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THE OPTIMAL CONTROL OF A MULTI-MASS VIBRATION PROPULSION SYSTEM IN A VISCOUS INCOMPRESSIBLE FLUID

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Abstract. *The present work is devoted to the study of the optimal control of the two-mass vibration propulsion system in a viscous incompressible fluid. The study of the motion is carried out in two stages. At the first stage the simplified model of a viscous fluid is considered. On the basis of this model, the problem of the optimal control of the vibration system is solved in terms of minimizing energy consumption. The obtained optimal laws are studied at the second stage on the basis of the direct numerical simulation.*

1 INTRODUCTION

The design of the effective vibration propulsion systems is one of the key vectors of the development of the modern micro-robotics. Currently, there are several popular implementation concepts of such propulsion systems for moving micro-devices in a resistant medium. One of them is based on the use of the mobile internal mass. This type of devices usually called vibration-driven robots. They represent a multi-body system that are consisted of a closed shell, placed in a resistant medium, and moving internal parts. The periodic motion of the inner system of the bodies (the internal mass) causes motion of the shell, which can be used (in the case of a nonlinear medium resistance) for directional motion in space. The obvious advantages of such a propulsion are the shell tightness and lack of moving external parts, which allow to use such micro-devices for non-destructive inspection of miniature technical objects.

Research of movement capabilities of multi-body vibration propulsion systems were conducted earlier for medium with different resistance laws. In [1-3] the possibility of motion in an ideal fluid with related deformations of the outer shell was considered, articles [4, 5] were devoted to the study of the movement on a rough plane in the presence of a Coulomb friction, in [6] the movement on the liquid interface was studied. Movement in the Newtonian fluid was considered in [7-11].

Problems of optimal motion control of vibration multi-body systems in a viscous fluid were discussed in studies [9, 10]. In [9] the problem of optimization of the movement of the robot was solved in the presence of an arbitrary power law relationship between the velocity and the resistance forces, including the square law, which is often used as an approximation for the resistance forces resulting from the motion of a body in a Newtonian fluid. In [10] vibration-driven robot motion in a viscous fluid was optimized for the case, when the resistance law was designed on the basis of experimental data for a sphere moving in a viscous fluid.

However, all these problems of optimization were solved on the basis of quasi-stationary models, when the drag force is uniquely determined by the velocity of the body movement. In fact, the hydrodynamic resistance is defined by fluid flows that have been formed around the body during the whole time of motion. In general, the forces cannot be described solely in terms of instantaneous velocity and should be determined by the history of the entire movement. Developing the correct model of the drag description requires a large-scale study of the interaction of the vibration system with a viscous fluid.

In this paper, the research in this direction is carried out for the two-mass model of vibration-driven robot consisting of the outer spherical shell and a movable internal mass, which makes periodic oscillations along the axis passing through the center of the body.

As a first approximation, a simplified model of a viscous medium is considered, in which the hydrodynamic force is represented as the sum of the viscous and Basset history components. The viscous component is responsible for the viscous friction force. The history component is responsible for accounting of the movement history is selected in the form proposed in [12]. On the basis of this model the problem of optimal control of the vibration system is solved in terms of minimizing energy consumption.

The obtained optimal laws are studied in the second phase with on the basis of the direct numerical simulation. The complete problem of interaction of vibration system with a viscous incompressible fluid motion is considered, when the fluid flow is described by the full system of Navier-Stokes equations. Axisymmetric and three-dimensional formulations are discussed. Numerical model are implemented in the OpenFOAM package.

2 MODEL OF TWO-MASS VIBRATION SYSTEM

Let us consider a system of two bodies. The body of a spherical shape (shell) with mass M is in a viscous liquid, and a shell with mass m (hereinafter, the internal mass) moves inside it. The longitudinal periodic movements of the internal mass relative to the shell, in which the whole system moves as a whole, are investigated. Let us denote the body velocity through u and the movement and velocity of the internal mass relative to the shell as x and $v = \dot{x}$. The basic equation which describes the motion's velocity $u(t)$ of the shell under the given law $x(t)$ of the motion of the internal mass has the form

$$\left(m+M\right)\dot{u}+R(u)=-m\ddot{x} \tag{1}$$

Here, R is the liquid resistance force to the shell's movement. For any given periodic law $x(t)$ with period T (1) uniquely determines with the same period the periodic function $u(t)$, and $x(t)$ plays the role of the kinematic control.

Energy consumptions for the movement of the body with an internal propulsion (movable internal mass) is conveniently characterized by a the energy coefficient (EC)

$$\eta=\frac{N_0}{N_{vbr}},\quad N_{vbr}=\min_{\langle R(u)\rangle=0,\langle u\rangle=U}\left\langle N(u)\right\rangle,\quad N_0=\min_{\langle u\rangle=U}\left\langle N(u)\right\rangle$$

as the ratio of the minimum power N_0 required for movement of the body at an average velocity U to the power N_{vbr} consumed for moving at the same velocity of vibration-driven robot.

3 OPTIMIZATION PROBLEM

The formulation of the optimization problem consists of finding such a periodic law $x(t)$ of internal mass oscillations which for a fixed period T of oscillations and given average velocity U of the shell movement will minimize the power of the internal propulsion $\left\langle N(u)\right\rangle$. The convenience of this formulation is that the original problem is thus reduced to the problem of finding the periodic function $u(t)$ with period T that provides a minimum for function $\left\langle N(u)\right\rangle$ within the constraints $\left\langle u\right\rangle=U$ and $\left\langle R\right\rangle=0$.

As an approximation of the resistance law let us consider widely used in the case of high-frequency oscillations [16] dependence

$$R[u]=\frac{1}{2}C_x\pi\rho a^2\left|u\right|u+6\pi\rho\nu a^2\int_{-\infty}^{t}\frac{du/d\tau}{\sqrt{\pi\nu\left(t-\tau\right)}}d\tau \tag{2}$$

The difference between the quasi-stationary approximation (used earlier in [10, 11]) and (2) consist in the addition Basset forces. The resistance is determined by not only the current value of the velocity, but also by the whole history of the motion. We consider the important special case $C_x=\text{const}$ of the quadratic viscous resistance.

By normalizing velocity u on U and time t on period T, we write down the problem of the optimal control of the shell movements in the following form:

$$N_{\min}=\min\left(N_V[u]+sN_H[u]\right) \tag{3}$$

$$\left\langle u\right\rangle=1 \tag{4}$$

$$\left\langle u\left|u\right|\right\rangle=0 \tag{5}$$

$$N_V = \left\langle |u|^3 \right\rangle, \quad N_H = \left\langle uR_H \right\rangle, \quad R_H = \int_{-\infty}^{t} \frac{\dot{u}(\tau)d\tau}{\sqrt{(t-\tau)}}$$

The minimization in (3) is carried out on a set of periodic functions with a unit period that satisfy constraints (4) and (5). When writing (5), it is further taken into account that $\langle R_H \rangle = 0$ for any periodic function u. The only dimensionless parameter of problem (3)

$$s = \frac{12}{C_x U}\sqrt{\frac{\nu}{\pi T}} \tag{6}$$

sets the degree of the nonstationarity of the shell's motion by characterizing the ratio of Basset forces to viscous forces. If $s = 0$ the problem (3) - (5) of optimal control of body motion is quasi-stationary problem with the quadratic resistance law, the decision of which is described in [10].

The problem (3) - (5) was solved numerically with grid methods. The obtained optimal laws of motion are biphasic. They consist of a slow forward movement phase and a fast backward movement phase.

Figure 1 shows that the most significant change in the optimal motion law occurs in the range of s from 0.1 to 1. If s is less than 0.1, the law of motion is close to the quasistationary $u_0(t)$ obtained by neglecting the Basset forces. At s larger than 3, in contrast, viscous friction forces can be ignored. Here $u(t;s)$ practically coincides with $u_\infty(t) = u(t;\infty)$.

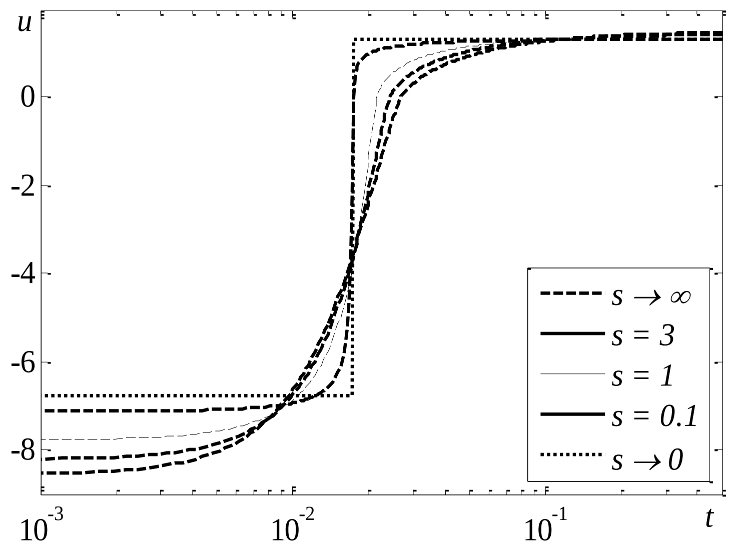

Figure 1. The optimal law of motion $u(t)$ for different values s.

In Figure 2 the solid line shows the dependence of the main integral characteristic, which is the energy coefficient $\eta_{\max}$, on parameter s. The dashed lines in this figure indicate $\eta_{\max}^{(0)} = 0.079$ and the asymptotic $\eta_{\max}(s) = \eta_\infty s^{-1}$ ($s \to \infty$). The value $\eta_\infty = 0.056$ is calculated by the power of Basset forces for $u_\infty(t)$ according to the formula $\eta_\infty = (N_H(u_\infty))^{-1}$. As might be expected, the energy coefficient decreases monotonically with an increase of parameter s, which corresponds to additional power losses of the propulsion to overcome Basset forces.

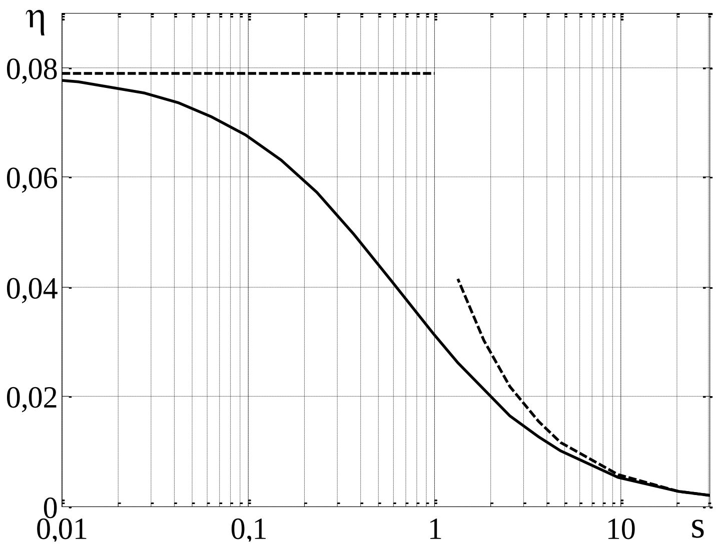

Figure 2. Dependence of the energy coefficient $\eta_{\max}$ on the parameter s - a solid line (dashed line - asymptotic behavior at the $s \to 0$ and $s \to \infty$).

4 NUMERICAL MODELING

4.1 Problem description

Approximation of the hydrodynamic forces that was proposed in the previous section contains only the simplest representation of Basset history term. To develop more accurate model of the interaction of the vibration system with a viscous fluid a more detailed analysis of the components of the hydrodynamic force is required. For this purpose in the second part of the research a numerical simulation of the movement of a vibration-driven robot in a viscous fluid is carried out on the basis of the full Navier-Stokes equations.

Motion laws are selected from the results of the optimization problem presented in the previous section. The velocity of the shell is defined up to a constant, which is determined from the conditions of periodicity of the internal mass and the shell motion.

The resulting problem depends on two dimensionless parameters: the Reynolds number (Re) and the dimensionless period of the movement (T_0) that are defined as follows

$$\mathrm{Re} = \frac{U_m D}{\nu}, \; T_0 = \frac{U_m T}{D}.$$

Here U_m is the maximal absolute value of velocity of the shell, D is the diameter .of the shell. The dimensionless parameters (Re, T_0) have the following interconnection with the parameter s of the optimization problem

$$s = \frac{12}{C_x}\frac{U}{U_m}\sqrt{\frac{1}{\pi T_0 \,\mathrm{Re}}}.$$

A numerical model for the simulation of the interaction of the vibration system with a viscous fluid is constructed in OpenFOAM package on the basis of the numerical schemes presented in [13, 14].

4.2 Results

The numerical modeling was carried out in the range 300 <Re <2500 of Reynolds numbers for the fixed value of dimensionless period $T_0 = 40$. The law of the shell motion was chosen on the basis of the data of analytical model, so that it provides a theoretically high efficiency in the whole study area. The corresponding dependence $u(t)$ is shown in Figure 3.

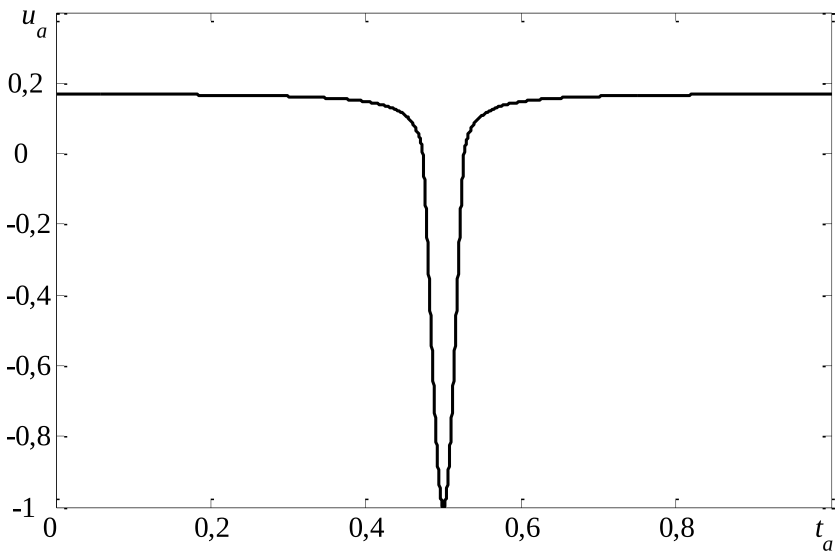

Figure 3. Motion law of the shell

Despite the high value of the maximum Reynolds number the flow structure around the vibration-driven robot retains axisymmetric in the investigated range of Re. This is due to the fact that the maximum speed of motion is reached in the time interval (see. Figure 3) that is too short for asymmetry development. At the same time, the velocity values of the direct motion phase are an order of magnitude smaller than the maximum, so all asymmetric disturbances are damped during this phase. In Figure 4 the main flow patterns observed in the investigated range of Re are presented. Visualization is made with colored weightless particles. In the first case (Figure 4, left), in the wake of the body there is irrotational fluid motion. Such a regime is observed in the range Re <2000. In the second case, the wake consists of gradually dissipating vortex rings (Figure 4, right).

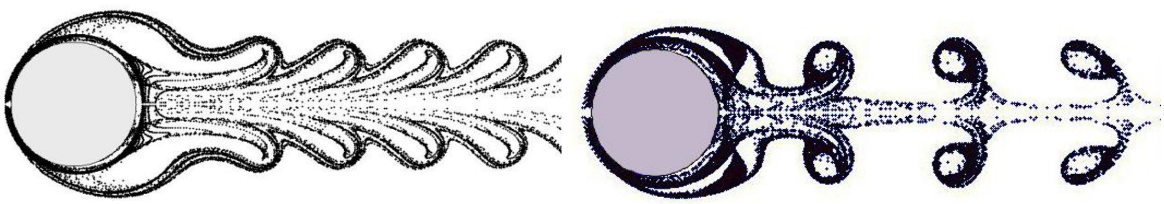

Figure 4. Flow patterns for parameter values $T_0 = 40$, Re=500 (left) and $T_0 = 40$, Re=2500 (right).

The efficiency of motion grows with increasing Reynolds number. Dynamics of change of the energy coefficient η is shown in Figure 5. However, values of η obtained in numerical simulation are more than two times lower than predicted by the theoretical model. To analyze the reasons for this discrepancy, the structure of the forces acting on the vibration-driven robot is studied.

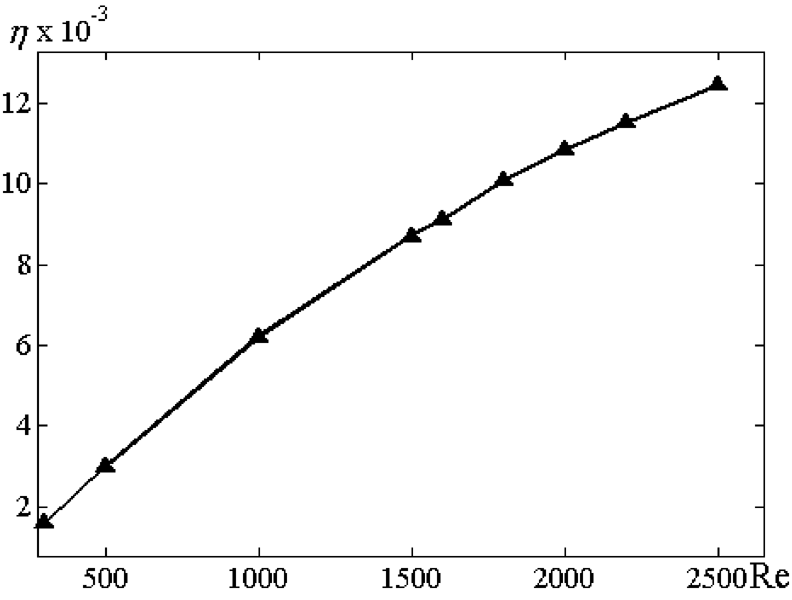

Figure 5. Dependence of the energy coefficient η on the parameter Re.

The resulting force is represented as the decomposition into viscous, inertial and history components in the following form

$$R(C_D, C_m, C_b, t) = C_D u(t)|u(t)| + C_m a(t) + C_b \int_{-\infty}^{t} \frac{a(\tau)}{\sqrt{t-\tau}} d\tau . \quad (7)$$

To do such decomposition using the assumptions $C_D = \text{const}$, $C_m = \text{const}$, $C_b = \text{const}$, that were applied in optimization problem, is impossible. Since we are dealing with two-phase regimes of motion, it is possible to assume that the coefficient of viscous forces C_D can take two values C_{d+} if $u(t) > 0$ and C_{d-} if $u(t) < 0$, and the coefficients of the inertia and Basset forces (C_m, C_b) are constants.

In order to determine the unknown coefficients the minimization problem is solved:

$$L(C_{d+}, C_{d-}, C_b, C_m) = \sqrt{\sum_{i=1}^{N} \left(R(C_{d+}, C_{d-}, C_b, C_m, t_i) - R_i^{\text{num}} \right)^2} \longrightarrow \min ,$$

$$\langle R \rangle = 0 .$$

Here L is a standard deviation function and R^{num} is computed force. Finally we find the combination $(C_{d+}, C_{d-}, C_b, C_m)$ for each Reynolds number. The proposed relationship (7) makes it possible to construct a good approximation of force R^{num} in the entire range.

Coefficient C_m almost independent on the Reynolds number. The values of coefficient C_b are quite small, so the third term of decomposition (3.21) has a very small influence. The values C_{d+} and C_{d-} change with increasing Reynolds number, they are shown on Figure 6. As we can see they are close to the experimental dependence [15] for the forces acting on a sphere in the stationary viscous flow.

The resulting force approximation differs from the one used in the section 3. This explains the differences in the estimates of the effectiveness of the resulting motion. The next step is to use it to solve the optimization problem. As the first results show, a new representation of forces does not introduce significant changes in the form of optimal laws of motion. That probably will allow to achieve a good consistency in the results of analytical and numerical models.

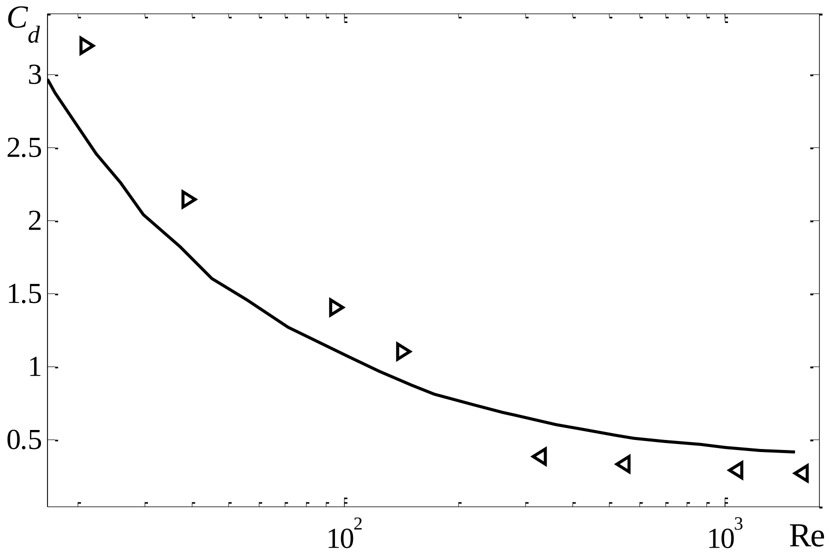

Figure 6 Resistant force. Markers are C_{d+} and C_{d-} values, solid line is the experimental dependence [15].

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