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HIGH-RESOLUTION SIMULATION OF INTERNAL WAVES ATTRACTORS AND IMPACT OF INTERACTION OF HIGH AMPLITUDE INTERNAL WAVES WITH WALLS ON DYNAMICS OF WAVES ATTRACTORS

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Abstract.

Internal wave attractors have received great attention since its discovery in 1995 by Leo Maas ([1, 2]). Now convectional theory describing the formation of attractors is generally accepted, and the principal interest of researchers in recent years is focused on nonlinear interactions. A number of theories had been proposed and now nonlinear interaction due to triadic resonance is accepted as a principle cause of instability of attractors, in which case the parent wave of large amplitude gives birth to two daughter waves, such that conditions of triadic resonance are fulfilled for all the three waves [3]. All this presumes that wave-wall interaction participate in the process only by focusing-defocusing of energy on the inclined boundaries. Here we address interactions of large amplitude internal waves with the boundaries in real laboratory conditions, where Prandtl-Schmidt number is equal to 700. We show that large amplitude waves produce folded structures which are clearly visible on density-gradient images, as if produced by "kneading the dough". These structures are not quickly dissipated due to high Schmidt number and with time they propagate to the interior of the domain, as was shown by our numerical simulation. These structures have visible impact on the instability of attractor and the whole picture of turbulent motion for large amplitudes. The mechanism of interaction of these structures with internal gravity waves structures is the subject of further research. Numerical simulation is quite challenging due to high Prandtl-Schmidt number and small scale of the folded structures in highly nonlinear regimes. Also study is performed on large time scales. As a consequence most of conventional computational approaches give unreliable results. We have applied spectral element approach base on code nek5000 by Paul Fischer. It allowed us to carefully follow the fine space structures on large time scales.

1 INTRODUCTION

Global and local atmospheric motions continue to be pivotal subjects of research, since only deep understanding of such processes may give insights on climate change, pollutants concentrations, and forecasts, short-term as well long-term. In contrast to the atmosphere of the Earth, the driving force of the Ocean is not a classical heat engine. Vertical transfer mechanisms of energy due to thermal processes and wind over the surface play significant role mostly in the vicinity of the surface of the Ocean. Meanwhile the global dynamics of the Ocean is greatly affected by deep water processes and mixing [4, 5, 6, 7, 8], which were studied in much a lesser extent, experimentally and theoretically.

Internal waves in stratified media give an important class of energy transfer in the Ocean. They may form due to tidal motions or flows past orography [9]. In 1995 Leo Maas discovered an amazing feature of internal gravity waves: in some geometries the waves, emitted by source of constant frequency, may form after multiple reflections from boundaries certain pathways [1]. These pathways, which were named *internal waves attractors*, accumulate almost all the energy of the wave motions in the containers. The works by Leo Maas were followed by a great amount of works devoted to formations and behavior of the attractors, so now we have conventional theory for the appearance of small amplitude internal wave attractors.

This is why the principal focus of researches is now on the nonlinear properties of large amplitude internal wave attractors. For such cases the motion can not be described with the help of linearised equation. Internal wave attractors may become unstable, turbulent, they may change structure with time, change background stratification and manifest other nonlinear properties.

In recent years in ENS de Lyon a number of methods for analysis of experimental results have been developed ([10, 11, 12, 13, 3]). Despite remarkable success in experimental study of wave attractors there are significant intrinsic constraints in convectional experimental approaches like PIV or Schlieren. These constraints can be overcomed by direct numerical simulation, provided that it is reliable. The experimental setup possess very large scales interval because of very high Prandtl-Schmidt number. This is why till recently such numerical research were not possible, and numerical modelling dealt with linear modes only, as in ([14, 15]). In these works finite volume method was used for numerical simulation, and realisation of finite volume method was based on MIT code of general circulation model. These papers supported the theory of formation of internal wave attractors. At the same time the authors confessed that their numerical approach could only give insight on formation of the attractor but it is not reliable on large time intervals or in strongly nonlinear modes due to high numerical viscosity. Only recently careful 3D direct numerical simulation was carried out, and for the first time hydrodynamical fields of the experiments and direct numerical simulation were in correspondence within 10% for linear, as well for nonlinear modes ([16, 17]). Now direct numerical simulation is planned to model more complicated configurations and highly nonlinear modes with overturning and mixing.

Internal waves in continuously stratified fluid differ from more "conventional" types of waves, such as electromagnetic or acoustic waves, by very peculiar dispersive relations: first – they are highly anisotropic, and second – temporal frequency depends only on direction (i.e. angle with vertical direction), and don't depend on magnitude of the waves vector. Internal waves may seem unimportant from "practical point" of view since usually they are considered in linear approximation with quite small amplitude. But actually the may have influence on background stratification and on transport of admixtures and pollutants. The influence on background stratification may occur due to interaction with the boundaries, which is always nonlinear. If the amplitude of the waves is high enough, the overturning may happen. Probability of such events may be estimated with the help of Miles – Howard necessary condition. In recent years understating of internal waves dynamics was greatly improved due to numerous experimental laboratory works. Usually internal waves are modelled with the density stratification due to salt solution in a tank filled using double-bucket method. In 1995 [1] and 1997 [2], first theoretically, and next experimentally the internal waves attractors were discovered. Later numerous works were devoted to theoretical investigation, laboratory and numerical experiments. Most of them were devoted to very small amplitudes almost corresponding to linear equations. In internal waves of larger amplitudes vorticity becomes significant. Interaction with the boundaries becomes much more complicated. In this work we show that this interaction produce folded structures which may propagate inside the domain and interact with the main flow.

2 MATHEMATICAL SETUP

In mathematical setup we will try to model as close as possible the experimental setup [3]. The sketch of the experimental installation is shown in Figure 1.

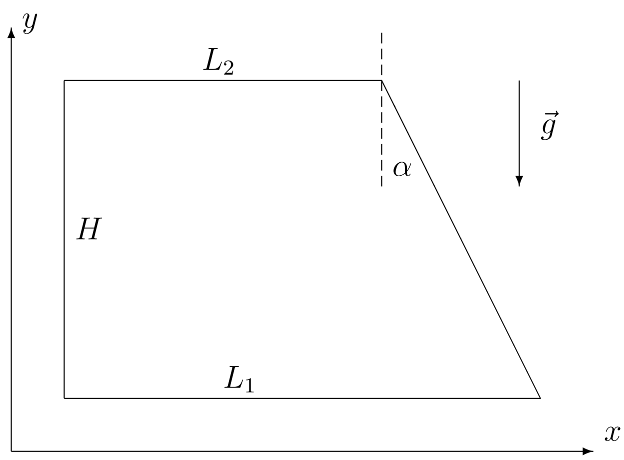

Figure 1: Computational domain in 2D case.

A salt solution is confined in the shown trapezoidal domain, all the boundaries are fixed except for the left one, which oscillates around mid-height point and has the shape of demi-cosine of vertical coordinate.

We will represent density as $\rho = \rho_m + \rho_s(t, x, y, z) = \rho_m + \rho_{s,0}(z) + \rho'(t, x, y, z)$, where ρ_m – density with minimal salinity and $\rho_s = \rho_{s,0}(z) + \rho'_s(t, x, y, x)$ – density increase due to increase of salinity, $\rho_m + \rho_{s,0}(z)$ – initial state. It can be easily shown that such conditions can be modeled by Boussinesq approximation of full Navier – Stokes equations, so we will use the following system of equation for mass, impulse a salt conservation:

$$\frac{\partial \vec{v}}{\partial t} + v^k \nabla_k \vec{v} = -\nabla \frac{\tilde{p}}{\rho_0} + \nu \Delta \vec{v} + s\vec{g} \tag{1}$$

$$\frac{\partial s}{\partial t} + v^k \nabla_k s = \lambda \Delta s \tag{2}$$

$$\mathrm{div}\vec{v} = 0 \tag{3}$$

The most important assumption for Boussinesq approximation $\rho_s/\rho_m << 1$ is obviously satisfied.

Viscosity and diffusion are assumed to be constant.

Initially, the fluid is in rest, and concentration of salt decreases with height.

If fluid density decrease with height, fluid particle that does get displaced vertically tends to be restored to its original level; it may then overshoot inertially and oscillate about this level.

Let the unperturbed state of the layer have the density

$$\rho(x, y, z, t) = \rho_m + \rho_0(z), \tag{4}$$

where ρ_m is the space average of the unperturbed state. Then the net gravitational force on a fluid particle after vertical displacement ζ is $-\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\zeta\vec{\mathrm{g}}$, so

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2} = \frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{\mathrm{g}}{\rho_0}\zeta, \tag{5}$$

which is actually an equation of oscillator so there naturally appears a notion of buoyancy frequency, (or Brunt-Väisälä frequency), the angular frequency at which a vertically displaced parcel will oscillate within a statically stable environment $\rho(z) = \rho_m + \rho_0(z)$:

$$\mathrm{N(z)} = \sqrt{-\frac{\mathrm{g}}{\rho(z)}\frac{\mathrm{d}\rho(z)}{\mathrm{d}z}} \tag{6}$$

To understand the basic properties of the system we may linearise to see how it behaves when the amplitudes of oscillations are small.

In case of constant buoyancy frequency the linearized system of equations is:

$$\rho_m\frac{\partial\vec{\mathrm{v}}}{\partial t} = -\nabla\mathrm{p} + \rho'\vec{g} \tag{7}$$

$$\frac{\partial\rho'}{\partial t} + \mathrm{v_z}\frac{\mathrm{d}\rho_{s,0}(z)}{\mathrm{dz}} = 0 \tag{8}$$

$$\mathrm{div}(\vec{\mathrm{v}}) = 0 \tag{9}$$

$\rho = \rho_m + \rho_s(t, x, y, z) = \rho_m + \rho_{s,0}(z) + \rho'(t, x, y, z)$.

If we will concider wavelike solution, periodic in space and time

$$f = F\exp i(\omega t - (\vec{k}, \vec{r})) \tag{10}$$

$$\omega^2 = N^2\left(1 - k_z^2/k^2\right) = (N\,|\mathrm{sin}\theta|)^2, \tag{11}$$

where θ is the angle between the *vertical* axis and $\vec{k} = e_x k_x + e_y k_y + e_x k_z = e_x k \mathrm{sin}\theta + \ldots$ Waves exist for any value of the angular frequency from zero up to N.

Here we see two most important features of linear waves in weakly stratified fluid of constant bouyancy frequency: direction of propagation of waves is derermined by frequency of perturbations, and after reflection from rigid surface the modulus of angle with vertical is preserved.

A tank is filled with salty water using double-bucket method. The stratification is almost perfectly linear, except for the 2 cm. layer near the top of tank. The size of tank is 46 cm length, 32 cm. height, 17 cm. In our recent work [17] we examined 3D effects on the dynamics of attractor and we showed very good correspondence of experiments and 3D numerical simulation, and we explained discrepancies observed in previous 2D numerical simulation [14].

2.1 Boundary conditions

The upper boundary is stress free, the bottom and right boundary are rigid with no-slip condition, the left vertical boundary oscillate according to a given law.

$$x_b(0, y, t) = a\cos(\pi y/H)\cos(\omega_0 t). \tag{12}$$

If amplitudes are small this condition can be replaced by condition on velocity on fixed boundary:

$$u(0, y, t) = a\omega_0 cos(\pi y/H) sin(\omega_0 t), v = 0. \tag{13}$$

At $t = 0$ the fluid is in rest and stratified with constant buoyancy frequency N.

3 DIRECT NUMERICAL SIMULATION

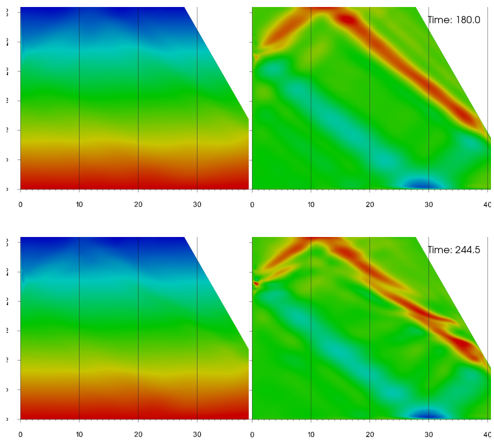

Figure 2: Results of DNS with moderate amplitude $a = 0.25cm$. Density field and horizontal component of velocity before and after PSI.

Numerical computations are performed with the help of spectral element methods [18, 19]. The geometry of the numerical setup closely reproduces the experimental one. The full system of equations being solved consists of the Navier-Stokes equation in the Boussinesq approximation, the continuity equation and the equation for the transport of salt. Typical meshes used in calculations consist of 50 thousands to half-million elements, with 8 to 10-order polynomial decomposition within each element, time discretization from 10^{-4} to 10^{-5} of the external forcing period.

As shown in [17], comparison of our experimental and numerical results are in beautiful agreement, not only qualitative but also quantitative.

Small amplitude behaviour is well predicted by linear theory [1, 2, 20]. The resulting motion is similar to the shown in first row of Figure 2: the internal wave attractor is formed. With higher amplitude we will get unstable wave attractor, which is shown in Figure 2: the first row shows the attractor just after it's formation, and the pictures in second row show the developing triadic resonance. At the same time no significant interaction with the walls, except for well predicted reflections, can be noted.

In Figure 3 you can see sequence of snapshots for twice higher amplitude $a = 0.5\text{cm}$. Here you can see appearance of folding in the wall regions. Layers of high concentrations are first formed due impermeability of the walls and rotational motion of waves. Next they begin to propagate inside the domain, where they interact with the background flow.

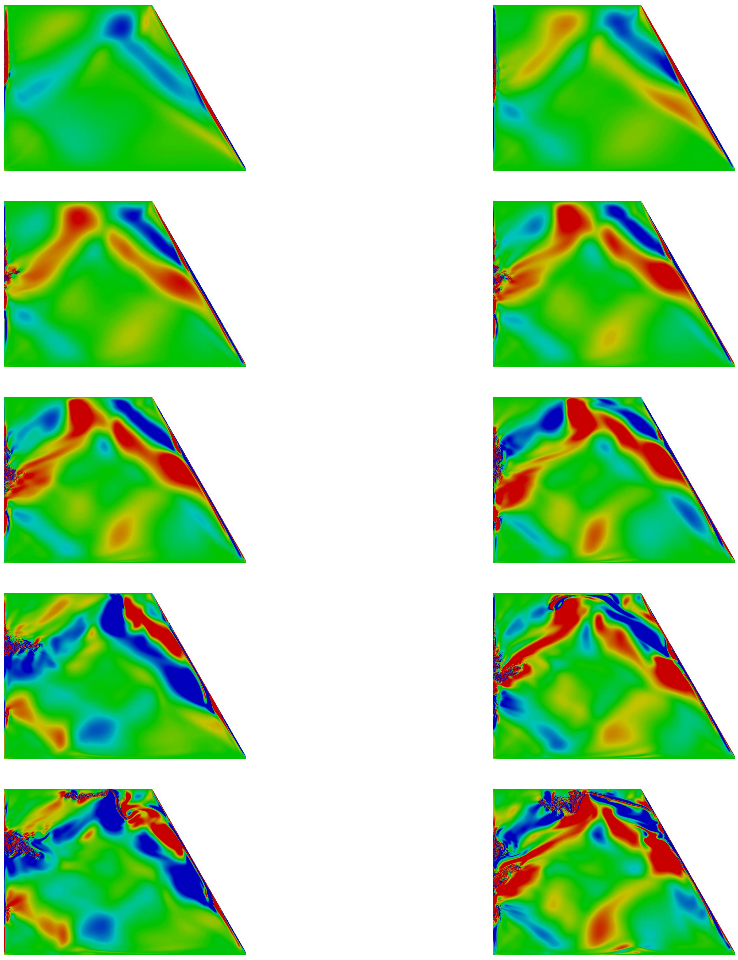

Figure 3: Results of DNS with large amplitude $a = 0.5cm$. Time-evolution of horizontal component of density gradient $\partial\rho/\partial x$

4 CONCLUSIONS

Direct numerical simulation of large amplitude internal wave attractors have shown the presence of folding structures in the wall regions of the container. They appear as a result of oscillating wave-like rotating motions of fluid near the boundaries. Since the boundaries are impermeable for salt and ratio of diffusion of motion and of concentration is very high (Sm=700), these folding structures are not quickly dissipated and propagate to the interior of the domain, where they interact with the background flow and influence mean stratification on long time periods. Since the precess is highly nonlinear and application conventional numerical methods is often criticized, careful experimental investigation is now needed fully describe this phenomena. Qualitative assessment of vertical transport and impact on background stratification is needed to describe interaction of the wave attractor and folded structures.

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