STABILITY OF STEEL STRUCTURES WITH CLEARANCES AND IMPERFECTIONS

Katarzyna Rzeszut¹, Andrzej Garstecki²

¹ Faculty of Civil and Environmental Engineering, Poznan University of Technology
  Piotrowo 5, 60-965 Poznan, Poland
  e-mail: katarzyna.rzeszut@put.poznan.pl

² Polytechnic Institute, Stanislaw Staszic, University of Applied Sciences in Pila
  Podchorazych 10, 64-920 Pila, Poland
  e-mail: Andrzej.garstecki@put.poznan.pl

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Abstract. This paper deals with statics and stability problems of steel structures with the emphasis on their sensitivity to initial imperfections and clearances. The main aim is to performed an indication as the state of stress, displacement and the critical load. The paper presents theoretical studies involving linear and nonlinear behavior of thin-walled steel structures with a special reference to the interactive buckling, initial geometric imperfections and clearances which is in the scope of the modern approach to design. For this purpose authors proposed model structures composed of rigid bars where strains are concentrated in connecting elastic joints These model structures enable to derive nonlinear algebraic equilibrium equations which strictly describe pre- and post-buckling behavior of structures with various combinations of imperfections and clearances. Numerous examples demonstrate variable types of structural response depending on the modes and amplitudes of imperfections in relation with clearances. In the case of large clearances the model structure behaves like a typical Euler column characterized by a stable and symmetric bifurcation point. However, clearances combined with certain patterns of imperfections can result in unstable post buckling behavior and also in snap through phenomena. The developed model structures, which illustrate complex structural stability response accounting for initial geometric imperfections and clearances, due to its simplicity makes possibility to describe the exact close form solutions for both linear and nonlinear stability analysis. Based on the proposed model structure it is shown, that interaction between initial clearances and initial geometric imperfections can strongly affect the structural stability response.
1 INTRODUCTION

In the last decades permanent development of light-weight structures is observed. They are aesthetic and economic. Important is also that small weight of structural materials well matches the requirements of sustainable development. From among structural materials steel is particularly attractive in application to light weight-structures. A great variety of hot- and cold-formed steel sections is at designers’ disposal. Modern highly automated cutting, drilling, welding and corrosion protection make possible to design and prefabricate optimal structural elements, which can be easily assembled using pre-stressed bolts. It must be underlined here that two special aspects in design of light-weight steel structures must be seriously taken into account, namely fire protection and protection against local and global instability.

The fact that steel structures are apt to fail by loss of stability belongs to the textbook’s knowledge for decades. This tendency towards instability is a direct consequence of high slenderness of steel structural elements. Designers can find practical recommendations in design codes [3, 4, 5] with theoretical and experimental background in the literature [6, 12] how to overcome the problems of instability. Professional computer programs supporting design of steel structures implement many of these recommendations. However, modern light-weight steel structures gave rise to new stability problems. Application of thin-walled cold formed sections and welded I sections with slender webs increased the importance of local instability phenomena which often appeared at a similar load level as global instability. The case when two or more different modes in stability and dynamic analyses are associated with the same or similar eigenvalues is termed a bimodal or multimodal solution. Designers’ concern is that these solutions are very sensitive to imperfections. Design codes and scientific papers [1, 2, 7, 13] contain recommendations with respect to forms and magnitude of global geometric imperfections which can be taken into account. There is definitely not so much information and recommendations with respect to local imperfections.

New stability problems were also originated when roof cladding elements were assumed to provide support for compressed top flanges of purlins or beams of frames [8, 9, 10, 11]. Roof cladding are often made of profiled steel plates. They are point-wise connected to the beams by self-tapping screws or by rivets. Thickness of steel plates is small, therefore variable loads and hence variable interaction forces often result in increasing diameter of the holes in thin walls of the plate. In this case we have slotted connections with unilateral constraints. Similar type of unilateral constraints can also appear in a bolted connection with a clearance between structural elements. These imperfect connections can strongly influence the structural response. Moreover, there can be unfavorable combinations of clearances and initial geometric imperfections.

Economic and safe design must be based on a reliable structural analysis where all essential imperfections are taken into consideration. On the other hand a numerical model of the actual structure cannot be too complicated. Otherwise computer time would be too great. Keeping this in mind, the author presents in this book stability behavior of structures with various combinations of global and local initial geometric imperfections, combinations of clearances and simultaneous appearance of geometric imperfections and clearances. Of course, in order to demonstrate these phenomena nonlinear stability problems must be formulated and solved. The author proposed model structures composed of rigid bars where strains are concentrated in connecting elastic joints. These model structures make possible to derive nonlinear algebraic equations which strictly describe pre- and post-buckling behavior of structures with various combinations of imperfections and clearances. By the way of numerous examples influence of imperfections and clearances on the structural behavior is discussed.
2 THEORETICAL BACKGROUND

In order to illustrate the phenomenon of the interaction of initial geometric imperfections and clearances a model structure is developed. It consists of \( n \) incompressible rods connected by elastic hinges with the rotational stiffness \( k_n \) [N·m/rad]. At the position of internal hinges the elastic intermediate supports with the stiffness \( k_{n,n+2} \) [N/m] and clearances \( \Delta_0 \) are imposed. Now \( \Delta_0 \) and \( \Delta \) denote, respectively, the clearance and the total transverse displacement of the structures at the position of internal hinges. This model enables the derivation of the exact nonlinear equilibrium equations, taking into account large displacements. At the same time, this simple model well illustrates the real structural response in terms of its stability problems. The geometry of the proposed model structure shows Figure 1.

![Figure 1: Geometry of the proposed model structure](image)

2.1 Eigenvalue problem

At the first stage, the initial clearance is very large and the gap remains open: \( |u_n| < \Delta_0 \). Then the supports positioned at the elastic hinges do not switch on to cooperate with the bar, which is supported only at its ends. At this stage, the equation describing displacement \( u_n \) and rotation angles of nodes \( \phi_n \) may be derived from geometries that are illustrated graphically in Figure 2.

![Figure 2: Geometric relation of the model structure](image)

Hence we obtain the following geometrical relationships:

\[
\varphi_n = \arcsin \frac{u_n - u_{n-1}}{L_n} \quad \text{and} \quad \phi_n = \arcsin \frac{u_n}{L_n} - \arcsin \frac{u_{n+1}}{L_{n+1}},
\]

(1)
where: $\varphi_n$ – rotation of angle n-th bar. The set of equilibrium equations takes the form:

$$k_n \cdot \phi_n^e = P \cdot u_n.$$ (2)

Equation (2) can be written in matrix form:

$$(K - PLI)\bar{u} = 0,$$ (3)

where $I$ is a unit matrix. The solution of (3) are the eigenvalues $\lambda_n^{cr}$ and associated with it the normalized eigenvectors $u_n^{cr}$.

### 2.2 Initial imperfections

The initial geometric imperfections $u_n^i$ are introduced to the model as a linear combination of eigenvectors $u_n^{cr}$ derived from the solution of the linear eigenvalue problem:

$$u_n^i = \alpha_m \cdot u_n^{cr}.$$ (4)

By changing the value of the proportionality factor $\alpha_m$, associated with various forms of buckling modes, enables to obtain all possible combinations of imperfection. It is worth to note that imperfections developed as the linear combinations of the eigenvectors are considered as the most dangerous.

The displacement caused by geometrical imperfections $u_n^i$ induce initial rotations of the bars $\varphi_n^i$ and rotations of the hinges $\phi_n^i$. These deformations are kinematically admissable and does not induce any stress.

As a result of the action of the external load elastic part of displacements and rotations, represent by the superscript $e$, appeared. They are inducing elastic internal moments at the elastic hinges $M_n = k_n \phi_n^e$. In the initial phase, when the external load $P = 0$, the geometric relationships shown in Figure 3 take the form:

$$\varphi_n^i = \arcsin \frac{u_n^i - u_{n-1}^i}{L_n} \quad \text{and} \quad \phi_n^i = \arcsin \frac{u_n^i - u_{n-1}^i}{L_n} - \arcsin \frac{u_{n+1}^i - u_n^i}{L_{n+1}}.$$ (5)

![Figure 3: Geometric imperfections in model structure](image)

In the next phase, when the load $P > 0$, it is obtained:

$$u_n = u_n^i + u_n^e \quad \text{and} \quad M_n = k_n \cdot \phi_n^e = P \cdot \left( u_n^i + u_n^e \right).$$ (6)

$$\varphi_n = \varphi_n^i + \varphi_n^e \quad \text{and} \quad \phi_n = \phi_n^i + \phi_n^e.$$ (7)
Now the set of nonlinear equilibrium equations takes the form:

\[ k_n \cdot \phi_n^\varepsilon = P \cdot (u_n^i + u_n^e) \]  

(8)

of unknowns elastic part of displacements \( u_n^e \). Elastic part of hinge rotation \( \phi_n^e \) is determined using the following relationship:

\[ \phi_n^e = \phi_n^i - \phi_n^j. \]  

(9)

### 2.3 Nonlinear equilibrium equations

In the second stage, depending on the value of the initial clearance \( \Delta_0 \) and total lateral displacement of structures \( u_n \), at the points of elastic hinges presence the cooperation with the intermediate supports is observed. When the gap is closing \( (|u_n| \geq \Delta_n) \), then the appropriate intermediate support with the stiffness \( k_{n\Delta} \) is switched on to the cooperation. Hence, the equilibrium equation should be written in the general form. Until now, the bending moment in the elastic joint has been described as \( M = k_n \cdot \phi_n^\varepsilon = P \cdot u_n \). Now, in equilibrium equations must be taken into account the reactions of intermediate supports \( R_n = k_{n\Delta} (u_n - \Delta_0) \). Equilibrium equations will be written in the form of an equation of virtual work. For this purpose, the virtual displacement \( \delta u_n \) is introduced to the model, so that the hinge No. \( n \) moves vertically and horizontally, while the hinge No. \( n+2 \) moves only in the horizontal direction (Fig. 4).

![Figure 4: Geometric relation of the model structure - virtual displacement \( \delta u_n \)](image)

Thus, the virtual displacement of the hinge No. \( n+1 \) in the vertical direction \( \delta^v u_{n+1} = 0 \). The vertical \( \delta^v u_n \) and horizontal \( \delta^h u_n \) components of virtual displacement of the hinge No. \( n \) can be determined using the following relationship:

\[ \delta^v u_n = L_n \cos \phi_n \delta \phi_n, \]  

(10)

\[ \delta^h u_n = L_n \sin \phi_n \delta \phi_n = u_n \delta \phi_n. \]  

(11)

The horizontal component of virtual displacement \( \delta^h u_{n+1} \) of hinge No. \( n+1 \) is calculated from the kinematic chain relation in the following form:

\[ \delta^h u_{n+1} = \delta^h u_n + L_{n+1} \sin \phi_n \delta \phi_{n+1}. \]  

(12)
The unknown value of the virtual rotation \( \delta \phi_{n+1} \) can be determined from the kinematic chain of vertical displacement:

\[
\delta^* u_{n+1} = L_n \cos \phi_n \delta \phi_n + L_{n+1} \cos \phi_{n+1} \delta \phi_{n+1}.
\]  

(13)

Hence:

\[
\delta \phi_{n+1} = -\frac{L_n \cos \phi_n}{L_{n+1} \cos \phi_{n+1}} \delta \phi_n.
\]  

(14)

For \( L_n = L_{n+1} \) equation (14) takes the form:

\[
\delta \phi_{n+1} = -\frac{\cos \phi_n}{\cos \phi_{n+1}} \delta \phi_n.
\]  

(15)

Thus, the horizontal displacement hinge No. \( n+1 \) can be written as:

\[
\delta^* u_{n+1} = u_n \delta \phi_n + (u_{n+1} - u_n) \left( -\frac{\cos \phi_n}{\cos \phi_{n+1}} \right) \delta \phi_n.
\]  

(16)

The equilibrium equations are derived in a week form and take into consideration the moments in hinges, reactions \( R_n \) and compressive force \( P \):

\[
M_n \delta \phi_n + M_{n+1} \delta \phi_{n+1} = P \delta^* u_{n+1} - R_n \delta^* u_n,
\]  

(17)

where: \( M_n = k_n \phi_n \) i \( M_{n+1} = k_{n+1} \phi_{n+1} \) and \( \delta^* u_n \) is vertical displacement of hinge No. \( n \) which amplitude exceeds the respective value of the clearance \( \Delta_0 \).

Total rotation of the hinge can be expressed as \( \phi_n = \phi_n - \phi_{n+1} \), thus similarly total virtual rotation of the hinge can be obtained as:

\[
\delta \phi_n = \delta \phi_n - \delta \phi_{n+1} = \left( 1 + \frac{\cos \phi_n}{\cos \phi_{n+1}} \right) \delta \phi_n
\]  

(18)

and

\[
\delta \phi_{n+1} = \delta \phi_{n+1} = -\frac{\cos \phi_n}{\cos \phi_{n+1}} \delta \phi_n.
\]  

(19)

Thus, the equation of virtual work takes the form:

\[
k_n \phi_n \left( 1 + \frac{\cos \phi_n}{\cos \phi_{n+1}} \right) \delta \phi_n + k_{n+1} \phi_{n+1} \left( -\frac{\cos \phi_n}{\cos \phi_{n+1}} \right) \delta \phi_n =
\]

\[
P \left[ u_n \left( 1 + \frac{\cos \phi_n}{\cos \phi_{n+1}} \right) \delta \phi_n - u_{n+1} \frac{\cos \phi_n}{\cos \phi_{n+1}} \delta \phi_n \right] - R_n L_n \cos \phi_n \delta \phi_n.
\]  

(20)

It is worth to note that equation (20) is a set of \( n \) equilibrium equations. The type of equilibrium path can be evaluated using the energy approach involving the analysis of the variation of potential energy:
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\[ \delta \Pi = 0 \quad \text{oraz} \]
\[ \delta^2 \Pi \left\{ \begin{array}{ll}
> 0 & \text{stan stateczny} \\
= 0 & \text{stan krytyczny} \\
< 0 & \text{stan niestateczny.}
\end{array} \right. \quad (21) \]

3 NUMERICAL EXAMPLES

Numerical examples are solved for various amplitudes of initial imperfections \( \alpha_m \), normalized amplitudes of clearances \( \bar{\Delta}_0 = \Delta_0 / L \) and elastic supports stiffness \( k_A \), where \( L \) is taken as the unit value.

The first example refers to a structure with initial imperfection pattern developed according to first buckling mode with small amplitude \( \alpha_1 = 0.001 \) (Fig. 5). At nodes 1, 2 and 3 with the initial intermediate support clearances of two different amplitudes \( \bar{\Delta}_0^{1} = 0.13 \bar{u}_2 \) and \( \bar{\Delta}_0^{2} = 0.5 \bar{u}_2 \) were introduced. The curve \( m1 \) shows the equilibrium path obtained for the structure without intermediate supports (solid line - stable; dashed line - unstable equilibrium path). On the other hand, curves 1a, b, c and 2a, b, c represent behavior of structures with intermediate supports in distances of a clearance \( \bar{\Delta}_0^{1} \) and \( \bar{\Delta}_0^{2} \). Symbols \( a, b \) and \( c \) refer to increasing value of intermediate support stiffness, expressed as a relation \( \eta = k_i / k_A \), where \( k_i \) is elastic rotational hinge stiffness and \( k_A \) is elastic intermediate supports stiffness (\( a - \eta_1 = 0.4, \ b - \eta_2 = 0.5, \ c - \eta_3 = 1 \)). In the first case the clearance \( \bar{\Delta}_0^{1} \) is so small that the intermediate supports switch on before the bifurcation point, so all equilibrium paths are stable.

In the second case the the clearance \( \bar{\Delta}_0^{2} \) is large, so there is possible stable and unstable post-buckling behavior as well. It is worth to note that for both small and large clearance the cooperation with intermediate supports resulting in increased stiffness of whole structure in proportion to the supports stiffness.

![Figure 5: Equilibrium path for the initial geometric imperfections \( \alpha_i = 0.001 \) and clearances \( \bar{\Delta}_0^{1} = 0.13 \bar{u}_2 \), \( \bar{\Delta}_0^{2} = 0.5 \bar{u}_2 \); displacement of the hinge: a) No. 1, b) No. 2, c) No. 3](image-url)

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Figure 6 shows the displacement obtained for the model structure with geometric imperfections pattern developed according to second buckling mode with small amplitude $\alpha_2 = 0.001$. Since the displacement of the bar is asymmetrical due to the asymmetric shape of imperfections, positive and negative value of the clearances have to be taken into consideration. One can notice that in this case both stable and unstable equilibrium path may occur. The last example refers to the initial geometric imperfections developed according to three different buckling modes with amplitudes $\alpha_1 = 0,001, \alpha_2 = 0,1$ and $\alpha_3=0,045$. The curve $(m1d2s3)$ shows the equilibrium path obtained for the structure without intermediate supports. In this case several bifurcation points associated with the three buckling modes and stable and unstable post-buckling behavior is observed. Small amplitude of clearance ($\Delta_0$) result in switching on the intermediate supports into cooperation before the bifurcation point, then all equilibrium paths are stable paths and increase of whole structure stiffness occurs (Fig. 7).

![Figure 6: Equilibrium path for the initial geometric imperfections $\alpha_2 = 0.001$ and clearance $\Delta_0 = \pm 0.18u$; displacement of the hinge: a) No. 1, b) No. 2, c) No. 3](image1)

![Figure 7: Equilibrium path for the initial geometric imperfections $\alpha_1 = 0.001, \alpha_2 = 0.1, \alpha_3=0.45$ and clearance $\Delta_0=0.5u$; displacement of the hinge: a) No. 1, b) No. 2, c) No. 3](image2)
4 CONCLUSIONS

In the paper interaction of initial geometric imperfections and clearances for the structural model consisting of \( n \)-compressible bars connected by means of elastic hinges is discussed. The proposed model illustrates the behavior of real structures in terms of stability and enables the derivation of exact nonlinear equilibrium equations taking into account large displacements.

In summary the following conclusions can be drawn:

- In the first stage, when the initial clearance is very large, intermediate supports do not switch on to cooperate and the stable equilibrium path is observed.
- For the very small amplitudes of geometric imperfections, developed according first, second or third buckling mode, single points of bifurcation accompanied by the unstable equilibrium path appeared.
- The increase in the amplitude of the initial geometric imperfection results in an increase in the value of the critical load defining the bifurcation point, where it is possible the snap through to the configuration described by unstable equilibrium path.
- The observed snap through is caused by transition of the structure from a high to a low level of potential energy. In this case, the initial geometric imperfections play a positive role resulting in a stable post-buckling behavior.
- In the case of small clearance, depending on the shape of initial geometric imperfections and intermediate supports stiffness, different equilibrium paths are possible. If clearance is very small and intermediate supports switch on before the bifurcation point, all equilibrium path are stable. When the intermediate supports switch on after bifurcation point both stable and unstable post-buckling behavior of the structure is observed.
- It is shown, that interaction between initial clearances and initial geometric imperfections can strongly affect the structural stability response. Therefore, the proper design of clearance, taking into account the initial geometric imperfection, is extremely important in order to obtain a safe the structure characterized by stable equilibrium paths.

REFERENCES

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