosion). This suggestion leads to a model in which the internal wall of the shell is dissolved uniformly over the entire surface, and its thickness h decreases with time t. The rate of this process is equal to the corrosion rate. Experimental data for various metals and steels, received for pure elastic stressed state, have shown that corrosion rate, besides the temperature T, depends essentially from the stressed state of the material, which in the case of a spherical shell is characterized by compressive stress σ, caused by the external uniform pressure. As a result, the corrosion rate v follows the relation $v = f[\sigma(t), T]$. It was shown in [5] that this dependence can be approximated by the exponent function. Similar to [6, 7], it is possible to write this law in an explicit form, taking into account specific conditions of corrosion and using experimentally established parameters.

Below stability loss of a thin-walled elastic closed spherical shell, subjected to simultaneous action of uniform external pressure and internal uniform corrosion, is studied. The formulas for the critical time (live-time), corresponding to the moment of stability loss, are derived.

3 SHELL STABILITY LOSS IN THE ABSENCE OF CORROSION

Consider the problem of stability loss for a thin elastic closed spherical shell with thickness $h(t)$ and radius of the middle surface $R(t)$, subjected to external pressure of intensity Q. Assuming that changes of the wall thickness and, consequently, the radius $R(t)$, are quasi-static, the equations of static stability loss in linear approximation can be applied. Spherical coordinates of the shell middle surface are characterized by the radius $R(t)$ and the angles ϑ and ϕ

(Fig.1). It is assumed that the initial stress state of the shell is membrane like [8] and, therefore, from equations of the thin elastic shells theory it follows that the loadings, arising from the external pressure with intensity Q, in all normal cross-sections of the shell have the form [2, 4]:

$$S = S_1 = S_2 = -\frac{QR}{2} \tag{1}$$

Assuming that the loading S and the corresponding stress σ are compressing, we obtain from Eq. (1):

$$S = \frac{QR}{2}; \quad \sigma = \frac{QR}{2h} \tag{2}$$

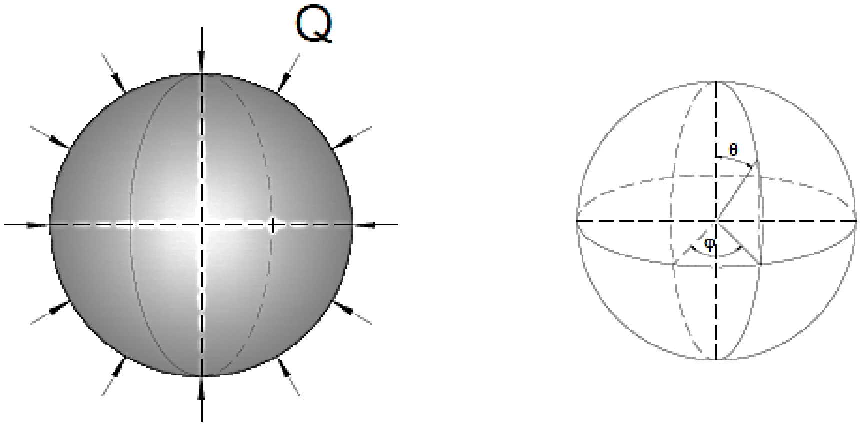

Figure 1: Spherical coordinate system and scheme of the shell's loading. Q - intensity of external loading, φ - polar angle, θ - spherical angle.

At solving of the linear problem for stability loss of elastic closed spherical shell we assume that on the middle surface of the shell many dents are formed. According to [2, 4, 8], the changes in the middle shell surface within one dent can be considered as minor. Taking into account Eq. (2) and using equations of the stability problem for thin elastic shallow spherical shells, we obtain the following homogenous differential equation of stability loss for a closed spherical shell, subjected to external uniform pressure of intensity Q [2, 4]:

$$\frac{Eh^2}{12(1-\nu^2)}\nabla^6 w + \sigma \nabla^4 w + \frac{E}{R^2}\nabla^2 w = 0 \tag{3}$$

Here E is the Young module, ν is the Poisson coefficient, and w is the normal deflection of the middle surface of the shell. Differential operators ∇^2, ∇^4 and ∇^6 are defined as follows:

$$\nabla^2 F(\theta,\phi) = \frac{1}{R^2}\left(\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\phi^2}\right)F, \quad \nabla^4 F = \frac{1}{R^4}\left(\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\phi^2}\right)^2 F, \quad \nabla^6 F = \frac{1}{R^6}\left(\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\phi^2}\right)^3 F \tag{4}$$

where the function $F = F(\theta,\phi)$ is supposed to have partial derivatives of a corresponding order.

Following [9], we accept that solution of Eq. (3) must satisfy the relation:

$$\nabla^2 w = -\lambda^2 w \tag{5}$$

where λ is an indefinite parameter. Then we receive from Eq. (3):

$$\frac{E h^2}{12(1-\nu^2)} \lambda^4 - \sigma \lambda^2 + \frac{E}{R^2} = 0$$

which yields the relation:

$$\sigma = \frac{E h^2}{12(1-\nu^2)} \lambda^2 + \frac{E}{R^2 \lambda^2} \tag{6}$$

The value of λ, corresponding to minimum of σ, is equal to:

$$\lambda^2 = \frac{\sqrt{12(1-\nu^2)}}{R h} \tag{7}$$

Substituting Eq. (7) into Eq. (6), we obtain the upper critical stress σ^* and the corresponding upper critical pressure Q^*:

$$\sigma^* = \frac{E h}{R\sqrt{3(1-\nu^2)}}, \quad Q^* = \frac{2E h^2}{R^2\sqrt{3(1-\nu^2)}} \tag{8}$$

Note that the upper critical stress σ^* for a spherical shell, defined by Eq. (8), coincides with that one for a thin elastic circular cylindrical shell with the radius R of the middle surface, subjected to longitudinal compressive forces and external general corrosion [7].

Eq. (8) was derived at the assumption that the shell wall thickness is specified, whereas the critical value of the external pressure Q is the unknown quantity. It follows from Eq. (8) that if to assume, on the contrary, that the external pressure Q value is specified, then the shell wall thickness $h*$, corresponding to stability loss, will be equal to

$$h* = \frac{\sqrt[4]{3(1-\nu^2)} \cdot \sqrt{Q} \cdot R}{\sqrt{2E}} \tag{9}$$

4 STABILITY LOSS OF THE SPHERICAL SHELL IN THE PRESENCE OF HOMOGENEOUS INTERNAL CORROSION

It is assumed below that the shell is subjected to simultaneous action of a constant external pressure Q and homogeneous internal corrosion. The rate of the thickness decrease at each moment of time t is equal to the corrosion rate

$$\frac{dh}{dt} = -f(\sigma, T) \tag{10}$$

where $f(\sigma, T)$ is a sufficiently smooth function of the compressive stress σ, defined by the Eq. (2), and the temperature T.

The corrosion rate dependence on the stress value, according to experimental data, can be approximated by exponential function suggested in [5], while the temperature effect is described usually by the Arrhenius type law [10]. Combination of both functions yields the following relation:

$$f(\sigma, T) = v_0 \exp[\bar{E}_{c0}(1-\bar{E}_c/\bar{T}) + V\sigma/(R_g T)], \quad \bar{E}_{c0} = E_{c0}/R_g T_0, \; \bar{E}_c = E_c/E_{c0}, \; \bar{T} = T/T_0 \quad (11)$$

Here R_g is the molar gas constant, V is the material molar volume, E_c, E_{c0} - the effective activation energy of the corrosion process and its reference value, respectively; T, T_0 - the absolute temperature and its reference value. The preexponential factor v_0 in (11) has the meaning of corrosion rate for a non-stressed shell ($\sigma = 0$) at $E_c = E_{c0}$ and $T = T_0$. Similar to [6,7], the critical time t^*, corresponding to the moment of the shell stability loss, can be obtained from the Eqs. (10), (11):

$$t^* = (h_0\sigma_0/v_0)\exp[-\bar{E}_{c0}(1-\bar{E}_c/\bar{T})]\cdot\int_{\sigma_0}^{\sigma^*}\sigma^{-2}\exp[-V\sigma/(R_g T)]\mathrm{d}\sigma \quad (12)$$

where

$$h_0 = h\big|_{t=0}, \quad \sigma^* = \frac{Eh_0}{R\sqrt{3(1-\nu^2)}}, \quad \sigma_0 = \frac{QR}{2h_0} \quad (13)$$

Here it is accounted for that $R \approx const$. Note that in Eqs. (12), (13) is supposed also that the external pressure Q for a given initial wall thickness h_0 and shell's parameters satisfies the inequality:

$$\sigma_0 \le \sigma^* \quad (14)$$

If we introduce the dimensionless variables according to

$$D^* = (t^* v_0)/R, \quad \tau = \sigma/\sigma^*, \quad N^* = \sigma_0/\sigma^*, \quad \varepsilon = h_0/R \quad (15)$$

then Eq. (12) takes the form

$$D^* = \varepsilon N^* \exp[-\bar{E}_{c0}(1-\bar{E}_c/\bar{T})]\int_{N^*}^{1}\tau^{-2}\exp[(-V\tau\sigma^*)/(R_g T)]\mathrm{d}\tau \quad (16)$$

Here, similar to [6, 7], the value of D^* can be treated as a relative durability of the shell, and value of N^* - as a safety coefficient for the stability.

5 NUMERICAL RESULTS

Numerical simulations were performed on the basis of Eq. (16) in order to study the relative durability D^* dependency on the safety coefficient for stability N^* for a thin–walled closed spherical shell, made from carbon steel, at different values of temperature and initial wall-thickness. The changes of D^* with N^* and ε are illustrated by the plots on the Fig. 2. Data for the Young modulus E temperature dependency for carbon steel (carbon content < 0.3%) were taken from the engineering toolbox [11] according to ASME B31.1-1995:

$$1)\, E = 2.049\times10^{11}\ \mathrm{N/m^2} \quad (-20^0\,\mathrm{C})\,; \quad 2)\, E = 1.971\times10^{11}\ \mathrm{N/m^2} \quad (100^0\,\mathrm{C})$$

Carbon steel for the both temperature values is characterized by $\nu = 0.295$, $V = 7\times10^{-6}\ \mathrm{m^3/mol}$; the molar gas constant is equal to $R_g = 8.317\ \mathrm{N\cdot m/(mol\cdot K)}$. The effective activation energy for corrosion was chosen as $E_c = E_{c0} = 50\ \mathrm{kJ/mol}$; the reference temperature $T_0 = 293\mathrm{K}$.

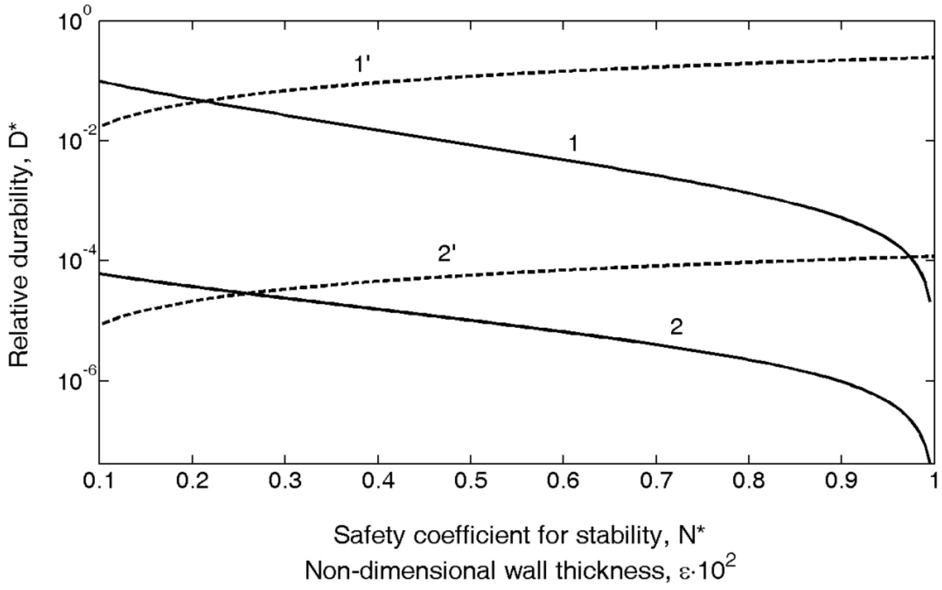

Figure 2: Dependency of the relative durability D^* from the safety coefficient for stability N^* (plots 1, 2) and the non-dimensional wall thickness ε (plots 1', 2') at different temperatures.

The curves 1, 1' on the Fig. 2 correspond to $T = -20^0\,C$, the curves 2, 2' - to $T = 100^0\,C$. The plots 1, 2 were calculated with a fixed initial non-dimensional wall-thickness $\varepsilon = 0.01$; the plots 1', 2' - with a fixed external load $Q = 5\cdot10^4\ \mathrm{N/m^2}$. It follows from the Fig. 2 that the relative durability decreases with the growth of the safety coefficient for stability, while the increase of the initial shell thickness has an opposite effect on the vessel life-time. The temperature increase yields the relative durability reduction because of two reasons – the corrosion enhancement and the elasticity modulus decrease. Simulations have shown that the first one is the most important factor.

6 CONCLUSIONS

- The critical time of stability loss of a thin-walled elastic closed spherical shell, subjected to uniform internal corrosion at a permanent uniform external pressure, can be predicted from the solution of the corresponding problem of static stability for the same shell, with account for the shell's wall thickness changes due to homogeneous corrosion, accelerated by mechanical stresses in the metal.
- The relative durability of the shell depends on the safety coefficient for stability; its value is highly influenced by the temperature and the initial shell thickness.
- It was shown that the relative durability decreases with the growth of the safety coefficient for stability.

- The temperature increase yields the relative durability reduction because of two reasons – the corrosion enhancement and the elasticity modulus decrease.
- Thicker shells at the same external load and temperature are characterized by larger life-time.

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