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NATIVITY CHURCH IN BETHLEHEM: FULL 3D NON-LINEAR FE APPROACH FOR STRUCTURAL DAMAGE PREDICTION

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Abstract. *This study presents some results of advanced numerical investigations carried out on the Nativity Church in Bethlehem in order to study the seismic response of the structure and identify possible causes of damage. Detailed three-dimensional finite element models of the church are developed and a damage plasticity material model, exhibiting softening in both tension and compression, is used for masonry. Non-linear bidirectional dynamic analyses are first performed on the model of the entire church in the actual configuration. From an overall analysis of the numerical results, it is possible to observe damage in the vault system, in the semi-domes and near the interlocking of the orthogonal walls. Then, the narthex is separated from the church and is analyzed under seismic excitation applied only in the longitudinal direction. The numerical simulations carried out provide some results that fit reasonably with the actual deformed configuration and can be considered as a useful tool for future rehabilitation interventions.*

1 INTRODUCTION

Very little is known about the history of the transformations of the narthex of the Nativity Church since its construction. The narthex that we see now, or at least the narthex that we can imagine on the basis of the current volumes or of the traces of some ancient and still visible openings on the walls, replaced in the second half of the VI century AD a larger cloister belonging to the previous Church, wanted by Queen Helena, mother of Emperor Constantine, in the IV century. From the outcomes of some archaeological excavations made in the thirties of the last century by R.W. Hamilton and from the results of the studies that have followed, it was possible to go back to the original form of the narthex that, unlike as it is now, was composed of a single volume, had a bigger central door flanked by two smaller side doors in both longer walls and two windows placed symmetrically with respect to the front door, Figure 1.

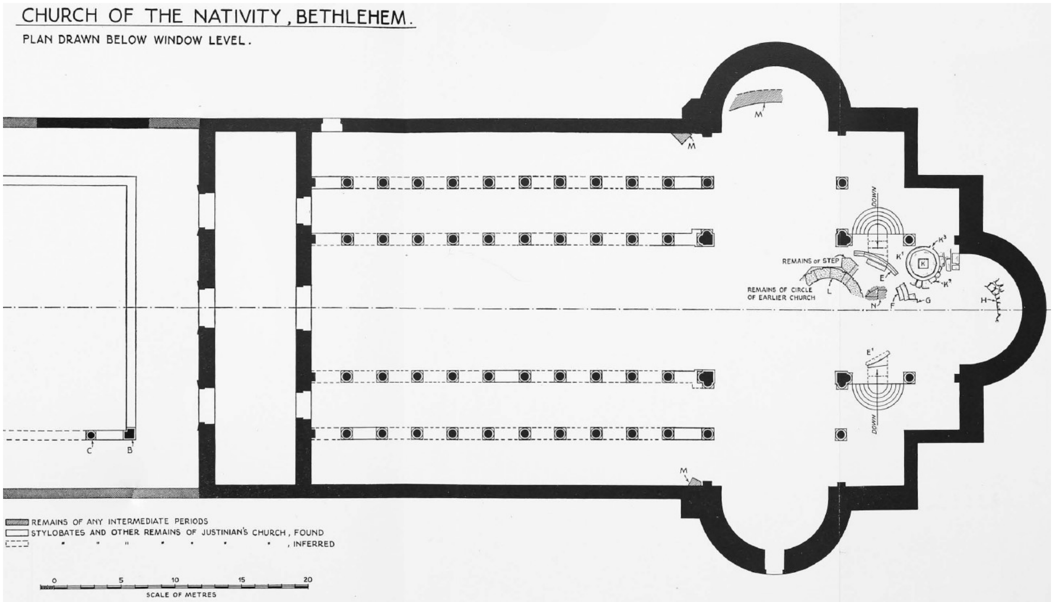

Figure 1: Original narthex of the Church of the Nativity.

From some traces found in the walls during the restoration works still in progress, it was possible to deduce that the original roof of the narthex was made of timber, with a single pitch, as in almost all early Christian and Byzantine basilicas with narthex. The typology of this wooden roof is still unclear, although the traces left on the walls suggest that it consisted of a series of simply supported beams arranged according to the slope of the roof. Anyhow it was a light roof. Over the centuries, especially during the Middle Ages, the Church increasingly took on the appearance of a fortress equipping itself with walls and towers for defense. Some traces of these additions are visible today even in the narthex, in particular in a small protective parapet above the façade and in two thick interior walls, perpendicular to the façade, probably the base of two outer towers, Figure 2 and Figure 3. Maybe, in order to allow these transformations for protective purposes, or in consequence of some destructive earthquakes, the original timber roof was replaced by a system of masonry cross vaults and some openings were infilled or reduced in size. The two internal walls are made of masonry of poor quality, or at least lower than that of the external walls, and they are simply adhering to the façade walls of the narthex and Church without any effective connection.

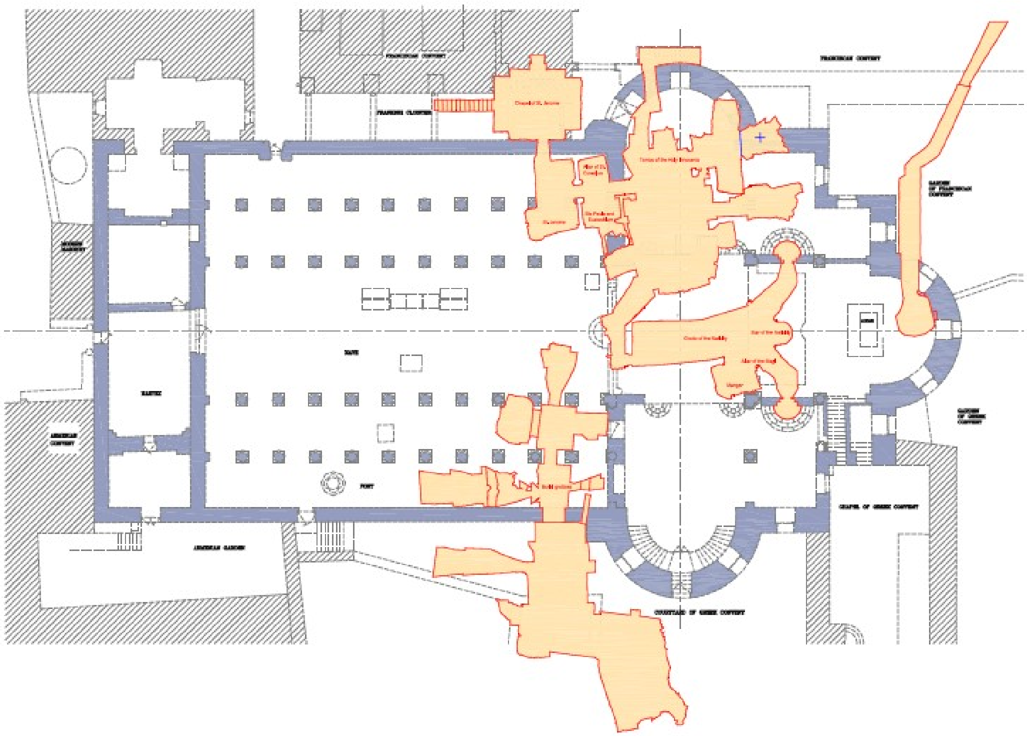

Figure 2: Plan of the Church with the current narthex.

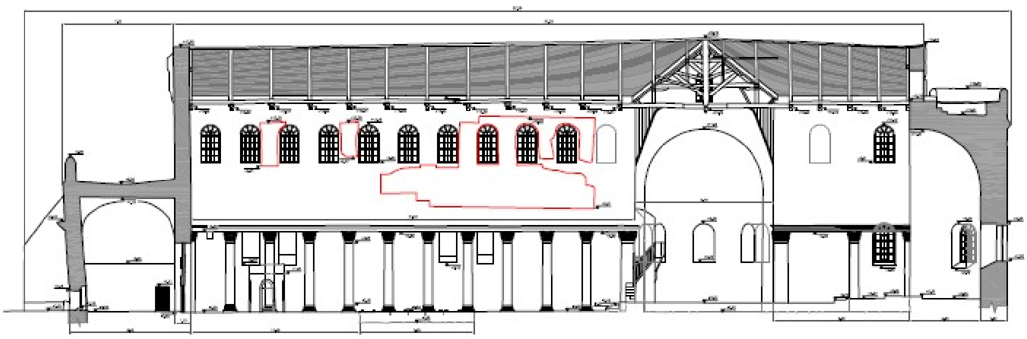

Figure 3: Longitudinal section of the Church

Also the vaults, made of irregular and roughly cut stones, are connected to the façade walls of both narthex and Church only in correspondence of the corbels at the base of the arches and for a height of about 1,10 – 1,30 m. Instead, such vaults are well connected with each other in the direction East-West (the longitudinal direction of the Church) by means of arches, orthogonal to the façade and built with greater and more regular stones; such stones have two different lengths, which alternate to allow for a greater connection between arches and vaults. Even the diagonal arches of the vaults are made with more regular stones, although not as the ones of the arches mentioned before. The thickness of the vaults is not constant everywhere and it varies between 35 – 40 cm. Before starting the restoration the space between the external paving and the extrados of the vaults was filled with sand and remains of a medieval paving. Some inspections made on site have shown that the walls of the narthex continue downwards with a constant thickness up to a depth of 94 cm from the floor; then they have an enlargement of 20 cm on each side up to a depth of 141 cm and continue with a compact layer of stones and mortar for other 150-165 cm before reaching the bedrock. The two side walls of the narthex are 1,00 m thick and have an inner core of undefined thickness; the façade wall, like the façade of the Church, is 1,15 thick, made of regular stones and with limited or null inner core. The narthex is now connected with the monasteries of the Franciscans Friars (North) and of the Armenian religious community (South), Fig. 2, who, together with the Greek-Orthodox one, manage the activities in the Church according to the current *status quo*. On the external front, towards the square, there is a big buttress probably built after the XVI century: in fact some drawings by Fr. Bernardino Amico, dating back to 1609, show the façade of the narthex still without the buttress, Figure 4.

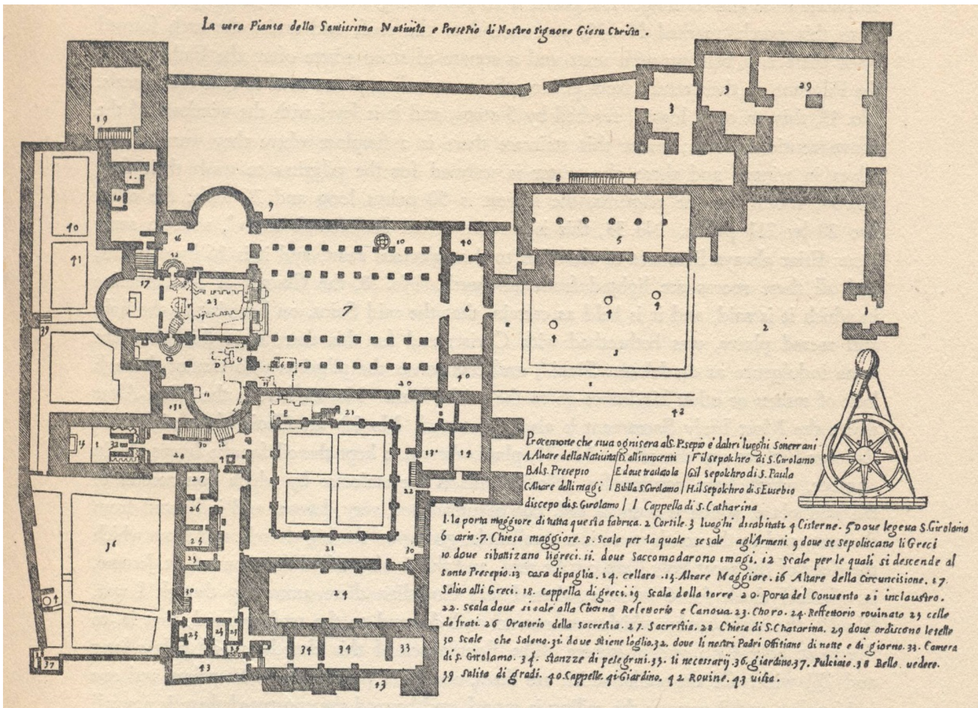

Figure 4: Plan of the Nativity Church (1609).

2 STATE OF STRUCTURAL DAMAGE AND CAUSES

The structure of the narthex is today strongly deformed. The façade wall exhibits a rotation towards the square more pronounced in the middle of its length and starting approximately from the architrave of the Humility door, about one meter above the ground. In consequence of this rotation, the façade wall has undergone a maximum horizontal displacement at the top, approximately in the middle of the wall, of about 40 - 42 cm measured from the base of the parapet. It is a very high value when compared with the height of the wall, about 8.70 meters. If some fractures have occurred in the past, these are now closed due to local interventions of cleaning and consolidation made over the centuries, which shows that the damage evolution is now over and that it was probably over even when the external buttress, still perfectly vertical, was added. The façade of the Church, opposite to the one of the narthex, exhibits a light out-of-plane deformation which starts from the roof of the narthex and achieves 10 cm at the top of the tympanum. In consequence of the rotation of the façade of the narthex the three central vaults, and in particular the one in the middle, underwent vertical displacements downwards in the central zones, detachments from the façade walls and cracks both at the extrados and the intrados. Especially the central vault, propped since the thirties of the last century, exhibits detachments 17-19 cm wide from the narthex façade and 10-11 cm wide from the Church façade Figure 5. Moreover big cracks, parallel to the façades, are visible at the extrados Figure 6.

Figure 5: Detachments from the wall.

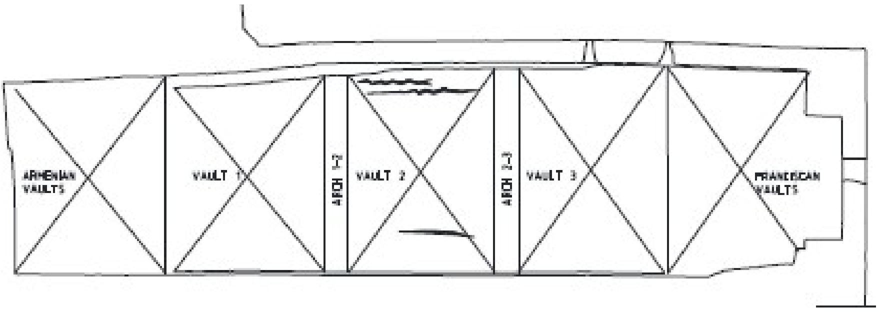

Figure 6: Cracks at the extrados of the central vault.

The arches connecting the vaults to each other (in the East – West direction) and also the ones along the diagonal planes of the vaults are strongly deformed. The causes of these damages have never been clarified. The replacement of the original wooden roof with a set of thrusting masonry vaults probably gave a significant contribution to the rotation of the façade towards the square. It is believed, however, that such thrusts alone cannot have been able to cause maximum horizontal displacements as large as those measured. Therefore, since Bethlehem is in a seismic area which over the centuries underwent several earthquakes, some of them even severe, it is very probable that a seismic event, occurred before the construction of the buttress, was one of the main causes of damage. The external buttress, added in the last centuries, as mentioned above, shows no sign of rotation and no differential vertical displacement.
The aim of this work is to identify, through a proper numerical simulation, the seismic event that, together with the thrust of the vaults, might have caused damage in the vault system and the present out-of-plane deformation of the narthex façade and of the tympanum of the main entrance. To this purpose, a 3D finite element model of the entire Church was built and analyzed by using the commercial software ABAQUS. The model reproduces the actual configuration of the church, but without the external buttress and without connections with the monasteries of the Franciscans Friars and of the Armenian religious community. The vaults at both ends of the narthex (North and South) were not modelled because their survey had not been yet carried out at the time of the present analysis. The presence of the three internal walls was taken into account. It is worth noting that a three-dimensional structural analysis of the entire Church is significant when a good level of interlocking among the structural elements is guaranteed. If such connections are not reliable, as in the case of the Church of the Nativity, it is, on the other hand, advisable to perform numerical analyses on smaller portions that may be affected by local collapse mechanisms.
A detailed finite element model of the narthex was then created by using a finer discretization. The model reproduces, with some approximations and simplifications, the structure of the narthex as depicted by Fra Bernardino Amico at the beginning of the XVII century, without the external buttress, with a unique access door (the Humility door) and only two small

windows on the façade, with masonry cross vaults instead of the original wooden roof. The side walls of the narthex are supposed clamped to the Church, the height and thickness of the vaults have values computed as an average of the real ones and the vaults are connected to the façade walls of both narthex and Church. Moreover they are connected to each other in the East - West direction (longitudinal direction of the Church) by means of arches similar to the real ones, Figure 6, and clamped to the façades of the narthex and Church. The façade of the Church is supposed to be clamped at the intersection with the orthogonal walls of the nave and the aisles. Figure 7 shows a view of the system of masonry cross vaults in the model of the narthex.

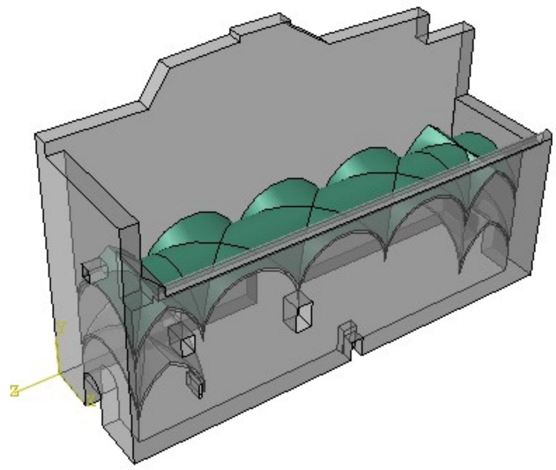

Figure 7: The system of masonry cross vaults in the model of the narthex.

3 FINITE ELEMENT NUMERICAL MODELS

With the aim of reproducing the actual state of damage on the narthex it is chosen to study the effects of a possible earthquake by non-linear dynamic analyses by applying spectrum compatible acceleration time-histories at the base of the structure. Such a method of analysis is very time consuming, but it is much more accurate and reliable than other approaches for its capability to identify in- and out-of-plane, as well as local and global failure mechanisms. Different three-dimensional finite element models of the church and the narthex are implemented in Abaqus, taking into consideration geometrical (large displacement effects) and material non-linearity (elastic-plastic with damaging behavior of the masonry material). Despite the general drawbacks regarding non-linear dynamic analyses, the use of sophisticated 3D models requires a relatively reasonable computational effort on sufficiently powerful workstations, if the discretizations are not excessively refined and the material models do not exhibit strong softening.

On the other hand, Italian Guidelines for the built heritage specify that such analyses can be conducted using finite element models and considering suitable constitutive models. Such models should be capable of reproducing the typical strength and stiffness degradation exhibited by the masonry material in the inelastic range. Italian Code on Constructions NTC 2008 and subsequent explicative notes underline that the aim of dynamic analyses would be the evaluation of the structure in the non-linear range under an expected accelerogram, allowing for a comparison between required and available ductility. Also, it is possible to verify the integrity of the structural elements which can potentially show brittle behavior.

When dealing with material properties assumed and available in Abaqus software code, the so-called “Concrete Damage Plasticity” model was adopted. Such a model is based on the assumption of a scalar isotropic damage with distinct damage parameters in tension and compression. It is particularly suitable for applications in which the material exhibits damage, especially under load-unload conditions and hence for dynamic analyses. An elastic-plastic behavior in both tension and compression can be also taken into account.

To describe the multi-dimensional behavior in the inelastic range, masonry is assumed obeying a Drucker-Prager strength criterion with non-associated flow rule. A value equal to 10° is adopted for the dilatation angle, which seems reasonable for a masonry material subjected to moderate-low levels of vertical compression, also in agreement with experimental evidences available in the literature. To avoid numerical convergence issues, the tip of the conical domain of the Drucker-Prager strength domain is smoothed with a function having hyperbolic shape. Abaqus code allows ruling smoothing by means of the so-called eccentricity parameter, which represents the length of the segment between the points of intersection of the cone and of the hyperbola with the p axis in the p-q space. A value equal to 0.1 is adopted for the eccentricity parameter.

Experimental results reported by Page on regular masonry wallets and successive numerical models show that such a material exhibits a moderate orthotropy ratio (around 1.2) under biaxial stress states in the compression-compression region. Obviously, such feature cannot be taken into account when an isotropic model, like the present one, is used. However, the use of isotropic models (like concrete smeared crack approach available in both Ansys and Adina) is commonly accepted in the literature after adapting the parameters to fit and average behavior between vertical and horizontal compression. A suitable model should also take into account the ratio between the ultimate compression strength in biaxial stress states and in uniaxial conditions. Such a ratio, which exhibits some similarities between concrete and masonry, is reasonably set equal to 1.16.

In tension the stress-strain response follows a linear-elastic relationship until the peak stress σ_{t0} is reached. Then, micro-cracks start to propagate in the material, a phenomenon which is macroscopically represented by softening in the stress-strain relationship. Under axial compression, the response is linear up to the value of the yield stress σ_{c0}. After the yield stress, the response is typically characterized by hardening, which anticipates compression crushing, represented by a softening branch beyond the peak stress σ_{cu}.

Damage variables in tension and compression are defined by means of the following standard relationships:

$$\begin{aligned} \sigma_t &= (1-d_t)E_0\left(\varepsilon_t - \varepsilon_t^{pl}\right) \\ \sigma_c &= (1-d_c)E_0\left(\varepsilon_c - \varepsilon_c^{pl}\right) \end{aligned} \qquad (1)$$

where σ_t (σ_c) is the mono-axial tensile (compressive) stress, E_0 is the initial elastic modulus, ε_t (ε_c) is the total strain in tension (compression), ε_t^{pl} (ε_c^{pl}) is the equivalent plastic strain in tension (compression). When strain reaches a critical value, the material elastic modulus degrades in the unloading phase to $E < E_0$. In particular, within the simulations, a reduction equal to 5% of the Young modulus with respect to the initial value is assumed for a plastic deformation equal to 0.003.

The issue of mechanical properties to adopt for the constituent materials results particularly difficult, especially in the absence of ad hoc experimental campaigns and specific indications by local building codes. It is common opinion, however, that the major damages registered in historical churches are a consequence of very poor mechanical properties of joints, whereas clay bricks exhibit a quite high strength. In the absence of specific indications, it was chosen to refer to what stated by Italian Code for existing masonry buildings.

As a matter of fact, masonry is a material which exhibits distinct directional properties due to the mortar joints, acting as planes of weakness.

Considering the well-known limitation of use of both micro-modelling and homogenization at large scale, isotropic macro-models are adopted for masonry. The reason for adopting an isotropic material stands in the impossibility to evaluate many parameters necessary for anisotropic materials in the inelastic range, in the absence of ad-hoc experimental characterizations. Finally, it is worth noting that commercial codes rarely make available to users anisotropic mechanical models suitable to describe masonry with regular texture in the non-linear range.

According to Italian Code NTC 2008 and subsequent Explicative Notes, the mechanical properties assumed for masonry material depend on the so-called knowledge level LC, which is related to the so-called Confidence Factor FC. There are three LCs, labeled from 1 to 3, related to the knowledge level about the mechanical and geometrical properties of the structure. The knowledge level LC3 is the maximum, whereas LC1 is the minimum. For the cases at hand, a LC1 level is assumed in the absence of specific in-situ test results.

Confidence Factor FC summarizes the knowledge level regarding the structure and the foundation system from a geometric and mechanical point of view. It can be determined defining different partial confidence factors FCk (k=1,4), on the basis of some numerical coefficients present in Italian Code (Table 4.1 Italian Guidelines). Due to the limited knowledge level achieved in this case, the highest confidence factor (FC = 1.35) was used.

The values adopted for cohesion and masonry elastic moduli are taken in agreement with Table C8A.2.1 of the Explicative Notes, assuming a masonry typology with very poor mechanical properties of the joint and quite regular courses. With the lowest knowledge level LC (confidence factor FC=1.35), Italian Code requires to select, in Table C8A.2.1, the lower bound values for strength and the average values between lower and upper bound for elastic moduli.

The application of the acceleration time-history occurs at the second step, where the structure is unrestrained using trailers, thus allowing it to move along the direction of the seismic action. The numerical analyses are carried out by means of a dynamic approach with implicit integration in the time domain, using a time step equal to 0.005 s, which corresponds to the accelerogram registration time interval. The results of the analyses conducted in this study are reported in the following sections.

4 NON-LINEAR DYNAMIC ANALYSES OF THE CHURCH

A detailed three-dimensional finite element model of the entire Church was developed by means of ABAQUS code, Figure 8, with the aim of investigating the seismic behavior of the structure through non-linear dynamic analyses. A finite element discretization, based on four nodes tetrahedral elements presenting an average size of 0.40 m, was adopted. The total number of elements of the model was equal to 131551. In particular, the mesh was refined near the regions where the main failure mechanisms were likely to start. Since not negligible out of plane displacement may occur during a seismic event, large displacement effects were considered. Perfect connection was assumed between perpendicular walls.

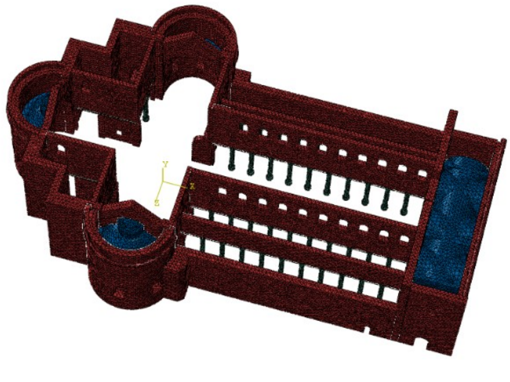

Figure 8: Finite element model of the entire Church of the Nativity.

The bidirectional non-linear dynamic analyses were performed by using artificial accelerograms generated by means of the software code SIMQKE. In this study the Church was subjected to spectrum compatible accelerograms corresponding to a peak ground acceleration equal to 0.15g according to Eurocode 8. The area where the Church is located is a medium-intensity seismic area with a_g = 0.15g, where a_g is the maximum horizontal acceleration occurring in the area, with a 10% of probability of exceeding this value over 50 years. Due to the complexity of the model and the high computing time of this type of analysis, the duration of the accelerograms was set equal to 10 s. In Figure 9 the contour plots of tension damage (red color is associated to full (1) damage and blue color to zero (0) damage), are shown from different points of view at the end of the numerical simulation. From an overall analysis of the numerical results, it is possible to notice that the Church exhibits significant damage under the expected earthquake excitation. It is evident that the damage spreads very quickly in the vault system, in the semi-domes and near the interlockings of the orthogonal walls. The damage in the vaults starts in the lateral right corner and then propagates towards the middle.

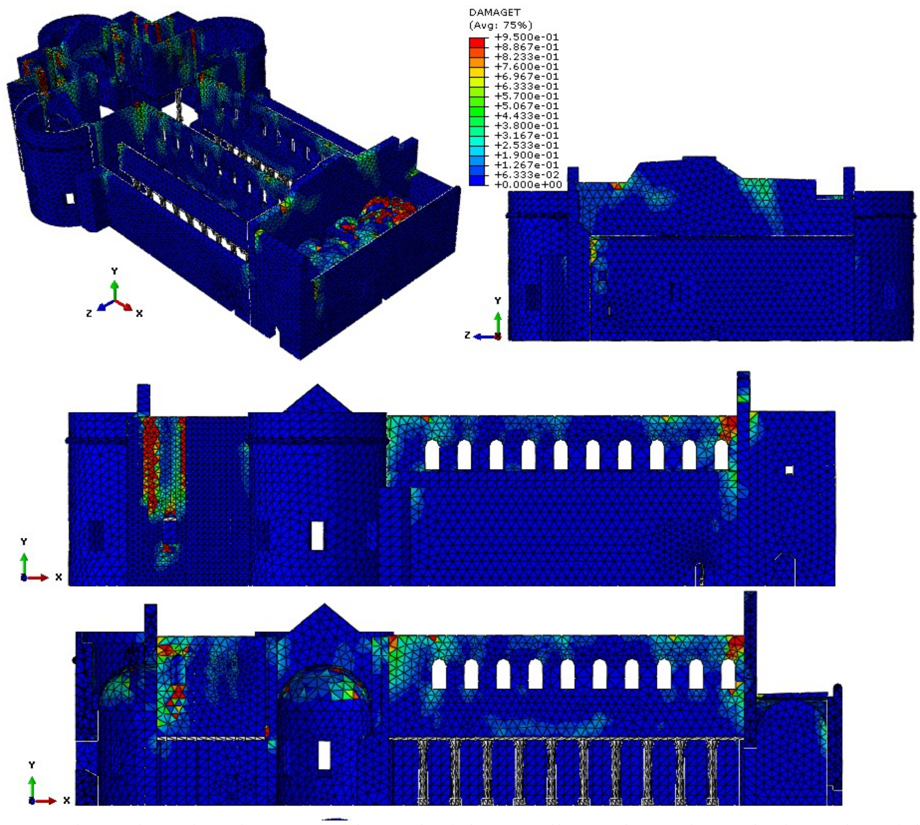

Figure 9: Contour plots of tension damage at the end of the non-linear dynamic analysis (red, 1: full damage; blue, 0: no damage).

Figure 10 shows the horizontal displacement time history for various control nodes of the Church along both the longitudinal and transversal directions. The main control nodes of the Church model monitored during the numerical simulations are shown in the same figure. In the longitudinal direction a maximum top displacement equal to 6 cm was computed for the top of the tympanum (control node P1) after about 7 s from the beginning of the ground motion. The stiffening presence of the internal walls of the narthex have the effect of reducing the out-of-plane rocking of the narthex façade during the application of the accelerograms. The maximum top displacement measured on the top of the narthex façade is about 5.5 cm.

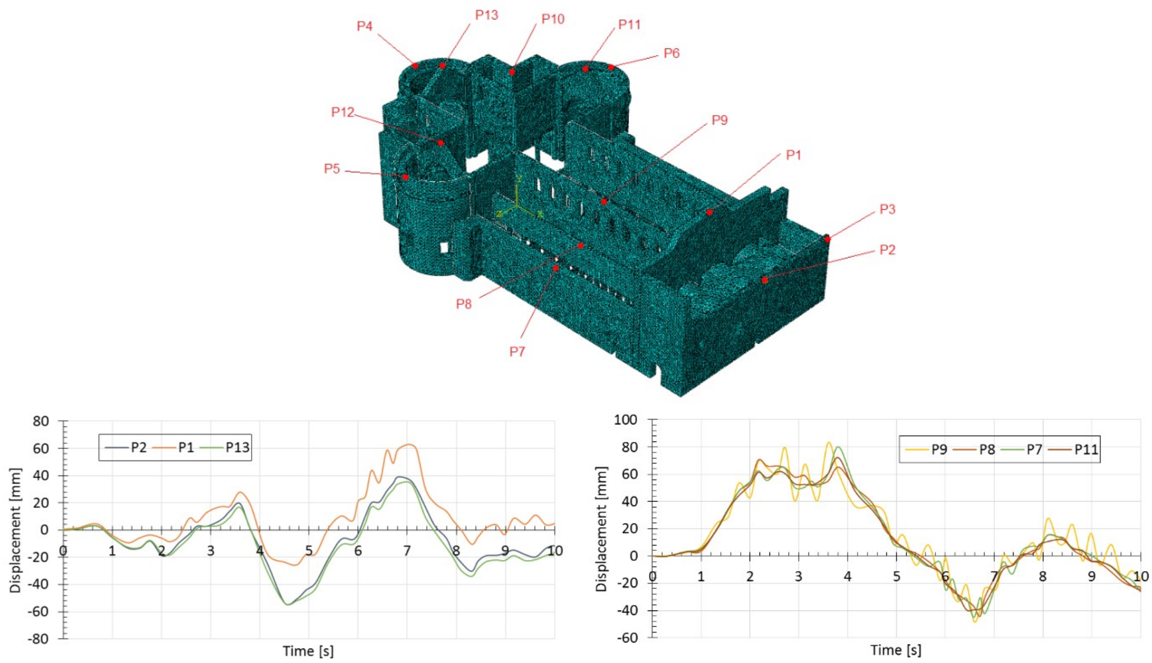

Figure 10: Displacement time history of some control nodes along the longitudinal and transversal directions during non-linear dynamic analyses.

5 NON-LINEAR DYNAMIC ANALYSES OF THE NARTHEX

The results of the non-linear dynamic analyses performed on the model of the whole Church showed the vulnerability of the structure under horizontal loads pointing out some critical regions. It is worth noting that the use of sophisticated three-dimensional finite element models requires a large computational effort and the model of the entire Church proved to be very-time consuming under bidirectional non-linear dynamic analyses. A finite element model of the narthex only was created in order to perform different non-linear dynamic analyses with various peak ground accelerations.

Then, the seismic performance of the narthex was assessed through non-linear dynamic analyses with response spectrum-compatible artificial accelerograms. A finer mesh, consisting of about 85000 tetrahedron elements and depicted in Figure 11, could be used for the narthex in this model. The internal walls were not modelled in order to reproduce the original configuration of the narthex and to neglect the stiffening effect provided by these elements.

The evaluation of the seismic response of the narthex was carried out using an artificial accelerogram, with duration equal to 20 s. For sake of simplicity, numerical analyses were carried out only in the longitudinal direction. The accelerogram used was generated by means of the software code SIMQKE in order to match the Eurocode 8 response spectrum. Different peak ground accelerations ranging between 0.15g and 0.25g were considered for the analyses.

This finite element model was used to identify the possible different collapse mechanism that can occur in the narthex. A ground motion applied in the longitudinal direction can determine the detachment of the narthex façade from the façade of the church, creating tensile stresses in the vault system. The vault system represents a critical element for the narthex and it is expected to undergo significant damage at the beginning of the numerical simulation.

Figure 11 describes the tensile and compression damage evolution, respectively, during the non-linear dynamic analysis. The results are reported at the end of the analysis (20 s). It can be noted that damage concentrates mainly in the cross vault system and, to a small extent, on both the façades. In particular, the tensile damage starts at the base of the vault system and then propagates to the middle part involving the whole system of vaults.

On the contrary, the compression damage starts at the top of the vault system and then propagates to the base. It can be noted that the interlocking of the longitudinal walls with the western façade of the Church tends to experience considerable damage.

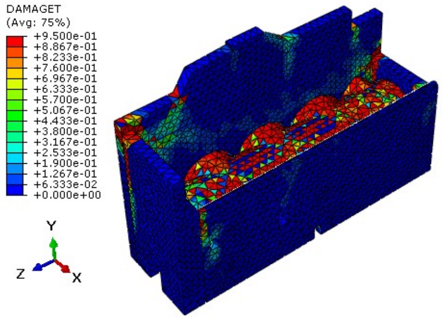

Figure 11: Contour plots of the tensile damage in the narthex at the end of the numerical simulation (red, 1: full damage; blue, 0: no damage).

The horizontal and vertical displacement time histories for some control nodes are reported in Figure 12. All the control nodes of the narthex model monitored during the numerical simulations are shown in the same figure. The horizontal displacements of three meaningful control nodes are measured during the numerical simulations: they are located on the top of the narthex façade, of the tympanum and of the central cross vault. The maximum horizontal displacement (about 10 cm) is observed for the control node N14 at the top of the tympanum of the main entrance. The narthex façade exhibits a peak horizontal displacement equal to about 6 cm. As regards the vertical displacement, the maximum value, equal to 9 cm, is registered for the central vault.

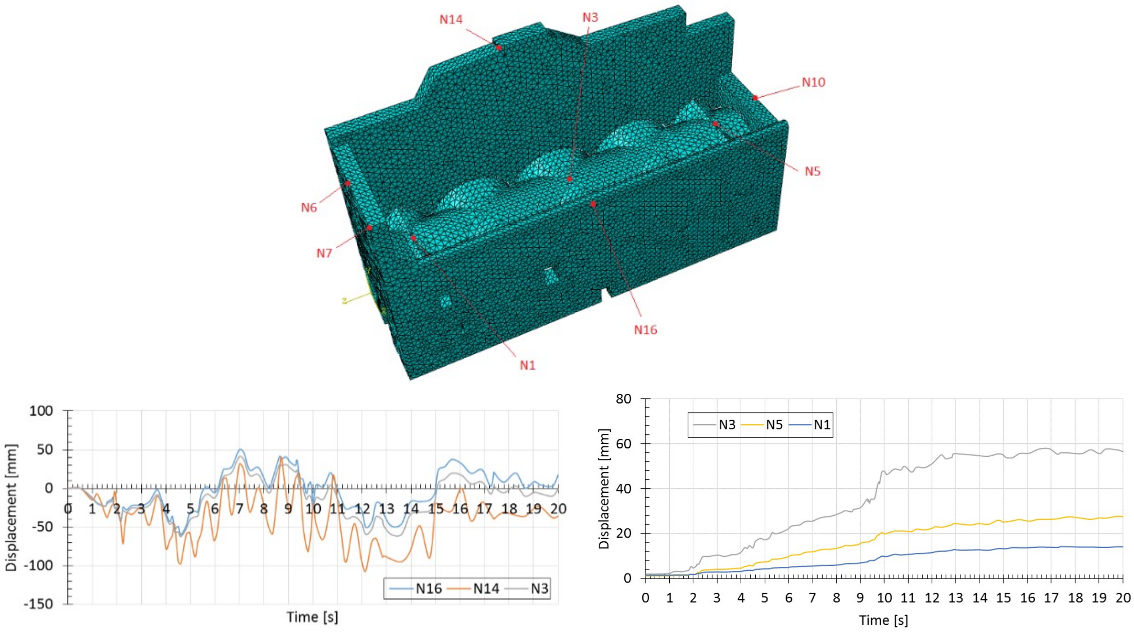

Figure 12: Horizontal and vertical displacement time-history of some control nodes during non-linear dynamic analyses.

The maximum values of the vertical displacements of the control nodes of the cross vaults are shown in Figure 13 for different peak ground accelerations. As can be noted, the maximum vertical displacements of the central vault is always larger than the other vaults. For higher values of seismic intensity levels (a_g=0.25g), the central vault presents a maximum value of vertical displacement equal to 15 cm.
The maximum values of the horizontal displacement of three meaningful control nodes are shown in Figure 14 for different peak ground accelerations. It can be noted that the peak values of the horizontal displacement of the narthex façade are always smaller than the top of the tympanum of the western façade. Under a seismic intensity level of 0.25g the tympanum exhibits peak displacements equal to about 35 cm, whereas the maximum displacement registered for the narthex façade is about 20 cm.

Non-linear dynamic analyses are able to provide, among other information, residual displacements of the control nodes. The maximum values of the residual displacements (35 cm) are registered for the western façade of the Church when subjected to seismic excitation with a_g=0.25g. As can be observed, the residual displacement found for the façade of the narthex is about 20 cm, still lower than the real ones, but indicating that a seismic event may be the cause of the present state of degradation. Probably larger residual displacements might be achieved by simulating the occurrence of a sequence of ground motions, as confirmed by the chronicles of the earthquakes in the area.

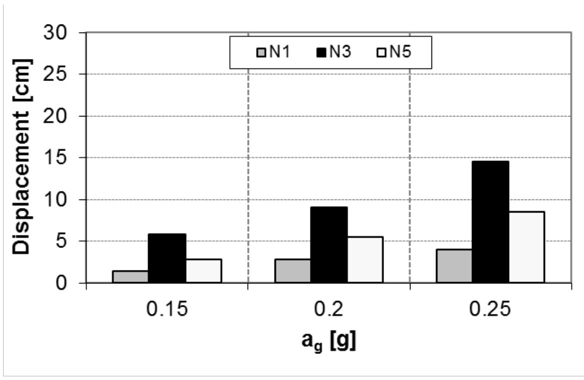

Figure 13: Maximum values of vertical displacement of some control nodes for different peak ground accelerations.

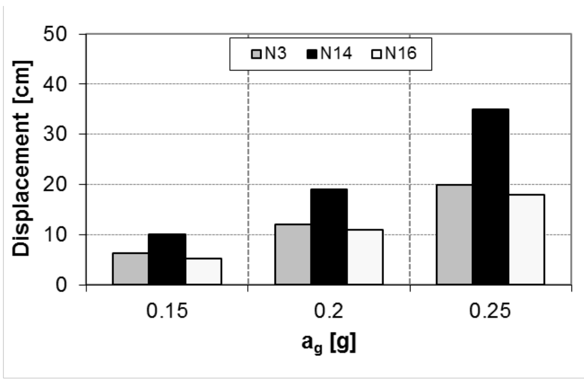

Figure 14: Maximum values of horizontal displacement of some control nodes for different peak ground accelerations.

6 CONCLUSIONS

This study summarizes the numerical investigations carried out on the narthex of the Nativity Church in Bethlehem. Advanced finite element numerical models accounting for

masonry damage and softening have been used within demanding non-linear dynamic simulations in order to identify the possible causes of damage in the narthex.
Non-linear bidirectional dynamic analyses were first performed on the model of the entire Church in the actual configuration. From an overall analysis of the numerical results, it is possible to notice that the Church exhibits damage for the expected earthquake excitation. It is evident that damage spreads very quickly in the vault system, in the semi-domes and near the interlocking of the orthogonal walls. The damage in the vaults starts in the lateral right corner and then propagates towards the middle.

Then, the narthex was separated from the Church and was analyzed under seismic excitation applied only in the longitudinal direction. Numerical results show the behavior of the narthex is considerably affected by the presence of the vaults that act as connecting element between the western façade of the Church and the façade of the narthex. During a moderate intensity earthquake, the vault system is subjected to significant stresses and consequently to severe damage. The second critical element of the narthex is the western façade that tends to show a local overturning mechanism due to the gradual accumulation of damage near the base of the vault system. Moreover, the façade of the narthex can reach significant displacements under seismic actions with a_g=0.25g. The results seem to indicate that the rotation of the narthex façade, with a consequent maximum out-of-plane displacement of 40 cm approximately, is probably due to a seismic event of great intensity or to several seismic events occurred in sequence over time. Certainly, results closer to the measured data can be obtained by introducing proper unilateral contact conditions at the interface between vaults and façade walls or between longitudinal walls and façade walls.

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