ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

EVOLUTIONARY LEVEL SET METHOD FOR CRASHWORTHINESS TOPOLOGY OPTIMIZATION

Mariusz Bujny[1], **Nikola Aulig**[2], **Markus Olhofer**[2], **Fabian Duddeck**[1]

[1]Technische Universität München
Arcisstraße 21, 80333 Munich, Germany
e-mail: {mariusz.bujny, duddeck}@tum.de

[2] Honda Research Institute Europe GmbH
Carl-Legien-Straße 30, 63073 Offenbach/Main, Germany
e-mail: {nikola.aulig, markus.olhofer}@honda-ri.de

Keywords: Level Set Method, Evolutionary Algorithms, Topology Optimization, Crashworthiness.

Abstract. *Vehicle crashworthiness design belongs to one of the most complex problems considered in the design optimization. Physical phenomena that are taken into account in crash simulations range from complex contact modeling to mechanical failure of materials. This results in high nonlinearity of the optimization problem and involves remarkable amount of numerical noise and discontinuities of the objective functions that are optimized. Consequently, the sensitivity information, which is necessary for the majority of Topology Optimization approaches, can be obtained analytically only for considerably simplified problems, which, in most cases, excludes the use of the gradient-based optimization methods. As a result, in the state-of-the-art methods for crashworthiness Topology Optimization, strong and thus arguable assumptions about the properties of the optimization problem are made and heuristic approaches are used. This problem can be solved with use of Evolutionary Algorithms, where no assumptions about the optimization problem have to be made and which perform well even for highly nonlinear and discontinuous problems. We propose a novel approach using evolutionary optimization techniques together with a geometric Level-Set Method in crashworthiness Topology Optimization. Both standard Evolution Strategies and the state-of-the-art Covariance Matrix Adaptation Evolution Strategy are used. In order to evaluate the proposed method, an energy maximization problem for a rectangular beam, fixed at both ends and impacted in the middle by a cylindrical pole, is considered. The results show that the evolutionary optimization methods can be efficiently used for an optimization of crash-loaded structures, while defining the objective function explicitly.*

1 INTRODUCTION

In the recent years, a lot of effort has been put in the automotive industry to shorten product cycles as much as possible. This is one of the most crucial demands of the car market that is also pushing car companies to broaden the offer of their products and increase the efficiency of development and production. Also the complexity of the vehicles produced today is increasing rapidly, which, together with rising material costs and strict CO_2 emission reduction targets for new cars, forces car companies to use numerical simulation and optimization in the vehicle design. The number of factors to be taken into account and fields to analyze is rising constantly, as well. However, crashworthiness optimization, due to its complexity, poses the main difficulty and affects the character of the whole vehicle design process [11].

The wide spectrum of physical phenomena incorporated in crash simulations, ranging from complex contact modeling to mechanical failure of materials, results in high nonlinearity of the optimization problem, numerical noise and discontinuities of the optimized objective functions. As a result, in general case, analytical gradients are not available and the gradient-based optimization methods cannot be applied. Therefore, development of alternative approaches for crashworthiness optimization is necessary.

The main focus of this paper is crashworthiness Topology Optimization, which is used to determine the best structural concept at early stages of the vehicle development process. Due to the complexity of crash phenomena mentioned above, in the state-of-the-art methods for crashworthiness Topology Optimization, very strong assumptions about the properties of the optimization problem are made and heuristic approaches are used to optimize structures. In the Equivalent Static Loads Methods [6, 7, 12, 29], equivalence of static and dynamic loads is assumed. In the Ground Structure Approaches [14, 31], simplified crash models, involving considerable calibration effort, are used. Graph and Heuristic approaches use rules derived from the expert knowledge [28]. Finally, both in the Hybrid Cellular Automata [30, 25] and the Hybrid Cellular Automata for Thin-Walled Structures [13, 22] homogeneity of energy density either all over the structure or in larger macro-structures is required. As a result, in each of those methods, the assumptions are arguable and convergence to the true optima cannot be guaranteed.

An alternative approach is to use evolutionary optimization methods, which are gradient-free and work very well even for highly nonlinear, noisy, discontinuous problems. In evolutionary methods, the optimization process is carried out solely through evaluating values of the objective function in different points of the design space, thus no additional assumptions have to be made. Therefore, the objective function can be precisely defined and solutions are exclusively judged by their objective values.

In this paper, we propose a novel approach for crashworthiness Topology Optimization, which uses an implicit parametrization of mechanical structures with geometric level-set functions [15]. In the Level-Set Methods [10], the material interface is precisely defined by the iso-contours of the level-set function, which is a crucial property from the point of view of manufacturability and the accuracy of crash simulations [5], where the presence of intermediate densities in the finite element model might lead to severe deviations from the real crash behavior. Moreover, by introducing the parameterization with the geometric level-set functions [15], the number of design variables can be reduced significantly, while allowing for a relatively high flexibility of topological changes. As a result, Evolutionary Algorithms, whose numerical costs highly depend on the dimensionality of the optimization problem, can be used efficiently for this type of parametrization.

Evaluation and validation of the proposed method is realized for a simple, 2D crash case, where a cylindrical pole impacts a clamped structure defined within a rectangular design domain. An energy maximization problem with mass constraint is considered. The optimization is carried out with use of standard Evolution Strategies (ES) [4] and the state-of-the-art Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [17]. Finally, performance of the structures optimized with use of the above mentioned methods and the Hybrid Cellular Automata technique is compared.

The paper has the following structure. Section 2 formulates the optimization problem and presents an overview of the optimization methods used in this research. In Section 3 the implicit parametrization with the geometric level-set functions is described. The experimental setup, optimization results and a corresponding discussion is presented in Section 4. The final conclusions are described in Section 5.

2 OPTIMIZATION PROBLEM

2.1 Problem formulation

In this paper, the problem of energy absorption E_{abs} maximization[1] at a given point of a crash event and under mass constraint is considered. Similar optimization problems were investigated by Hunkeler [22] and Aulig et al. [3]. In the most general form, the optimization problem can be formulated as follows:

$$\begin{aligned} &\min_{\mathbf{z}} \left(-E_{abs}(\mathbf{z})\right) \\ &\qquad \text{s.t.} : \mathbf{r}(t) = \mathbf{0}, \\ &\qquad\qquad m(\mathbf{z}) \leq m_{req}, \end{aligned} \tag{1}$$

where the residual $\mathbf{r}(t) = \mathbf{0}$ expresses the dynamic equilibrium at time t, m is mass of the structure and m_{req} is the required mass of the final design. The vector of design variables is denoted by $\mathbf{z}$.

Since this is a constrained optimization problem, an appropriate constraint-handling technique has to be chosen. In the field of evolutionary optimization, the most commonly used approaches for constrained optimization are the exterior penalty methods [9, 32]. For simplicity, static penalties [24] were used in this work, for both standard ES and the CMA-ES[2]. In the context of evolutionary computation a minimum of so-called fitness function has to be found. For the given problem, after introducing the penalty term, it takes the following form:

$$f(\mathbf{z}) = -E_{abs}(\mathbf{z}) + c \cdot \max\left(0, m(\mathbf{z}) - m_{req}\right), \tag{2}$$

where c is a weighting coefficient for the mass constraint.

2.2 Optimization methods

Biologically-inspired evolutionary optimization methods turned out to be very successful in many engineering applications [8, 27, 37, 21]. In particular, they showed very good capabilities

[1]Equivalent to minimization of negative energy absorption.

[2]Very little research on constraint handling for the CMA-ES has been done so far [23, 2] and behavior of the algorithm for more sophisticated constraint-handling techniques is not known. Therefore, the use of simpler approaches is favored.

for solving multimodal, noisy and discontinuous optimization problems. This makes their application to the crashworthiness Topology Optimization very promising. Below, a short overview of the evolutionary methods used in this paper is presented.

2.2.1 Evolution Strategy

Evolution Strategies were developed in Germany by the research group of Ingo Rechenberg and Hans-Paul Schwefel [33, 34]. Compared to genetic algorithms, the primary emphasis in Evolution Strategies lies on mutation instead of recombination. Most of evolution strategies are based on the following core structure:

```
t:=0;
initialize: P(0) := {a_1(0), ..., a_μ(0)} ∈ I^μ;
evaluate P(0) : {f(a_1(0)), ..., f(a_μ(0))};
while (ι(P(t)) ≠ true) do
    recombine: P'(t) := r_Θr(P(t));
    mutate: P''(t) := m_Θm(P'(t));
    evaluate P''(t) : {f(a''_1(t)), ..., f(a''_λ(t))};
    select: P(t+1) := s_Θs(P''(t));
    t:=t+1;
end
```

Algorithm 1: Standard Evolutionary Algorithm [4].

where $f : I \to \mathbb{R}$ denotes the fitness function to be minimized (I is the genotype space) and $\mathbf{a} \in I$ is an individual. The size of the parent and offspring population are denoted by $\mu \geq 1$ and $\lambda \geq \mu$, respectively. A population at generation t comprises all the parent individuals, i.e. $P(t) := \{\mathbf{a}_1(t), ..., \mathbf{a}_\mu(t)\}$. The recombination operator is a mapping $r_{\Theta_r} : I^\mu \to I^\lambda$, whereas mutation operator: $m_{\Theta_m} : I^\lambda \to I^\lambda$. Both recombination and mutation are controlled by sets of operator parameters: Θ_r and Θ_m. The selection operator defined as $s_{\Theta_s} : I^\lambda \to I^\mu$ is used to choose the individuals, which compose the parent population in the next generation.

The functioning of the algorithm can be characterized as follows. After the initialization and evaluation of the parent population, the main optimization loop begins. First of all, the new offspring is generated from the parent population in the recombination step. Recombination is usually performed both on design variables and strategy parameters. Depending on the type of recombination, it can be performed according to one of the following rules [4]:

$$z'_i = \begin{cases} z_{S,i} & \text{without recombination,} \\ z_{S,i} \text{ or } z_{T,i} & \text{discrete recombination,} \\ z_{S,i} + \chi \cdot (z_{T,i} - z_{S,i}) & \text{intermediate recombination,} \\ z_{S_i,i} \text{ or } z_{T_i,i} & \text{global, discrete recombination,} \\ z_{S_i,i} + \chi_i \cdot (z_{T_i,i} - z_{S_i,i}) & \text{global, intermediate recombination,} \end{cases} \tag{3}$$

where z_i is the i-th component of the vector of object variables, S and T denote two randomly selected parent individuals, whereas $\chi \in [0, 1]$ is a random variable from the uniform distribution. In case of the global recombination, parents S and T as well as the χ factor are chosen independently for each component of the $\mathbf{z}$ vector.

According to Bäck et al. [4], the best results were observed for discrete recombination on design variables and intermediate recombination on strategy parameters. Also historically, recombination in its intermediate and global form were applied for a constant value of $\chi = \frac{1}{2}$.

In the second step of the main optimization loop, the mutation operator is used. In the most general form, the mutation operator produces mutated individuals by deviating first the strategy parameters and then the design variables:

$$\begin{aligned} \sigma_i' &= \sigma_i \cdot \exp\left(\tau' \cdot N(0,1) + \tau \cdot N_i(0,1)\right) \\ \mathbf{z}' &= \mathbf{z} + \mathbf{N}\left(\mathbf{0}, \boldsymbol{\sigma}'\right), \end{aligned} \tag{4}$$

where τ' and τ are global and local learning rates, respectively.

For a single standard deviation for all object variables, (4) can be reduced to:

$$\begin{aligned} \sigma' &= \sigma \cdot \exp\left(\tau' \cdot N(0,1)\right) \\ \mathbf{z}' &= \mathbf{z} + \mathbf{N}\left(\mathbf{0}, \sigma'\right) \end{aligned} \tag{5}$$

According to Schwefel [36] a good choice of those parameters is the following:

$$\begin{aligned} \tau &= \frac{1}{\sqrt{2\sqrt{n}}} \\ \tau' &= \frac{1}{\sqrt{2n}} \end{aligned} \tag{6}$$

The mutation mechanism presented above allows the algorithm to evolve its own strategy parameters during the optimization process. As a result, it is often referred to as the "self-adaptation" mechanism and was first formulated by Schwefel [35].

Finally, after the evaluation step, the best individuals are chosen to form a new parent population. There are two main methods to do that. Either μ parents are selected from λ offspring individuals ((μ, λ)-ES) or the new parent population is selected out of parent and offspring populations combined ($(\mu + \lambda)$-ES). In this research, we use the (μ, λ)-ES since it can better deal with noisy quality evaluations [4].

2.2.2 Covariance Matrix Adaptation Evolution Strategy

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [19] is a derandomized Evolution Strategy, where the covariance matrix of the normal mutation distribution is adapted on the basis of the previous search steps [17]. This is a similar concept to the gradient-based quasi-Newton methods, where the Hessian matrix is estimated iteratively as the optimization process progresses. Initially, the method was designed for small populations and has proven to be a robust and efficient local search strategy [16]. Particularly, the CMA-ES can minimize efficiently unimodal functions [17]. The superiority of the method on non-separable and ill-conditioned problems and its applicability to real world problems has been also demonstrated [19]. An extension of the CMA-ES by the rank-μ-update [26, 20] allowed to use more effectively the information from large populations without influencing the performance when small populations are considered. As it is one of the state-of-the-art methods, its Python implementation [18] was used in this research, as well.

3 PARAMETRIZATION

Let us first define a global level-set function as follows:

$$\begin{cases} \Phi(\mathbf{x}) > 0, \mathbf{x} \in \Omega, \\ \Phi(\mathbf{x}) = 0, \mathbf{x} \in \partial\Omega, \\ \Phi(\mathbf{x}) < 0, \mathbf{x} \in D\backslash\Omega. \end{cases} \tag{7}$$

Analogically, we introduce a local level-set function of the i$^{\text{th}}$ elementary structural component having the following property:

$$\begin{cases} \phi_i(\mathbf{x}) > 0, \mathbf{x} \in \Omega_i \\ \phi_i(\mathbf{x}) = 0, \mathbf{x} \in \partial\Omega_i \\ \phi_i(\mathbf{x}) < 0, \mathbf{x} \in D\backslash\Omega_i \end{cases} \tag{8}$$

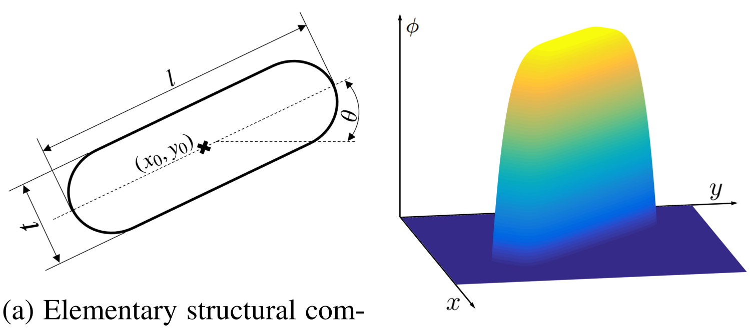
where Ω_i is a part of the design domain D occupied by an elementary component. As a result, the material phase is defined as:

$$\Omega = \bigcup_{i=1}^{e} \Omega_i, \tag{9}$$

where e denotes the number of elementary components. We introduce a level-set function (defining an elementary component) after Guo et al. [15], which for $D = \mathbb{R}^2$, $\mathbf{x} = (x, y)^T$, has the following form:

$$\begin{aligned} \phi_i(\mathbf{x}) = -\Bigg(& \left(\frac{\cos\theta_i\,(x - x_{0i}) + \sin\theta_i\,(y - y_{0i})}{l_i/2} \right)^q \\ & + \left(\frac{-\sin\theta_i\,(x - x_{0i}) + \cos\theta_i\,(y - y_{0i})}{t_i/2} \right)^q - 1 \Bigg), \end{aligned} \tag{10}$$

where (x_0, y_0) is the position of the center of a component (see Figure 1a) with length l and thickness t. The rotation angle of a component is denoted by θ. Similarly to the approach presented by Guo et al. [15], we take the exponent $q = 6$. The plot of the level-set function defined by (10) is presented in the Figure 1b.



(a) Elementary structural component.

(b) Level-set function.

Figure 1: Parametrization of the elementary structural component and the corresponding level-set function (where negative values are set to zero) [15].

As in the other Topology Optimization approaches [5], we use a standard density-based geometry mapping [10], where the dependency between the level-set field and the material density[3] $\rho(\mathbf{x})$ at position $\mathbf{x} \in D$ is given by:

$$\rho(\mathbf{x}) = H(\Phi(\mathbf{x})), \tag{11}$$

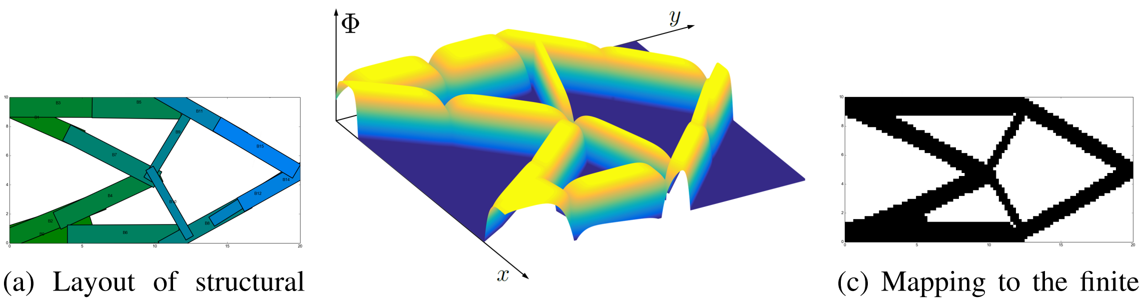
where:

$$\Phi(\mathbf{x}) = \max(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), ..., \phi_m(\mathbf{x})), \tag{12}$$

and $H(x)$ denotes the Heaviside function:

$$H(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0. \end{cases} \tag{13}$$

Figure 2 presents a possible layout of structural components for the compliance minimization of the standard cantilever beam benchmark case [5]. Plots of the corresponding (global) level-set function and its mapping to the finite element mesh are shown, as well.



(a) Layout of structural components.

(b) Corresponding level-set field.

(c) Mapping to the finite element mesh.

Figure 2: Possible layout of structural components for the compliance minimization for the cantilever beam problem [5].

4 NUMERICAL EXPERIMENTS

4.1 Setup

As a test problem, topology optimization of a rectangular aluminum beam is considered. The beam is fixed at both ends and impacted in the middle by a cylindrical pole. Dimensions of the beam, as well as initial and boundary conditions are shown in Figure 3.

The LS-Dyna FEM mesh is composed of 1600 eight-node solid elements and the problem is considered as a 2D crash case (displacements of all nodes in the direction perpendicular to the cross-section shown in Figure 3 are set to 0). A piecewise linear elastic-plastic material model is used. Exact setup of the test case and material properties are given in Table 1.

The initial layout of structural components and the mapping of the corresponding level-set field to the LS-Dyna FEM mesh are shown in Figure 4.

[3] In the finite element discretization, $\rho(\mathbf{x}) = 0$ results in deletion of the finite element from the mesh.

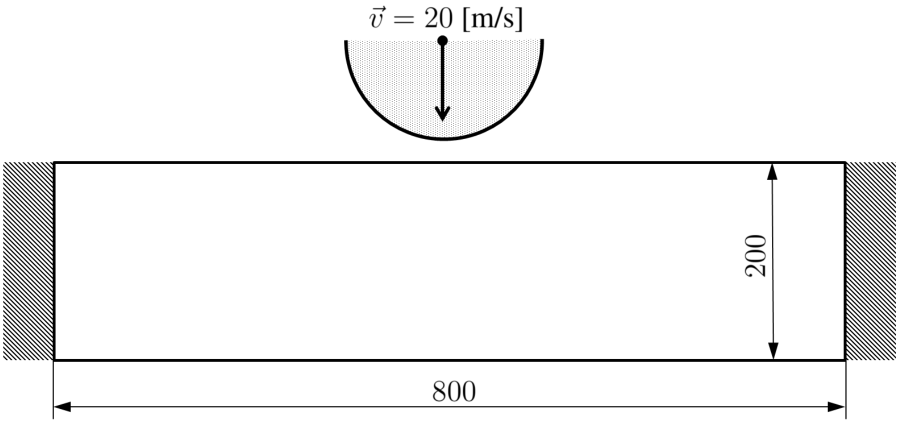
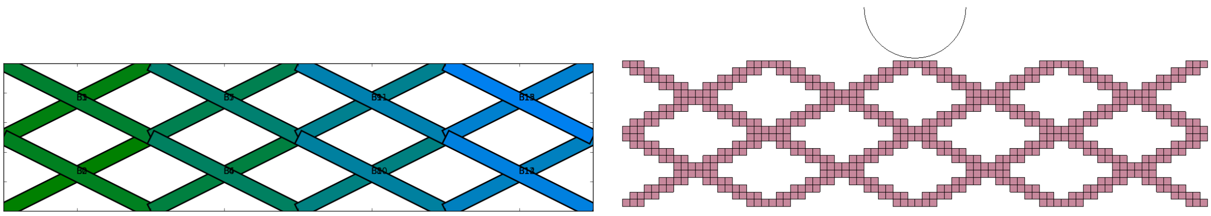


Figure 3: Design domain, initial and boundary conditions for the investigated crash case.

Property	Symbol	Value	Unit
Beam material density	ρ	$2.7 \cdot 10^3$	kg/m^3
Young's modulus	E	$7.0 \cdot 10^4$	MPa
Poisson's ratio	ν	0.33	-
Yield stress	R_e	241.0	MPa
Tangent modulus	E_{tan}	70.0	MPa
Friction coefficient	μ	0.1	-
Pole velocity	v	20	m/s
Pole mass	m_p	11.815	kg
Required structure mass	m_{req}	2.16	kg
LS-Dyna termination time	t_{end}	1.5	ms
LS-Dyna mesh resolution	-	80 x 20	-

Table 1: Configuration of the test case.



(a) Layout of structural components. (b) Corresponding finite element mesh.

Figure 4: Initial layout of structural components and the corresponding LS-Dyna finite element mesh.

4.2 Results

For evaluation of the proposed topology optimization approach, 10 optimization runs of both the standard Evolution Strategy (ES) and the state-of-the-art Covariance Matrix Adaptation Strategy (CMA-ES) were completed. In each case, the optimizations were stopped after 1000 generations. For both algorithms an offspring population of the size of 17 and a parent population consisting of 8 individuals[4] were used. In both cases, the initial value of σ was set to 0.1.

The results show that both ES and the CMA-ES can be successfully used for optimization of energy-absorbing structures. In Figure 5, averaged convergence of the fitness function for both ES and CMA-ES is presented. Figure 6 shows the statistical evaluation of both algorithms.

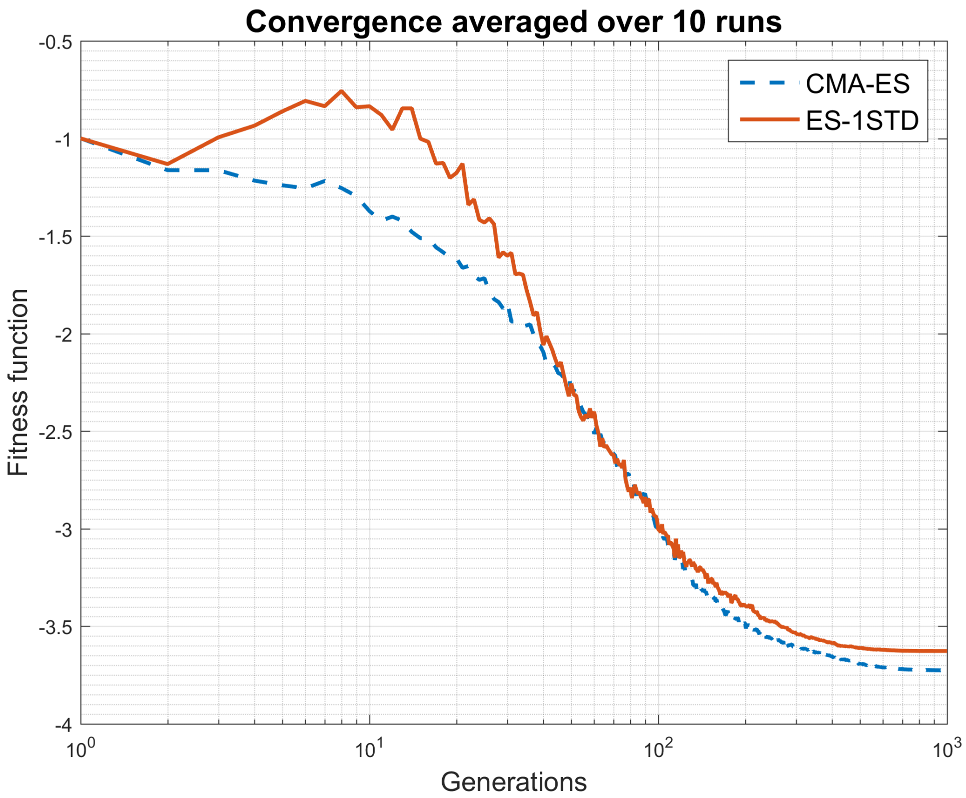


Figure 5: Convergence of the fitness function averaged over 10 runs of ES and CMA-ES.

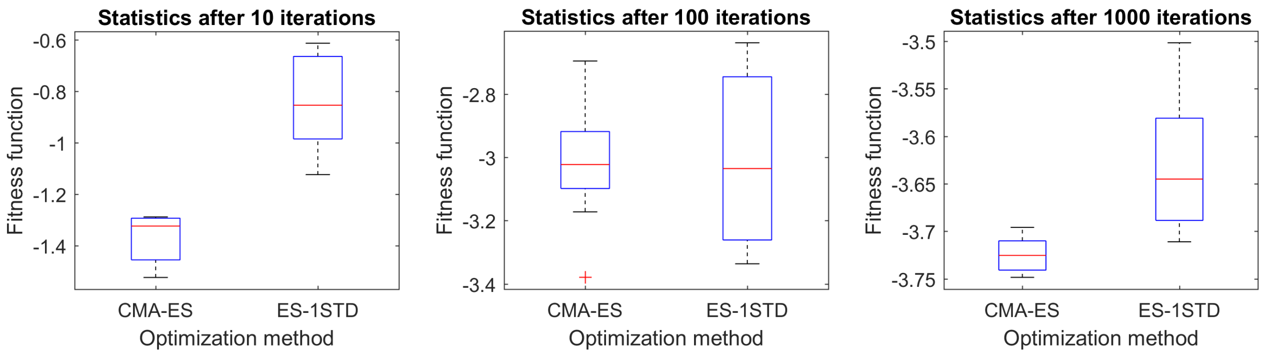


Figure 6: Box plots for 10 optimization runs of ES and CMA-ES after 10, 100 and 1000 generations, respectively.

[4] A default population size for a problem with 80 design variables, estimated and used by the Python implementation of the CMA-ES [18].

The CMA-ES exhibits considerably better performance at the beginning and the end of the optimization process. However, in the range especially interesting for practical applications (ca. 100 generations - equivalent to 1700 evaluations), the performance of both methods is similar. Nevertheless, in general, ES much more frequently results in inferior designs and is characterized by a relatively high variance of the fitness function.

The best designs out of 10 optimizations carried out with ES and CMA-ES are shown in Table 2. As a reference, a design obtained with the Hybrid Cellular Automata Technique is shown, as well.

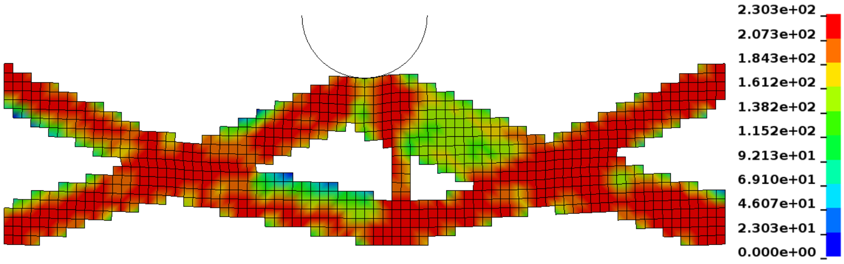
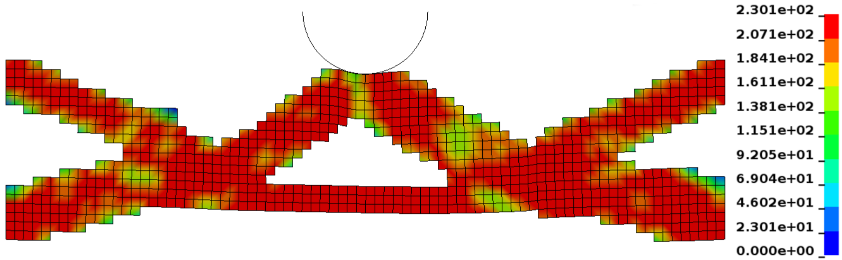
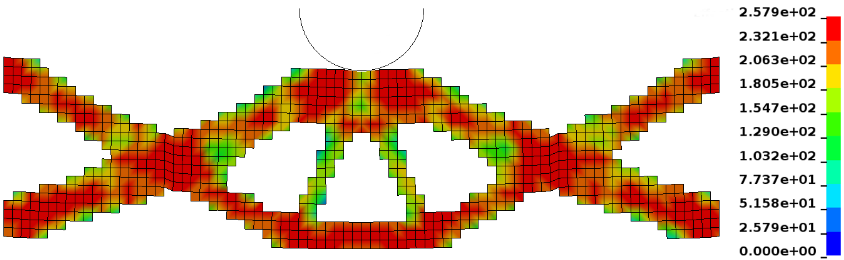
<table>
<tr><th>Method</th><th>Effective stress field (von Mises) at the final time step [MPa]</th><th>Energy [kJ]</th></tr>
<tr><td>ES-1STD</td><td>
</td><td>2.23</td></tr>
<tr><td>CMA-ES</td><td>
</td><td>2.25</td></tr>
<tr><td>HCA</td><td>
</td><td>2.24</td></tr>
</table>

Table 2: Best designs obtained within 10 optimization runs of ES and CMA-ES and a design optimized with the HCA method (with use of the LS-TaSC [1]). Von Mises stress field and energy absorption.

All of the designs exhibit similar performance with respect to energy absorption, but the design obtained with CMA-ES performs slightly better than the design obtained with the HCA method. This justifies the use of the proposed method, as a good alternative for the heuristic methods. Of course, the computational cost in case of evolutionary methods is considerably higher, but it may result in much better designs, especially when highly nonlinear cases are considered and the assumptions used in the HCA method are no longer correct. Another advantage of Evolutionary Algorithms is their good scalability on parallel machines, which can significantly reduce the optimization time and enable their use in industrial applications.

5 CONCLUSIONS

In this paper, a novel approach for crashworthiness Topology Optimization was presented. This technique uses Evolutionary Algorithms for optimization of crash structures parameterized implicitly with geometric level-set functions. For evaluation of the proposed method, optimization of the topology of a rectangular, clamped beam, impacted in the middle by a cylindrical pole, was considered. Performance of both standard Evolution Strategy and the Covariance Matrix Adaptation Evolution Strategy was compared.

The results show that the proposed approach can be successfully used as an optimization tool in the initial phase of development of crash structures. It can be considered as an attractive alternative for the state-of-the-art, often heuristic methods, such as the Hybrid Cellular Automata technique, since in evolutionary-based methods no additional assumptions have to be made and the optima can be precisely identified. Furthermore, unlike in the other approaches, handling of different objectives and constraints is straightforward, what broadens considerably the scope of possible applications. However, evolutionary optimization methods require much higher number of fitness function evaluations, which, especially in case of costly crash simulations, might pose a difficulty for industrial applications. Nevertheless, the excellent scalability properties of evolutionary methods make their use on parallel machines very efficient and thus, make them relevant also for the industry.

All in all, the future research in this field is promising, since the proposed methodology might be successfully used also in case of alternative objective functions even in case of highly nonlinear and noisy crash events, where the applicability of the standard crashworthiness optimization methods is limited. Further parallelization of simulations can help to overcome the problems associated with high computational costs and make the proposed approach an efficient method for crashworthiness optimization in early stages of the product development process.

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