ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

NUMERICAL VALIDATION OF EQUIVALENT-FRAME MODELS FOR URM WALLS

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Keywords: Masonry, URM walls, Equivalent-Frame models, seismic vulnerability.

Abstract. *In the last decades increasing attention has been devoted to masonry structures both from researchers and professionals. This is due to the awareness of the great importance of these structures in the historical urban context, together with the great risk that they suffer in seismic areas.*

Growing success has been obtained by a simplified approach that models masonry walls through "equivalent" plane frames, with concepts and procedures drawn from the study of reinforced concrete and steel frames. In this approach, known as Equivalent Frame Method (EFM), each masonry resisting wall is modelled as a system of linear (frame) elements representative of the behaviour of finite portions of the wall (pier and spandrel panels).

Up to now EFM has proven to be effective in the case of new buildings characterized by regular geometrical configurations and with openings' dimensions for which the frame-like assumption is suitable. Critical issues emerge from existing buildings in European historical centres, where irregularities are almost always present and geometrical anomalies can be detected even in the case of regular walls.

For the specific case of geometrically regular URM walls, this paper presents sample cases tested with linear and non-linear analyses, with the aim to explore the applicability of EF procedures and to identify its limits. A comparison between an EFM procedure with a fiber approach and a more detailed FEM method with plane elements, is adopted as a validation tool, to evaluate the accuracy of the results provided by the EFM for walls characterized by different geometrical configurations.

1 INTRODUCTION

Unreinforced masonry (URM) structures are a worldwide recurring element in the huge variety of building local traditions. Associated to different construction techniques and structural materials, URM buildings make up a large part of the built heritage, having at the same time an enormous cultural and historical value but also a high seismic vulnerability. In general, masonry structures were not designed for seismic resistance. Moreover, they may have had over the time modifications altering their structural performance and increasing their vulnerability.

With the aim to provide effective tools for the modelling of this heterogeneous set of existing structures, researchers and professionals have dedicated significant efforts to investigate the behaviour of masonry structures [1]. Among the various methods, the Equivalent Frame Method (EFM) has become one of the more successful ones due to its compromise between accuracy and efficacy [2-4].

EFM is a simplified modelling procedure based on the assimilation of masonry walls in-plane behaviour to a plane frame structure. In this approach, each resistant masonry wall is discretized in finite portions whose structural behaviour is simulated by means of one-dimensional beam elements connected to each other by fully rigid nodes.

The EFM has been employed till now both in the case of new and existing buildings. In the first case, the structure is characterized by regular geometrical configurations and by openings' dimensions such that the *frame-like* assumption is suitable for modelling the resisting walls. On the other hand, in the case of existing constructions, the structure may show significant irregularities making the *frame-like* assumption hard to be accepted. In addition to these geometrical aspects, the application of the EFM to existing masonry constructions faces critical points related to the quality of masonry materials, the presence of deformable diaphragms and the interaction with other structures in aggregate configurations.

Considering the great popularity of EFM in professional practice, there is the need to validate the EF approach by means of a numerical comparison with the more detailed FEM approach. In this paper the attention is focused on the regular geometrical configurations. For these types of walls the EF discretization is generally considered reliable and suitable. However, as discussed here, also in the case of regular URM walls it is possible to identify some limit cases that cannot be readily represented by the frame-like scheme.

2 EQUIVALENT-FRAME MODELS FOR URM WALLS

EFM is currently one of the most popular tools adopted for structural modelling of masonry structures in professional practice, due to its remarkable simplicity of application and interpretation of the results.

As it is well known, EFM is based on the discretization of each masonry wall in macro-elements, namely masonry panels distinguishable for mechanical and geometrical properties and modelled as beam-column elements connected by rigid nodes. The wall is studied as a plane frame, with a consequent remarkable reduction of the number of degrees of freedom and hence of the computational effort, with respect to more detailed and accurate methods, such as FEM.

The macro-elements discretization of masonry walls on which the EFM procedures is based is compatible with the observed damages suffered by masonry structures under seismic catastrophic events. It is in fact recurrent the concentration of damages in such parts of the wall modelled as deformable mono-dimensional elements (piers and spandrels). Only minor damage is usually observed in correspondence of the connection area between piers and spandrels (node panels), suggesting the idea of a greater strength and stiffness for these elements.

The accuracy of the EFM procedures is strictly connected to the capability to correctly simulate all the failure mechanisms and the damaging phenomena that can affect a masonry wall subject to in-plane actions, namely flexural and shear failure mechanisms.

However, and generally speaking, the EFM can be considered suitable and realistic only in the case of masonry walls characterized by a regular distribution of openings with "standard" dimensions. In these cases, the identification of the structural elements composing the ideal frame is straightforward, while in the case of walls characterized by an irregular arrangement of openings (irregular walls) or by very small windows, the definition of the frame configuration is more uncertain and problematic.

2.1 Discretization of walls in macro-elements and definition of their characteristic dimensions

In the EF idealization, masonry walls are considered as an assemblage of mono-dimensional structural components that can be geometrically identified by extending the contours of openings. In this way, it is possible to identify the following macro-elements (Figure 1):

- **Pier panels**: vertical structural panels whose function is to carry the gravitational and seismic actions and to transfer them to the foundation system;
- **Spandrel panels**: horizontal panels whose function is to transfer the floors' loads to the piers and to connect piers;
- **Node panels**, connecting spandrels and piers, modelled as fully rigid nodes.

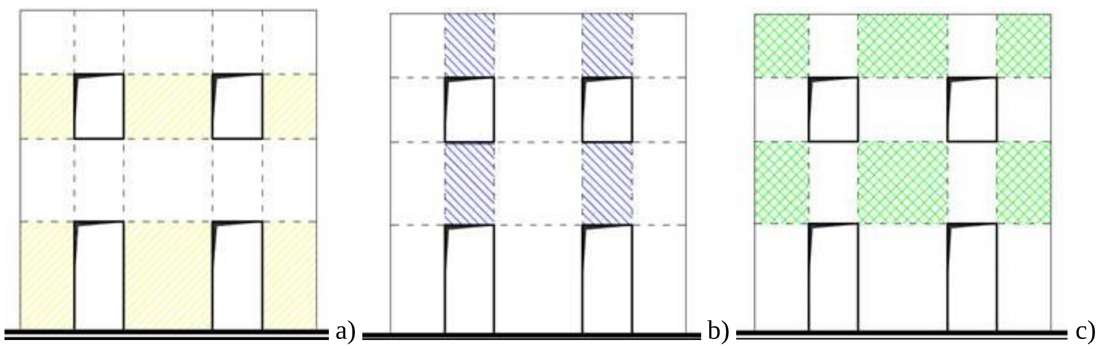

Figure 1: Identification of macro-elements in a masonry façade: piers (a), spandrels (b) and node panels (c).

Looking to the ideal frame configuration proposed in the EF approach, an important role in the walls' structural behaviour is played by spandrels. Such elements represent the horizontal resisting elements of the ideal frame configuration and can give an important contribution to the seismic performance of the wall by behaving as coupling elements and/or restraints for pier panels. The strength and stiffness of the spandrels, in fact, influence the mechanical behaviour of the adjacent piers, which can be modelled with static schemes ranging between the two extreme conditions of cantilever and double-fixed member. The activation of damaging mechanisms in the spandrels can also dissipate energy.

For what concerns the geometrical definition of spandrels, their characteristic width is usually assumed as equivalent to the average width of the openings that include it.

A more important role is played by pier panels, whose stiffness deeply influences the total stiffness and the global seismic behaviour of the wall. The definition of the effective height of the pier was actually one of the first important problems addressed in structural modelling of masonry buildings by the EFM.

The POR method [5] represents one of the first modelling procedures proposed for URM structures anticipating the EF approach, in which only the shear failure of piers is taken into account, with a simplified constitutive shear-deformation law. No clear rules are provided by the POR method about the piers' effective height, even though it strongly affects both their stiffness and shear strength.

A geometrical criterion was introduced by Dolce [2], taking into account the boundary conditions of each piers. The pier's behaviour and stiffness are in fact influenced by the mutual interaction with the surrounding spandrels and by their deformability.

Starting from these consideration and from a previous theoretical study carried out for coupling beams in reinforced concrete structures [6], Dolce proposed an empirical formula for the definition of the piers' effective height as a function of a geometrical parameter (*h'* in Figure 2a). This parameter is defined as the distance between the midpoints of the line connecting the vertices of two consecutive openings, fixing a limit value for the inclination of these lines equal to 30° [2]. The criterion proposed by Dolce was validated by numerical FEM simulation on different pier-spandrel systems. The formula provided by Dolce represents, with some simplifications, the main reference adopted in the most common EF methods actually available for structural modelling of URM structures [3, 4].

Recovering and updating the assumption of "strong spandrel and weak piers" contained in FEMA 356 [7], a different proposal was provided by Augenti [8] by assuming the piers' effective height equal to the height of the openings computed from the side of the earthquake loading. This assumption leads to the need for considering two different models for asymmetric walls, taking into account both the possible horizontal directions of the earthquake, as shown in Figure 2.

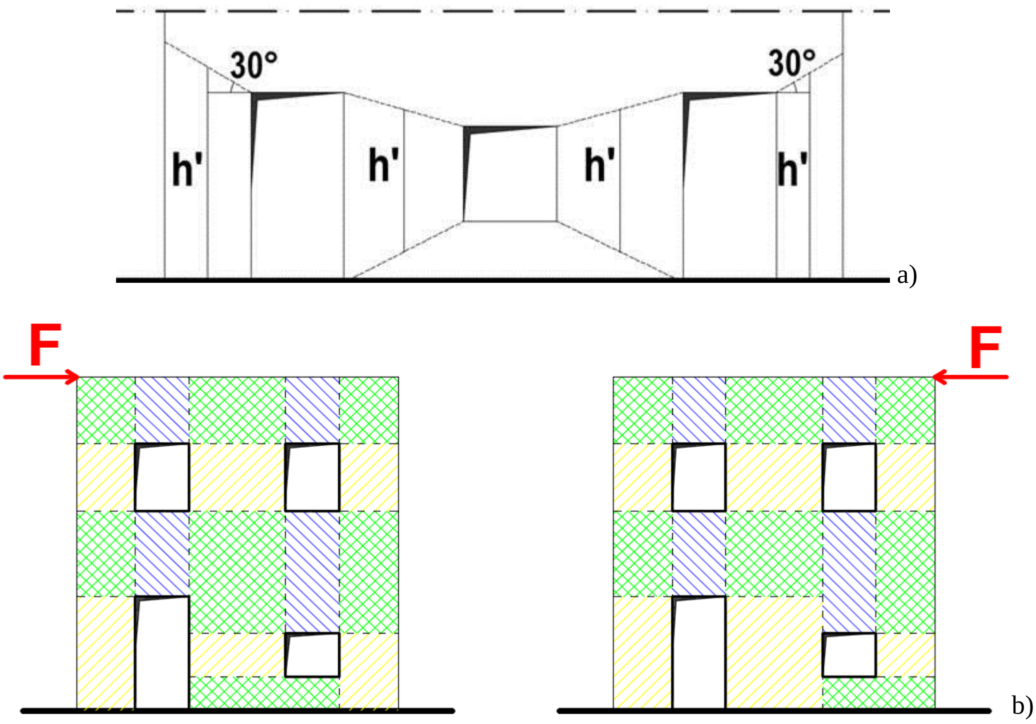

Figure 2: Proposals for piers' effective height by Dolce [2] (a) and by Augenti (b), as a function of the horizontal forces direction [8].

Augenti's proposal is based essentially on direct observation of recurrent damages and failure mechanisms caused by past earthquakes to ordinary masonry buildings. In most of the cases it is usually possible to observe diagonal shear cracks starting from the corners of the openings, which suggest the idea of different structural responses depending on the direction of the seismic actions. This proposal was validated by experimental tests carried out on masonry building prototypes, whose damage patterns appeared in good agreement with Augenti's assumptions [9].

A comparative analysis between the two proposals for piers effective height has been recently presented by Marques and Lourenço [10]. They have compared the well-known EF methods and then tested on simple regular URM structures. In particular, non-linear static analyses have been carried out by the authors on a two-storey masonry wall by using several EFMs (SAM [4], 3MURI [3] and RAN [11]) related to Dolce's and Augenti's assumptions for piers' effective height and with different boundary conditions (double-fixed and cantilever schemes). The results of the analysis were compared in terms of capacity curves. The comparison showed an important aspect that has been confirmed in the present work. Augenti's assumption presents a better accuracy in evaluating the base shear capacity and the forces distribution among the vertical elements, while the global stiffness of the wall was is strongly overestimated. On the other hand, Dolce's approach obtaines better results for stiffness approximation, while base shear capacity is underestimated.

Due to the fundamental importance of the definition of piers' effective height for structural modelling of URM buildings, the assumptions provided by Dolce and Augenti have been taken into account in the present work and assumed as alternative criteria for the implementation of EF models. The aim of the present work is the evaluation of the accuracy of the EFM in simulating the structural behavior of regular masonry walls. For this purpose, some sample cases of regular masonry walls have been tested in linear and non-linear field. The models studied with EF approach have been modelled considering both the assumptions for piers' effective height formulated by Dolce and Augenti. The results have been compared with reference to corresponding FE models. This comparison gives the possibility to evaluate the effect of different geometric modellings of the piers on the results of the EF models.

2.2 Equivalent-Frame model with fiber approach

In the present work the EFM approach is implemented with a recent formulation for the mechanical characterization of walls' structural components, namely a fiber model initially implemented for reinforced concrete structures [12] and then extended also to URM structures [13]. Fiber approach represents in fact a good compromise between the accuracy of a tri-dimensional FE procedure and the computational convenience of a simplified uni-dimensional modelling approach.

Differently from other currently available EFM procedures, the use of the fiber approach allows the implementation of the non-linear behavior with a spread plasticity model.

The fiber model is based on the discretization of the beam elements' cross-section into longitudinal fibers, which allows the simulation of the flexural behaviour of the structural element by means of *ad hoc* non-linear constitutive $\sigma-\varepsilon$ laws. The fiber is assumed as having only axial deformation and, for this reason, with this model it is not possible to simulate the flexural and the shear behaviour at the same time. Given the importance of the shear failure mechanisms in masonry structures, the model has been therefore completed by introducing a non-linear spring with shear constitutive law into the fiber discretization [13].

The resulting model is composed therefore of three springs working in series. The central one is a non-linear spring calibrated according to a multi-linear V-γ law, to simulate the possible shear mechanisms than can affect masonry panels, i.e. diagonal cracking and shear sliding. The two extreme springs are defined by fiber discretization, so by means of a spread plasticity model. For what concerns the simulation of the flexural behaviour, the uniaxial constitutive law provided by Kent and Park [14] has been adopted.

The models adopted in the present work for non-linear simulations have been implemented in the software Midas GEN © [15], where it is possible to combine the fiber discretization of beam elements with non-linear springs simulating shear constitutive laws.

3 REGULAR AND FRAME-LIKE MASONRY WALLS

The concept of structural regularity is usually referred to the global configuration of the structures. Making reference to the prescriptions provided by the Italian technical code [16], a generic construction can be defined regular if it fulfils requirements based on the stiffness, geometrical and inertial properties of the whole structural system.
In the case of URM buildings, namely structural systems composed by the intersection of vertical load-bearing walls and horizontal slab diaphragms, the global seismic performance is strongly dependent on the in-plane response of the single load-bearing walls, once ensured a low probability of activation of out-of-plane failure mechanisms. In this case, the global seismic vulnerability of a URM building is due to the in-plane performances of its load-bearing walls, each of them working for the horizontal forces transferred by the slab diaphragms. As a consequence, it is important to extend the concept of regularity to the single load-bearing wall and in particular to such structural features that can affect its in-plane response. The definition of regularity adopted in this work is therefore a purely geometrical concept referred to the configurations of single masonry walls. The arrangement of the openings in a URM wall is strictly connected to the definition of regularity. A URM wall can be defined regular if its openings are perfectly aligned in both vertical and horizontal directions [17], as for the sample schemes of Figure 3.

As already explained in section 2, the EFM is essentially based on the identification between a single load-bearing wall and an ideal plane frame. In this sense, it is possible to distinguish between configurations that appear clearly consistent with the frame idealization, here denoted as “frame-like”, and configurations for which the affinity with a frame system may be questionable, namely “non-frame-like” ones.

Two schemes are reported in Figure 3 to clarify the distinction between *frame-like* and *non-frame-like* walls, with reference to a two-story masonry wall. Both the sample schemes fulfill the definition of regular wall. However, only the scheme of Figure 3a is characterized by piers and spandrels having geometrical properties compatible with the assumption of mono-dimensional elements. For this scheme a good degree of affinity with a plane frame can be recognized, so the definition of *frame-like* is suitable to describe its configuration.

On the contrary, the extremely small dimensions of the windows in the scheme of Figure 3b induce anomalies in the geometrical properties of piers and spandrels. This scheme appears strongly in contrast with the frame idealization and hence with the previous definition of *frame-like* masonry walls.

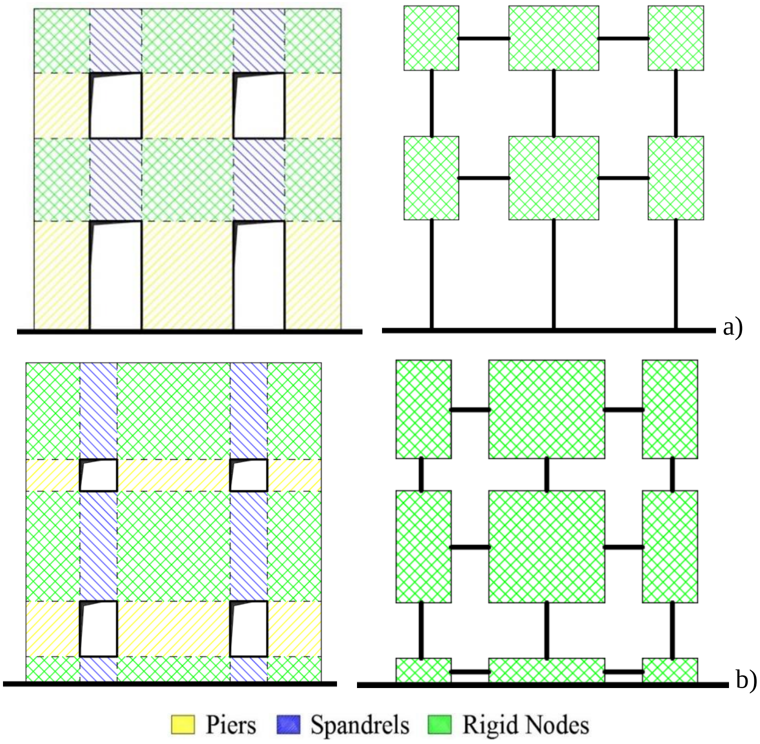

Figure 3: Identification of the equivalent-frame model for *frame-like* (a) and *non-frame-like* (b) URM walls.

4 NUMERICAL VALIDATION OF EQUIVALENT-FRAME METHOD FOR URM WALLS

This section discusses the accuracy of EFM for regular URM walls depending on their geometry. The study is part of a wider research on regular and irregular walls characterized by an increasing level of complexity [18]. The first results are discussed here with reference to the models introduced in Figure 4. Both the walls have been submitted to a linear static analysis under the same loading, mechanical and boundary conditions. A distribution of horizontal loads proportional to the first mode of vibration has been computed and distributed over the nodes of the wall in order to avoid any stress concentration.

The EF models have been built using the two aforementioned approaches for piers' effective height, i.e. the modelling scheme by Dolce [2] and the scheme of "strong spandrel and weak piers" by Augenti and co-workers [11].

The results given by the EF and FE models have been compared in terms of forces and displacements. The base shear reactions and the absolute horizontal displacements have been assumed for comparisons because of their importance in the evaluation of the wall's structural performance. The percentage difference between the predictions provided by EF models and by FE models, in terms of reactions and displacements, has been assumed as a measure of the accuracy of EFM in simulating the actual structural behaviour of URM walls.

Figure 4 shows the percentage differences between EF and FE results for the two sample schemes with regards to the base shear and the top displacement of one of the external piers of the wall.

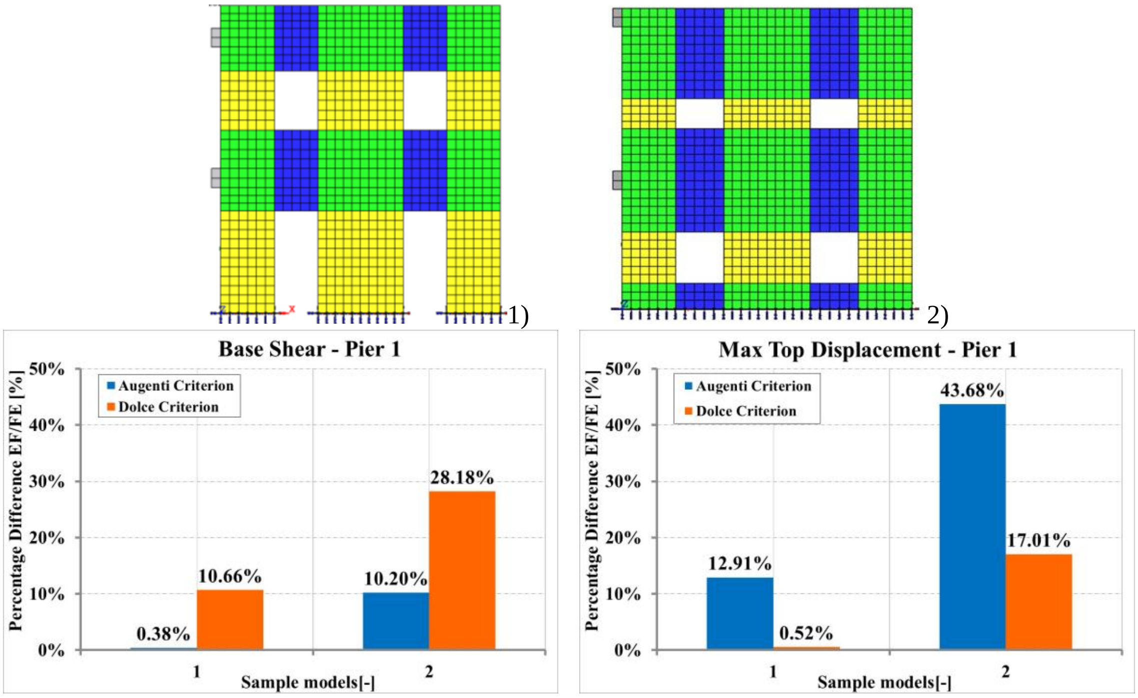

Figure 4: Percentage difference between EF and FE predictions of Base Shear and top Displacement for a frame-like (1) and a non-frame-like (2) URM wall.

Given the strong non-linearity of masonry structures, the validation of a numerical approach for the structural modelling of such structures cannot be limited only to the linear field. Masonry structures show in fact a non-linear behaviour also under low states of stress. Although the study of linear models can be helpful to check the capability of a numerical simulation to predict the elastic stiffness and the distribution of internal forces induced by gravitational loads, it is necessary to extend the study also to the non-linear field in order to have a complete overview of the critical points affecting the modelling method under consideration. For this reason, a non-linear analysis of the same regular schemes has also been performed. The results provided by FE models have been assumed as reference for the evaluation of the EFM accuracy. The FE models have been defined as two-dimensional systems composed by plane-stress rectangular elements implemented in MIDAS FEA © [15].

For what concerns the mechanical characterization of masonry non-linear behaviour, in the EF approach a fiber discretization has been adopted to simulate the flexural behaviour of the uni-dimensional beam elements (piers and spandrels). A non-linear element, located in the midsection of each structural element, has been also added to simulate the two possible shear mechanisms than can affect the masonry panels, i.e. diagonal cracking and shear sliding [13].

The definition of loading conditions has been made by steps, separating the application of gravitational and horizontal loads in both the modelling procedures adopted. An initial loading step has been defined to apply gravitational loads, that have been introduced by assuming a unit weight equal to γ =18.0 kN/m^3 and a slab load equal to 15.90 kN/m for both the levels. A second loading step has been defined to apply horizontal actions directly on the deformed shape obtained at the end of the first loading step. A monotonic incremental time function has been used to apply a distribution of horizontal forces proportional to seismic masses. Finally, a displacement controlled procedure has been adopted to carry out the non-linear static analysis by assigning the value of the horizontal displacement for a control node located at the second floor level of the wall. The iterative solution procedure applied is based on the “initial stiffness” method.

Figure 5 shows the comparison of the capacity curves obtained by the EFM and FEM approaches for the two sample schemes under consideration.

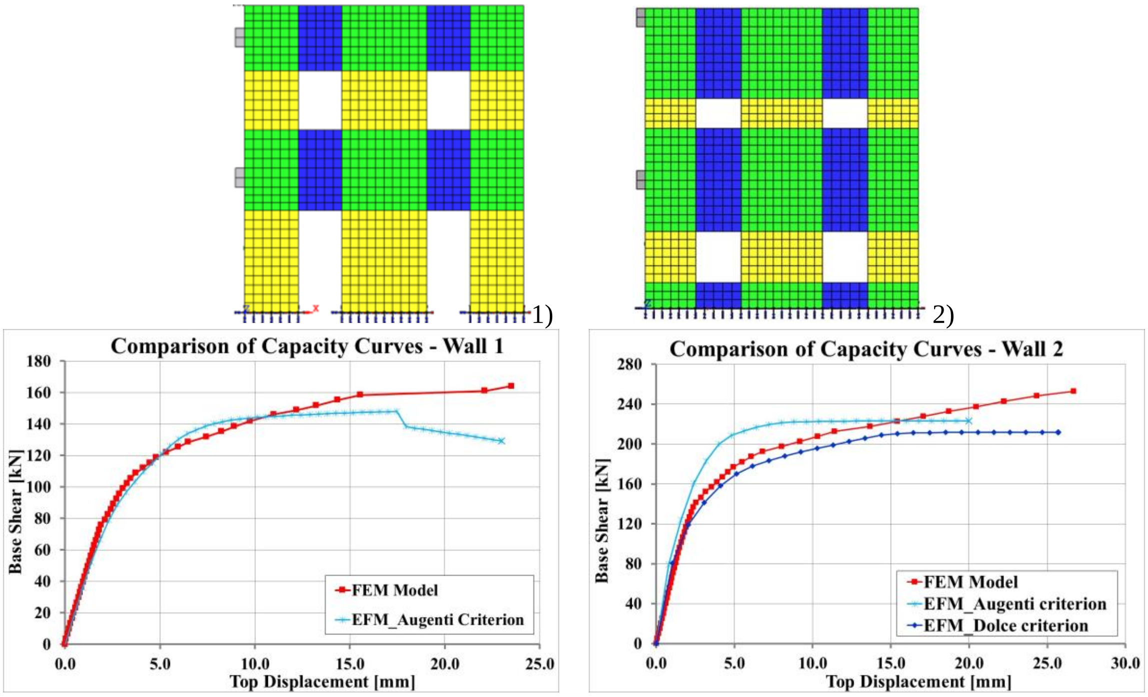

Figure 5: Comparison of the capacity curves obtained with EF and FE models for a frame-like (1) and a non-frame-like (2) URM walls.

4.1 Discussion of results

The two sample schemes analysed in this work show different levels of similarity with the ideal configuration of a frame structure. In particular, the analogy with a frame seems hardly applicable in the case of the second scheme. According to the distinction between *frame-like* and *non-frame-like* walls introduced in Section 3, the results obtained in both the linear and non-linear fields confirm that a regular geometry is not sufficient to justify the assumption of the EF discretization in the modelling of URM walls. As expected, the percentage differences associated to EF models predictions, with respect to FEM results, tends to increase for the second wall.

Figure 4 shows the different level of accuracy that characterizes the two assumptions for the piers' effective height in the EF models, namely the Dolce's and the Augenti's criteria (Section 2.1). As already observed by Marques and Lourenço [10], the criterion adopted by Augenti estimates better the actual distribution of forces among piers than the criterion provided by Dolce. As for the prediction of the base shear, the models defined according to Augenti's criterion provide percentage differences up to 10% with respect to the corresponding FEM results. An opposite trend is shown by the models defined according to Dolce's criterion. For such models a negligible error can be recognized with respect to FE models in the horizontal displacements predictions, while the difference increases significantly for what concerns the forces distributions among the piers.

Such observations about the prediction of shear distributions and horizontal displacements are confirmed by the theoretical bases of the two criteria. The rule provided by Dolce [2] to define the effective piers' height, in fact, is derived by equivalent stiffness considerations among EF and FE models. The effective height of masonry piers was defined following a principle of statistic equivalence between the elastic stiffness expressed by EF and FE models for several examples of pier-spandrel modules. On the other hand, Augenti's criterion is tailored on observed damage and therefore provides better results for the evaluation of the strength.

The observations made in linear field have been generally confirmed also by the non-linear analyses, although the differences between the results provided by EF models and the FE ones are smaller in the non-linear cases. The higher accuracy shown by the EF approach in the non-linear field demonstrates a better performance of the approach in reproducing the redistribution of forces among the structural elements as a consequence of incremental damaging conditions.

5 CONCLUSIONS

This work presents a comparison between Equivalent Frame Method (EFM) and FEM in the simulation of URM existing buildings. The accuracy of EFM is treated as a function of the degree of similarity between the effective masonry wall's geometry and an ideal frame configuration. A distinction between *frame-like* and *non-frame-like* configurations is introduced with the aim to delimit the set of URM walls' configurations for which EFM approach can be applied.

Two sample URM walls are considered, in order to provide a measure of the EFM applicability. For both walls the definition of regular configuration can be assumed, i.e. a configuration characterized by openings aligned in both horizontal and vertical direction, but they differ in terms of "frame-like" or "non-frame-like" geometry. A comparison is discussed between EFM and FEM results for both models, with the purpose of measuring the applicability of EFM with respect to the predictions provided by the more detailed FE models.

The models are investigated using both linear and non-linear static analyses. The results of these analyses, expressed in terms of shear distributions and displacements, are compared with predictions provided by FE models. The study shows that the differences in results by EF and by FE models increase as the URM walls and the frame schemes are unlike.

The effects of different assumptions for the definition of the piers' effective height are also evaluated following the procedures proposed by Dolce [2] and Augenti [11]. Augenti's criterion shows a greater accuracy in predicting the distributions of shear forces among the piers. On the other hand, Dolce's criterion yields more accurate predictions of the horizontal displacements.

The models presented in this work are part of a broader research [18] aimed at exploring the range of applicability of EF procedures in the case of regular and irregular URM walls. In particular, it is aimed at further verifying the capability of EFM to simulate the main irregularities that can affect the in-plane structural behaviour of existing URM walls, such as those caused by misalignments of the openings or anomalies in their dimensions.

ACKNOWLEDGEMENTS

CSP Fea is deeply acknowledged for providing the software MIDAS Gen© and MIDAS Fea© [16] used in this work for the implementation of the numerical models.

The financial support of University "G. D'Annunzio" of Chieti-Pescara (MIUR ex 60% funds) is acknowledged by the authors. Partial support by the ReLUIS program 2014-2016 is also acknowledged by the authors.

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