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MESH OBJECTIVE DAMAGE MODELING OF DUCTILE FRACTURE AT VISCO-PLASTIC CONTINUUM RESPONSE

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Abstract. *Local- continuum damage models, such as the JC-dynamic failure criterion are often combined with the constitutive JC-model to represent the material behavior during machining. The major drawback is that the failure criterion exhibit a pathological mesh dependence. In literature it has been argued that a viscous regularization of the continuum material model coupled to damage via visco-plasticity may remove the mesh dependence. We present results, based on the objective enhanced element removal model derived from previous contribution, form a mesh dependence study using a extended formulation of the constitutive JC- model covering visco-plasticity. The results show that, excluding the mesh objective enhancement the pathological mesh dependence still exist if the visco-plastic formulation is used. If the mesh objective enhancement is used a relatively good convergence is obtained for a set of mesh sizes for the damage model.*

1 INTRODUCTION

In order to model ductile fracture combining the constitutive JC-model with the JC- dynamic failure criteria as in [1] is a possible approach. The JC-model describes the damage evolution in a gauss-point of an element as a function of the failure strain in relation to the accumulated plastic strain. When the JC- dynamic failure criterion is fulfilled, the stress response in the gauss-point is instantaneously relaxed and the element is removed from the simulation. As for the failure strain, it is composed by three dependencies, the stress triaxiality, plastic strain rate and temperature similar to the JC- model. This approach refers to the standard JC-dynamic failure criterion. Despite its simplicity it is not possible to disregard that the JC-model, due to its softening effects, might result in a loss of solution uniqueness. This results in in so called pathological mesh dependence which is a problem for local damage models.

In previous contribution [1] a remedy was proposed for the observed inherent mesh dependence in ductile fracture models. To remove the dependence, a set of mesh objective damage models were developed based on a local-continuum damage formulation combined with concepts from a scalar damage phase-field [3], [4]. Independent of damage model, the JC- dynamic failure criterion was used to initialize a ”fracture state” which signal for damage initialization and damage evolution. One of the objective damage models relates to the traditional ”element removal” which is nothing else than the coupling between the JC- models, described by an instantaneous damage evolution.

In [5], [6] it is argued that a viscous regularization of the continuum material model coupled to damage via visco-plasticity may remove the mesh dependence. In this contribution, the rate dependent response was incorporated in the JC- model extending the mesh objective damage enhancement. The model was implemented in ABAQUS/Explicit as user subroutine based on the hypo-elastic inelastic framework which is computationally efficient [7]. In order to investigate the assertion regarding viscous regularization postulated, a mesh dependency study was conducted for highly refined mesh sizes of a 2D plane strain plate. The results show that, excluding the mesh objective enhancement the pathological mesh dependence still exist if the visco-plastic formulation is used. If the mesh objective enhancement is used a relatively good convergence is obtained for a set of mesh sizes.

2 CONSTITUTIVE MODELING BASED ON VISCO-PLASTIC CONTINUUM RESPONSE

In this section mesh objective element removal model formulated in [1] with the potential to describe ductile fracture in materials, for rate independent response, is evaluated in a visco-plastic/rate-dependent context. As for the considered model, the damage evolution takes place in one single step when the the damage criterion is fulfilled, representing a instantaneous stress relaxation. In order to represent the effective stress response the Johnson-Cook (JC) constitutive material model, extended to include visco-plastic response, is considered.

2.1 Visco-plastic continuum response and ductile damage models derived from the Johnson Cook failure criterionl

In the present context it is of significant interest to enhance the effective material in terms of a “scalar damage” measure α acting on the effective material so that

$$\psi = (1-\alpha)\hat{\psi} \text{ with } \hat{\psi} = \hat{\psi}\left[\bar{\mathbf{b}}, k\right] \tag{1}$$

where $\hat{\psi}$ is the free energy of the effective (undamaged) material, denoted by a superimposed hat. It is assumed that $\hat{\psi}$ depends on the elastic Finger deformation tensor $\bar{\mathbf{b}}$ and an isotropic hardening variable k. It turns out that the total dissipation rate $\mathcal{D}$ in the dissipation inequality can be formulated in terms of the effective dissipation rate $\hat{\mathcal{D}}$ as

$$\mathcal{D} = (1-\alpha)\hat{\mathcal{D}} + \hat{\psi}\dot{\alpha} \geq 0 \text{ with } \hat{\mathcal{D}} = \hat{\tau} : \mathbf{l}_p + \kappa\dot{k} \geq 0 \tag{2}$$

which corresponds to the constitutive state equations

$$\hat{\tau} = 2\frac{\partial\hat{\psi}}{\partial\bar{\mathbf{b}}} \cdot \bar{\mathbf{b}} \, , \, \hat{\kappa} = -\frac{\partial\hat{\psi}}{\partial k} \tag{3}$$

In (3), we introduced the effective Kirchhoff stress $\hat{\tau}$ and the hardening stress $\hat{\kappa}$ pertinent to the effective material. To ensure positive total dissipation $\mathcal{D} \geq 0$ it suffices to consider $\hat{\mathcal{D}} \geq 0$ as in (2) and $\dot{\alpha} \geq 0$ (since $\hat{\psi} > 0$ always). This corresponds to the constitutive state equations

$$\tau = (1-\alpha)\hat{\tau}, \kappa = (1-\alpha)\hat{\kappa}, A = -\frac{\partial\psi}{\partial\alpha} = \hat{\psi} \tag{4}$$

where A is the damage driving force. Upon assuming neo-Hookean isotropic elasticity, we assume for the stored elastic behavior

$$\begin{aligned} &A = \hat{\psi} = \hat{\psi}^{\text{iso}} + \hat{\psi}^{\text{vol}} + \hat{\psi}^{\text{hard}}[k] \text{ with} \\ &\hat{\psi}^{\text{iso}} = \frac{1}{2}GJ^{-\frac{2}{3}}\left(1 : \hat{\mathbf{b}} - 3\right) , \hat{\psi}^{\text{vol}} = \frac{1}{2}K(J-1)^2 \, , \hat{\psi}^{\text{hard}} = \frac{B}{1+n}(-k)^{1+n} \end{aligned} \tag{5}$$

where G is the shear modulus, K is the bulk modulus, B is the isotropic hardening parameter and n is the hardening exponent.

Moreover, we consider the visco-plastic Johnson Cook model for the inelastic effective continuum response. We thereby introduce a static yield function $\phi = \phi\left[\hat{\tau}_e, \hat{\kappa}\right]$ in terms of the the effective von Mises stress $\hat{\tau}_e$, and the evolution of the internal dissipative variables $\left\{\mathbf{l}_p, \dot{k}\right\}$

$$\mathbf{l}_p = \lambda\frac{\partial\phi}{\partial\hat{\tau}} = \frac{3}{2}\frac{\hat{\tau}_{\text{dev}}}{\hat{\tau}_e} \, , \, \dot{k} = \lambda\frac{\partial\phi}{\partial\hat{\kappa}} \tag{6}$$

where $\lambda \geq 0$ is the plastic multiplier determined by the *Johnson-Cook* (JC) overstress

$$\lambda = \dot{\epsilon}_0 \exp\left[\frac{\phi}{C\left(A+\hat{\kappa}\right)}\right] \text{ for } \lambda > \dot{\epsilon}_0 \tag{7}$$

where $\hat{\kappa} = B(-k)^n$ is the isotropic micro-hardening stress and ϕ the yield function explicitly formulated as $\phi = \hat{\tau}_e - (A+\hat{\kappa})$. We note that A, C and $\dot{\epsilon}_0$ are material parameters representing initial yield and rate sensitivity, respectively. The response becomes visco-plastic (or rate dependent) whenever $\lambda \geq \dot{\epsilon}_0$. To handle the rate independent case $\lambda < \dot{\epsilon}_0$, the plastic multiplier is controlled by the Kuhn-Tucker (KT) loading conditions

$$\phi \leq 0, \quad \lambda \geq 0, \quad \lambda\phi = 0 \text{ for } \lambda \leq \dot{\epsilon}_0 \tag{8}$$

Following the developments in [1], we outline here the mesh objective element removal model associated with the JC-material and failure models representing ductile fracture. To this end, we consider the JC- dynamic failure criterion, where it is postulated that ductile fracture occurs when the equivalent plastic strain approaches the fracture strain ϵ_f^p defined as

$$\epsilon_f^p = (d_1 + \exp[-d_2 r])\left(d_3 + \exp\left[d_4 \frac{\lambda}{\dot{\epsilon}_0}\right]\right) \text{ with } r = -\frac{\hat{p}}{\hat{\tau}_e} \text{ and } \hat{p} = -\frac{\mathbf{1} : \hat{\tau}}{3} \tag{9}$$

Hence, a fracture state is obtained when $k + \epsilon_f^p = 0$ at $t = t_c$, where t_c is time when local failure occurs. Please note that this critical time depends on the rate of loading $\lambda/\dot{\epsilon}_0$, the triaxialty r and initial failure strain. These inter-dependences are described by (9) along with the failure parameters d_1–d_4.

The total damage force is introduced as

$$A_T = \hat{\mathcal{D}}_T^f[t] + A \,,\, \hat{\mathcal{D}}_T^f[t] = \int_{t_c}^{t} \hat{\mathcal{D}}\, dt \tag{10}$$

We consider next the *damage-element removal model* based on the instantaneous damage evolution rule, as formulated by the Dirac-delta function $\delta_S[\bullet]$ for the damage evolution, i.e.

$$\dot{\alpha} = \mu \delta_S \left[t - t_c\right] \tag{11}$$

where $\dot{\alpha}$ is controlled by

$$\phi_\alpha = A_T - \frac{L_c}{L_e} A_c \leq 0 \,, \quad \mu \geq 0 \,, \quad \phi_\alpha \mu = 0 \tag{12}$$

In this case, the critical time is defined by when $t = t_c$ has been reached in (9) and $A_T - A_c = 0$.

We thus conclude that a "fracture state", $t = t_c$, is arrived at when the JC- dynamic failure criterion (9) is met. For the damage model, the stored free energy A_c in (12) to be released is evaluated at the Gauss-point level for each element size. For finer meshes with $L_e < L_c$ the continuous dissipation $\hat{\mathcal{D}}_T^f$ is integrated so that A_T can be computed from (10). For the considered damage model, the stress state is "removed" when the condition (12) is approached.

3 NUMERICAL EXAMPLES

In [5], [6] it is argued that a viscous regularization of the continuum material model coupled to damage via visco-plasticity may remove the mesh dependence. In order to investigate the assertion the rate dependent/visco-plastic material model described in previous sections will be considered together with the proposed mesh objective element removal damage model in a mesh dependency investigation for highly refined meshes.

3.1 Finite element modeling

As a point of departure for the mesh dependency investigation the same 2-D shear loaded plane strain plate as in [1] and [7] is considered with the material parameters for the JC material and failure models in Tables 1. The 50x50 mm square shaped plate is divided into three parts. On the right side of the top edge of the plate a prescribed vertical displacement is applied on the nodes. The right boundary of the plate is constrained in horizontal direction allowing vertical motion. Therefore a displacement of 25 mm is applied on the upper right edge for the displacement rate 500 mm/s. The left section is fully constrained in both directions as shown in Figure 1. Since the mid section is subjected to severe shear deformation it suffices to use a fine FE-representation here in order to model the damage with a reasonable accuracy, while using

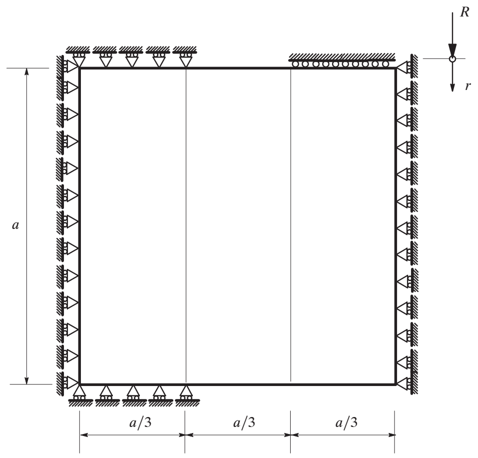

Figure 1: Considered geometry used for the analyses with corresponding boundary conditions

coarser elements for the remaining parts of the geometry. In this way the computational cost is reduced. The investigation is performed for the element removal model using the hypo-elastic inelastic framework implemented in the commercial software ABAQUS/Explicit using the user subroutine VUMAT.

Table 1: Johnson-Cook constitutive material and failure parameters

A [MPa]	B [MPa]	C	n	$\dot{\epsilon}_0$	d_1	d_2	d_3	d_4
550	500	0.0804	0.4	0.001	0.25994	0.61368	2.5569	-0.027652

To perform the mesh dependency investigation a set of six different FE discretizations of the plate are created. The 4-node plane strain bilinear elements described in previous section are used with different element sizes: 1, 0.75, 0.5, 0.25, 0.125 and 0.1 mm. Therefore, a wise discretization of the plate, regarding the finer mesh sizes, is beneficial from the computational point of view. Due to the severe shear deformation that occur in the mid-section of the geometry, we choose to enrich this region with the specific element sizes described. Doing so, highly resolved damage modes are achieved. Noteworthy, only a structured mesh is used in the mid-section while a free distribution of elements is used for the remaining geometry.

4 RESULTS

In this section, the performed simulations based on the suggested examples of different FE discretizations are analyzed with respect to mesh dependency. The simulations are conducted for the enhanced objective element removal model proposed in [1] with the extension of the JC - material model to include rate dependent behavior. The results are presented in form of force-displacement curves, where the magnitude of the force is computed at the edge where the displacement boundary conditions are applied.

4.1 Element removal model - Excluding mesh objective enhancement

If we consider the response without any mesh objective enhancement, using standard JC dynamic failure model, it could be argue that the pathological mesh dependency should decrease with the introduction of vico-plasticity in the material model. However, our simulations show that there is no tendency of minimization of the mesh dependency for the deformation rate chosen as illustrated in Figure 2. But as the mesh size is refined, the response becomes more and more brittle, eventually a convergence is obtained. It is also clear that as the FE discretization of the plate is successively refined, the kinematics allow for other possible damage modes which may be observed in Figure 2.

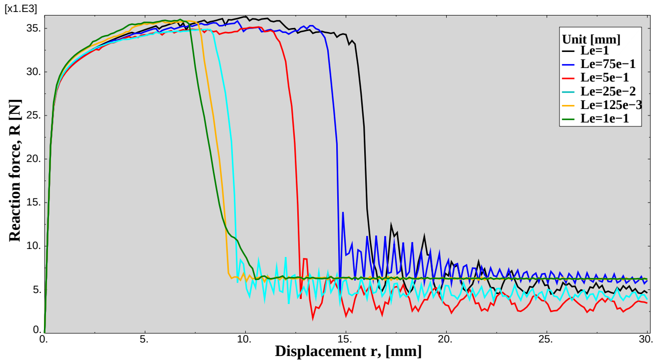

Figure 2: Force-displacement curves for different element sizes L_e and deformation rate v = 500 mm/s excluding objective enhancement in the element removal mode, representing standard JC dynamic failure model.

4.2 Element removal model - Including mesh objective enhancement

The results for the element removal model including the objective enhancement are presented for the chosen displacement rate where the reference element size of $L_c = 0.5$ mm was chosen. When the mesh objective enhancement is introduced a relatively good convergence is achieved for element sizes finer than the reference size. No indication of mesh size convergence is notable for $L_e > L_c$ as observed from Figure 3, which is similar to the rate independent case in [1].

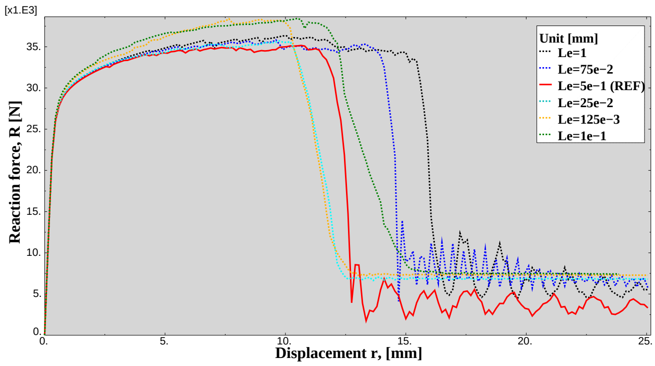

Figure 3: Force-displacement curves for different element sizes L_e and deformation rate v = 500 mm/s including objective enhancement in the element removal model.

5 CONCLUDING REMARKS

As a point of departure in the current contribution, the derived mesh objective element removal model, based on the local-continuum damage formulation combined with scalar damage phase-field concepts, in [1] is investigated. The constitutive JC- model was expanded to incorporate visco-plastic response, taking the rate dependence into consideration. As it was argued in literature that a that a viscous regularization of the continuum material model coupled to damage via visco-plasticity may remove the mesh dependence an investigation was conducted. The mesh objective damage models were implemented in ABAQUS/Explicit as user subroutines. In order to conduct the mesh dependency investigation the same 2D plane strain plate as in previous contribution was used with different structured FE discretizations. Our results show that the the pathological mesh dependence is till present when the mesh objective enhancement is excluded although visco-plasticity is incorporated in the constitutive model. Furthermore, when the mesh objective enhancement is included a convergence in the force-displacement curves is evident for a set of mesh sizes, independent of damage model. Nevertheless, the models do predict reliable response regarding the force-displacement curves and the pathological mesh dependence is removed for a set of mesh sizes.

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