

EFFECT OF PERIVASCULAR TISSUE ON INFLATION-EXTENSION BEHAVIOR OF ABDOMINAL AORTA

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Abstract. *The research in the field of cardiovascular biomechanics is focused primarily on the heart and blood vessels, but the surrounding tissues, on the other hand, are often overlooked in the literature. This study shows that the human perivascular adipose tissue can significantly affect mechanical response of the human abdominal aorta. Analytical model of incompressible bilayer thick-walled closed cylindrical tube has been created and subsequently used to simulate the inflation-extension response of the aorta surrounded with adipose tissue. The inner layer, abdominal aorta, was assumed to exhibit residual stresses. The material of this layer was considered as anisotropic and was described by hyperelastic nonlinear constitutive model. The outer layer, fat tissue, was considered to be a hyperelastic isotropic material which does not exhibit residual stress. The outer radius of the aorta and inner radius of the external fatty tube were considered to be equal during the pressurization and axial stretch. The inflation-extension response of bilayer tube was compared with one-layer representing only the abdominal aorta. The simulations showed that the abdominal aorta is more compliant in the circumferential and axial direction in comparison with the fat tissue. The Cauchy stress across the wall thickness of bilayer model and the one-layer tube was determined and compared. The plot of the Cauchy stress versus deformed radius proved that the radial stress satisfies the boundary condition $\sigma_{rr}(r_i^A) = -P$, $\sigma_{rr}(r_o^P) = 0$ and $\sigma_{rr}(r_o^A) = \sigma_{rr}(r_i^P)$. An abrupt change in the axial stress by approx. 80 kPa occurs on the contact between the aorta and perivascular tissue. The results of this study suggest that the surrounding tissue should not be neglected in the modeling of blood vessels.*

1 INTRODUCTION

In the last few decades, there has been a significant growth of interest in the mechanical properties and constitutive modeling of biological soft tissue (predominantly vessels), but research on the perivascular tissue has been neglected. The adipose tissue surrounds all blood vessels and organs and influences their mechanical state. The perivascular tissue should be taken into account in solving of various types of boundary value problems describing biomechanics of abdominal aorta such as computational simulations of an aneurysm or *in vivo* constitutive modeling.

In the case of vessels, most of the works focus on modeling only one-layer tubes, but Holzapfel et al. (2000) modeled healthy carotid artery of a rabbit as bilayered structure consisting of two layers corresponding to the media and adventitia. They showed results of deformation behavior during inflation and axial torsion, bending, with and without including the residual strain and after that, they presented the distributions of the principal Cauchy stress components $\sigma_{\theta\theta}$, σ_{zz} and σ_{rr} along the deformed wall thickness (media and adventitia layers). Waffenschmidt et al. (2014) adopted the material, structural and geometrical parameters for a carotid artery of a rabbit from Holzapfel et al. (2000) and published extremal states of the energy of a double-layered thick-walled tube with these parameters. Bilayered model of the pressurized tube was also used by Sommer and Holzapfel (2012) who identified material parameters of invariant-based exponential elastic potential for both intact (bilayer model) and layer-dissected (one-layer model) human carotid arteries.

In the present study, the analytical model of bilayer thick-walled closed tube (the human abdominal aorta and perivascular adipose tissue as outer layer) has been created as a subject of the inflation-extension test, by using geometrical data of human abdominal aorta from Labrosse et al. (2012), material parameters of aorta from Horny et al. (2014) and material parameters of human perivascular tissue from Vonavkova et al. (2015). To the authors knowledge, this is the first study where the analytical model with individual layers (abdominal aorta and fat tissue) is studied.

2 MATERIAL AND METHODS

All the computations performed within this study have been conducted in Maple (Maplesoft, Waterloo, Canada). The abdominal aorta represents inner layer of analytical model and was modeled as an incompressible, hyperelastic, anisotropic residually stressed homogeneous thick-walled closed tube. The perivascular adipose tissue surrounding the aorta was considered as incompressible, hyperelastic and isotropic without residual stress.

In what follows, constitutive model and geometrical data characterizing abdominal aorta of 38 years old male donor are adopted from Horny et al. (2014). They used results, originally published by Labrosse et al. (2013), to determine material parameters of the human abdominal aorta defined in the constitutive model suggested by Gasser et al. (2006).

Vonavkova et al. (2015) published preliminary results of the uniaxial tensile tests conducted with human perivascular adipose tissue. They used hyperelastic constitutive model based on Fung-Demiray strain energy density function. This stress-strain relationship is here adopted to model mechanical behavior of the adipose tissue (male donor 29 years old).

2.1 Kinematics of inflation and extension of a bilayer tube

After removal from a body, the abdominal aorta is in the load-free configuration Ω_{LF} . However, this state is not a stress-free (reference) configuration which is denoted as Ω_{SF} . It is known that cutting the arterial ring in the vessel axis leads to opening of the artery due to the release of residual stress. Here, it is assumed that the open sector is the stress-free (reference)

configuration Ω_{SF} (Sommer and Holzapfel, 2012), as depicted in Figure 1. Thus, the cylindrical coordinates of the tube (ρ, θ', z') in the reference configuration Ω_{SF} are defined in (1).

$$\rho_i \leq \rho \leq \rho_o, \quad 0 \leq \theta' \leq (2\pi - 2\alpha), \quad 0 \leq z' \leq \zeta \quad (1)$$

Here ρ_i , ρ_o , α and ζ denote the inner and outer radius of the abdominal aorta in undeformed configuration, opening angle and the length of the stress-free tube, respectively. The opening angle for reference configuration is shown in the Figure 1.

The adipose tissue is modeled as not subjected to residual deformation, and, therefore it can be said, that its reference configuration is the load-free configuration Ω_{LF} (Sommer and Holzapfel, 2012). See Figure 1.

The deformation χ (composition of the deformations χ_L and χ_{RES}) which maps Ω_{SF} into the deformed (loaded) configuration Ω_L is depicted in the Figure 1. The deformation χ_{RES} maps from Ω_{SF} to Ω_{LF} and can be understood as a bending of curved beam corresponding to the stress-free arterial strip. The residual stress is induced by means of χ_{RES} . The deformation χ_L is associated with axial stretch and inflation and leads to the deformed configuration Ω_L (Sommer and Holzapfel, 2012). In terms of the cylindrical coordinates (R, Θ, Z) , the region of Ω_{LF} is

$$R_i^A \leq R^A \leq R_o^A, \quad 0 \leq \Theta \leq 2\pi, \quad 0 \leq Z \leq L^A, \quad (2)$$

where R_i^A , R_o^A and L^A are the inner and outer radius and the length of the abdominal aorta in the load-free configuration Ω_{LF} , respectively. $\Theta = \frac{2\pi}{(2\pi-2\alpha)}\theta'$ and $Z = \delta z'$ holds in the mapping χ_{RES} . R_i^P , R_o^P , L^P ($L^P = L^A$), δ are the reference inner and outer radius, length and axial stretch of the perivascular tissue in the unloaded configuration.

The deformation χ_L (Sommer and Holzapfel, 2012) takes the load-free configuration Ω_{LF} into the deformed configuration Ω_L (Figure 1). In terms of cylindrical coordinates (r, θ, z) , the region of the current configuration is

$$r_i^A \leq r^A \leq r_o^A, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq l^A. \quad (3)$$

Here r_i^A , r_o^A and l^A denote the inner and outer radius and the length of the deformed abdominal aorta, respectively. r_i^P , r_o^P , l^P ($l^P = l^A$) are the deformed inner and outer radius, length and axial stretch of the perivascular tissue in the loaded configuration.

The kinematics of inflation and extension is described by the deformation gradient \mathbf{F}_L in (4). The deformation gradient \mathbf{F}_{RES} which considered closing of opened up circular sector as shown in the equation (4). The resulting kinematics is given as (5).

$$\mathbf{F}_{RES} = \begin{pmatrix} \frac{\partial R(\rho)}{\partial \rho} & 0 & 0 \\ 0 & \frac{\pi}{\pi-\alpha} \frac{R(\rho)}{\rho} & 0 \\ 0 & 0 & \delta \end{pmatrix}, \quad \mathbf{F}_L = \begin{pmatrix} \frac{\partial r(R)}{\partial R} & 0 & 0 \\ 0 & \frac{r(\rho)}{R} & 0 \\ 0 & 0 & \lambda_z^A \end{pmatrix} \quad (4)$$

$$\mathbf{F} = \mathbf{F}_L \mathbf{F}_{RES} = \begin{pmatrix} \lambda_r(r) & 0 & 0 \\ 0 & \lambda_\theta(r) & 0 \\ 0 & 0 & \lambda_z \end{pmatrix} = \begin{pmatrix} \frac{\partial r(\rho)}{\partial \rho} & 0 & 0 \\ 0 & \frac{\pi}{\pi-\alpha} \frac{r(\rho)}{\rho} & 0 \\ 0 & 0 & \lambda_z^A \delta \end{pmatrix} \quad (5)$$

Here λ_z^A ($\lambda_z^A = \lambda_z^P$) is the axial stretch of aorta which is identical with the stretch of the adipose tissue. λ_r , λ_θ and λ_z are the radial, circumferential and axial stretches, respectively.

During the deformation, the inner radius of the abdominal aorta and the inner radius of perivascular were expressed by means of the radius and the length via incompressibility condition (6) and (7), respectively.

$$\pi l^A (r_o^{2A} - r_i^{2A}) = (\pi - \alpha) L^A (\rho_o^2 - \rho_i^2), \quad \lambda_z^A = \frac{l^A}{L^A}. \quad (6)$$

$$\pi L^P (R_o^{2P} - R_i^{2P}) = \pi l^P (r_o^{2P} - r_i^{2P}), \quad \lambda_z^P = \frac{l^P}{L^P}. \quad (7)$$

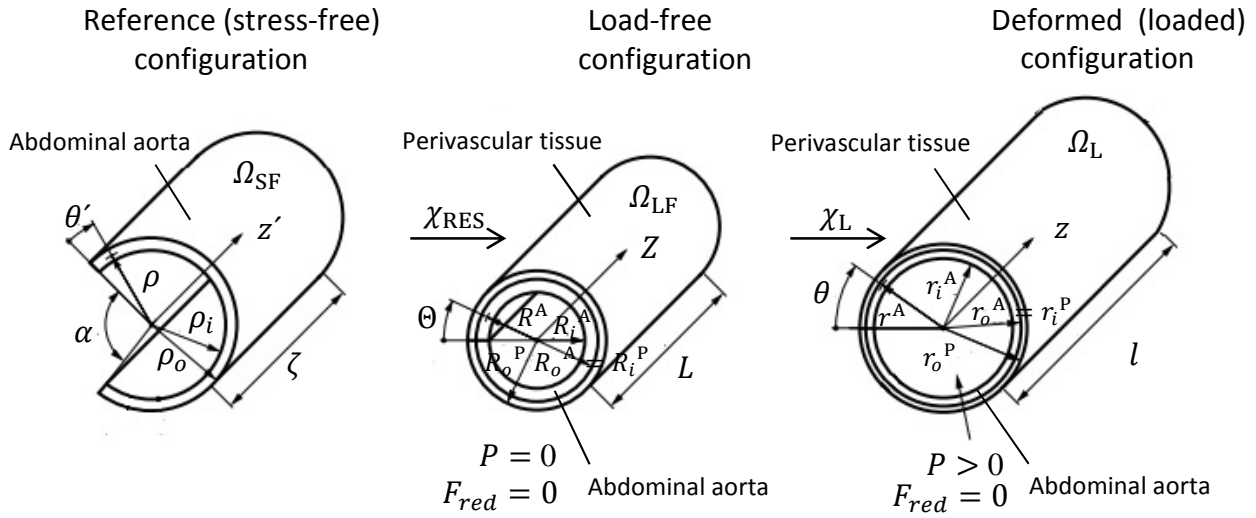


Figure 1: Kinematics of the abdominal aorta and the perivascular tissue.

The arterial ring with residual stress in the reference (stress-free) configuration Ω_{SF} .
The abdominal aorta with surrounding perivascular tissue in the load-free configuration Ω_{LF} but without internal pressure.
The bilayer tube in the deformed (loaded) configuration Ω_L after the application of internal pressure P .

2.2 Constitutive models

According to Ogden (1982), the constitutive equation for an incompressible hyperelastic material can be written in the form of (8).

$$\sigma = 2\mathbf{F} \frac{\partial W}{\partial \mathbf{C}} \mathbf{F}^T - \mathbf{I}p. \quad (8)$$

Here σ denotes the Cauchy stress tensor. \mathbf{C} is the right Cauchy-Green strain tensor, $\mathbf{C} = \mathbf{F}^T \mathbf{F}$. p plays the role of a Lagrange multiplier, which represents the hydrostatic contribution to σ , not captured by W , due to the incompressibility constraint.

2.2.1. Abdominal aorta

The abdominal aorta wall was modeled as a homogenous, anisotropic, incompressible, hyperelastic material. The strain energy density function W_{GOH} was used, proposed by

Gasser et al. (2006) for modeling general mechanical characteristics. The strain energy density function is expressed by equation (9).

$$W_{GOH} = \frac{\mu}{2}(I_1 - 3) + \sum_{j=4,6} \left(\frac{k_1}{2k_2} e^{k_2(K_j-1)^2} - 1 \right), \quad (9)$$

$$K_j = \kappa I_1 + (1 - 3\kappa)I_j, \quad j = 4, 6, \quad (10)$$

with

$$I_1 = \lambda_r^2 + \lambda_\theta^2 + \lambda_z^2, \quad I_4 = I_6 = \lambda_\theta^2 \cos^2 \beta + \lambda_z^2 \sin^2 \beta, \quad (11)$$

where $\mu = 15.9 \text{ kPa}$ is the infinitesimal shear modulus of the isotropic matrix. $k_1 = 78.488 \text{ kPa}$ is a stress-like material parameter and $k_2 = 4.911$ is a dimensionless parameter. The material parameters were adopted from Horny et al. (2014). K_4 and K_6 are generalised invariants related to imperfect fiber alignment. $\kappa = 0.189$ is a structural parameter which measures the degree of fiber dispersion ($0 \leq \kappa \leq \frac{1}{3}$). I_1 is the first invariant of the right Cauchy-Green strain tensor. I_4 and I_6 are additional strain invariants induced by the existence of preferred directions in a continuum. The parameter $\beta = 41.41^\circ$ denotes the angle between the (mean) fiber direction and the circumferential direction in the individual layers, and, therefore, acts as a geometrical parameter.

2.2.2. Perivascular tissue

For the perivascular adipose tissue, the exponential function of strain energy density W_D proposed by Demiray (1972) was used (12).

$$W_D = \frac{1}{2} \frac{\mu \left(e^{b(\lambda_r^2 + \lambda_\theta^2 + \lambda_z^2 - 3)} - 1 \right)}{b}, \quad (12)$$

with stress-like parameter $\mu = 18.397 \text{ kPa}$ and dimensionless parameter $b = 31.834$. These material parameters were taken from Vonavkova et al. (2015).

2.3 Thick-walled bilayer tube model

The equilibrium equations for the closed incompressible hyperelastic thick-walled tube in the radial and axial direction are expressed in (13) and (14) for the inner layer (the abdominal aorta) and in (15), (16) for the outer layer (the perivascular tissue). Their form is adopted from Holzapfel and Ogden (2010), Labrosse et al. (2013) and Horny et al. (2013). Detailed derivation can be found in Matsumoto and Hayashi (1996). The indices A, and P are used to distinguish between individual layers (aorta, perivascular tissue). The boundary conditions are considered in the form $\sigma_{rr}(r_i^A) = -P$, $\sigma_{rr}(r_o^P) = 0$ and $\sigma_{rr}(r_o^A) = \sigma_{rr}(r_i^P)$. Further, the condition $\lambda_z^A = \lambda_z^P$ was considered.

$$P^A = \int_{r_i^A}^{r_o^A} \lambda_\theta^A \frac{\partial \hat{W}^A}{\partial \lambda_\theta^A} \frac{dr^A}{r^A} \quad (13)$$

$$F_{red}^A = \pi \int_{r_i^A}^{r_o^A} \left(2\lambda_z^A \frac{\partial \hat{W}^A}{\partial \lambda_z^A} - \lambda_\theta^A \frac{\partial \hat{W}^A}{\partial \lambda_\theta^A} \right) r^A dr^A \quad (14)$$

$$P^P = \int_{r_i^P}^{r_o^P} \lambda_\theta^P \frac{\partial \hat{W}^P}{\partial \lambda_\theta^P} \frac{dr^P}{r^P} \quad (15)$$

$$F_{red}^P = \pi \int_{r_i^P}^{r_o^P} \left(2\lambda_z^P \frac{\partial \hat{W}^P}{\partial \lambda_z^P} - \lambda_\theta^P \frac{\partial \hat{W}^P}{\partial \lambda_\theta^P} \right) r^P dr^P \quad (16)$$

Here \hat{W} denotes the strain energy density function with eliminated explicit dependence on λ_r by substituting $\lambda_r = \frac{1}{\lambda_\theta \lambda_z}$. P indicates internal pressure and F_{red} is the reduced axial (prestretching) force acting on the closed end of the tube additionally to the force generated by the pressure acting on the end (Horny et al., 2013; Horny et al., 2014; Matsumoto and Hayashi, 1996).

The distributions of the principal Cauchy stress components $\sigma_{\theta\theta}$, σ_{zz} and σ_{rr} across the deformed wall thickness (aorta and perivascular tissue layers) are given equations (17), (18), (19).

$$\sigma_{rr}(r) = - \int_r^{r_o} \lambda_\theta \frac{\partial \hat{W}}{\partial \lambda_\theta} \frac{dr}{r} \quad (17)$$

$$\sigma_{\theta\theta} = \lambda_\theta \frac{\partial \hat{W}}{\partial \lambda_\theta} + \sigma_{rr} \quad (18)$$

$$\sigma_{zz} = \lambda_z \frac{\partial \hat{W}}{\partial \lambda_z} + \sigma_{rr} \quad (19)$$

2.4 Inflation-extension behavior of bilayer tube model

Horny et al. (2014) fitted experimental data from Labrosse et al. (2013) by W_{GOH} hyperelastic model and estimated material parameters $(\mu, k_1, k_2, \beta, \kappa)$ and subsequently determined variable radii in an undeformed configuration (ρ_i, ρ_o) and axial deformation which arose during closing of the aorta ring (δ).

The reference geometrical data ($R_i = 5.3 \text{ mm}, H = 1.22 \text{ mm}, \alpha = 117^\circ$) of human abdominal aorta (male age 38 years) from Labrosse et al. (2013), material parameters ($\mu = 15.90 \text{ kPa}, k_1 = 78.49 \text{ kPa}, k_2 = 4.991, \beta = 41.41, \kappa = 0.1875$) and geometrical parameters ($\rho_i = 16.20 \text{ mm}, \rho_o = 17.42 \text{ mm}$) from Horny et al. (2014) were chosen for purpose of this study.

The outer radius of the aorta and inner radius of the external fatty tube ($R_o^A = R_i^P, r_o^A = r_i^P$) were considered to be equal in the pressurization. The outer radius of adipose tissue was chosen ($R_o^P = 16.52 \text{ mm}$). The value of the outer radius corresponds approximately to the amount of fat in the selected donor (male age 38 years).

The inner and outer radius of perivascular tissue (r_i^P, r_o^P) and axial stretch ($\lambda_z^A = \lambda_z^B$) have been determined by means of nonlinear least squares regression method. The objective function Q (20) was minimized in Maple (Maplesoft, Waterloo, Canada) subjected to the constraint $F_{red} = 0$.

$$Q = \sum_{i=0}^{n=16} \left((P_i^{\text{exp}} - P_i^{\text{mod}})^2 - (F_{red,i}^{\text{exp}} - F_{red,i}^{\text{mod}})^2 \right) \quad \text{for } i = 0, 1, 2, \dots, 16 \text{ kPa} \quad (20)$$

where

$$P_i^{\text{exp}} = P_i^A + P_i^P, \quad F_{red,i}^{\text{exp}} = F_{red,i}^A + F_{red,i}^P. \quad (21)$$

3 RESULTS

The results of analytical model of bilayer thick-walled closed tube consisting of both the human abdominal aorta (index A) and surrounding perivascular tissue (index P) are summarized in the Figure 2. The bilayer model is compared with model containing only abdominal aorta. The plots of the pressure versus circumferential stretch (λ_θ) or axial stretch

(λ_z) are depicted in the Figure 2 A, B. The radial, circumferential and axial Cauchy stress ($\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$) versus deformed radius (r) at the pressure 16 kPa are showed in the Figure 2C.

The axial stretch of the abdominal aorta and adipose tissue was equal during inflation-extension, but the analytical model shows that the tangential and axial stress differs significantly depending on the thickness of the outer tube. The radial stress satisfies the boundary condition $\sigma_{rr}(r_i^A) = -P$, $\sigma_{rr}(r_o^P) = 0$ and $\sigma_{rr}(r_o^A) = \sigma_{rr}(r_i^P)$. On the contact with the aorta and perivascular tissue is an abrupt change in the axial stress by 80 kPa.

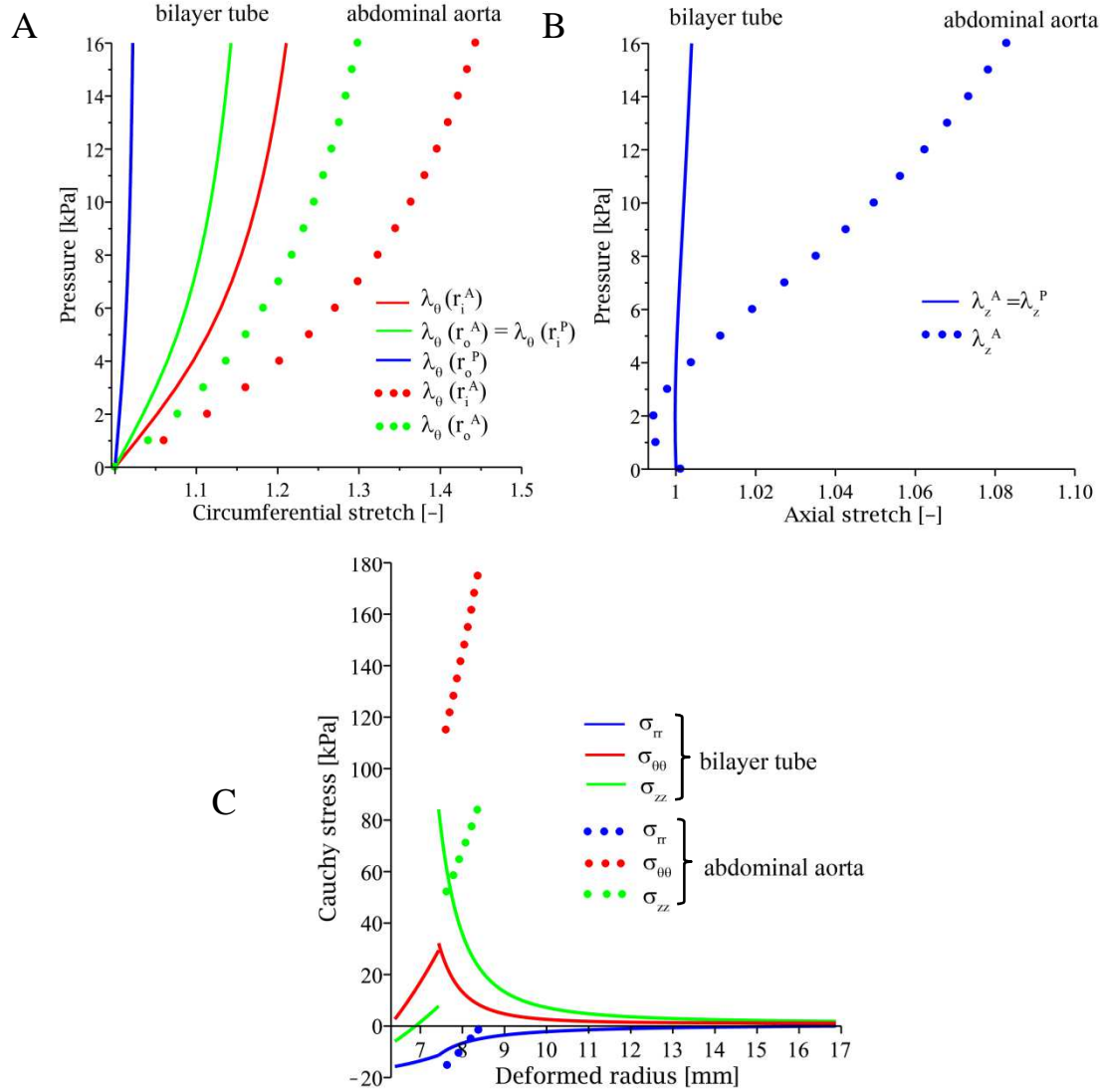


Figure 2 A The plot of the pressure versus circumferential stretch (λ_{θ}) and comparison of bilayer tube (lines) with abdominal aorta (points).

B The plot of the pressure versus axial stretch (λ_z) and comparison of bilayer tube (lines) with abdominal aorta (points).

C The radial, circumferential and axial Cauchy stress ($\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$) versus deformed radius for bilayer tube (lines) and abdominal aorta (points) at the pressure 16 kPa.

4 DISCUSSION

The objective of this study was to compare the mechanical behavior of the bilayer thick-walled close tube, where the inner layer was a human abdominal aorta and the outer layer was perivascular tissue, with abdominal aorta tube. The abdominal aorta and adipose tissue were

modeled as a homogeneous, anisotropic, incompressible and hyperelastic material. The strain energy density functions W_{GOH} , W_D were used for the description of mechanical response of inner and outer layer of the model.

The simulations showed that the abdominal aorta is much more compliant in the circumferential and axial direction in comparison with the fat tissue. The stress along the wall thickness of bilayer model and only aorta tube was determined and compared.

The results of this study show that the mechanical responses are significantly different, therefore it is uncertain, whether a monolayer model without the surrounding tissue may correspond to *in vivo* reality.

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