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FINITE-ELEMENT MODEL OF INTRAOCULAR PRESSURE MEASUREMENT BY MAKLAKOV APPLANATION TONOMETER

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Abstract. *The stress-strain state of the corneal shell of a human eye loaded with a flat base stamp of an arbitrary weight for different values of the intraocular pressure is analyzed. In the Maklakov method the intraocular pressure (IOP) is measured by deformations under loading. Similar approach, for which the stress-strain state is considered as the result of loading with a flat base stamp, is applied in [1, 2, 3].*

This paper discusses the three dimensional finite-element model of the contact problem, when the multilayer corneal layer is loaded with a flat base stamp. This mathematical model is constructed in software package ANSYS. The cornea is modelled as an orthotropic spherical shell of variable thickness consisting of four layers: the outer layer (the epithelium), Bowmans membrane, the core layer (stroma of the cornea), and the inner layer (Descements membrane). Moreover, all layers have their specific elastic modules, that differ signicantly both in tangential direction and in transverse directions. The analysis is conducted of four flat base stamps of different weight and of different values of intraocular pressure (from 10 up to 30 mm Hg) for each stamp. The deformations of the corneal shell depends substantially on these factors. Also, the possibility of the contact detachment under loading is examined.

Finally, the estimation of different fundamental characteristics is made, including diameter of the contact zone, with the help of which the intraocular pressure is measured. The results of the mathematical modeling obtained in research are compared with the clinical data.

1 INTRODUCTION

Determination of IOP is important in the diagnosis eye diseases and in monitoring during the postoperative period. IOP measurement is made by recording deformation of the cornea in response to mechanical impact under the load of a flat base stamp.

In the work [1] a two-dimensional model of measuring IOP using the Maklakov's tonometer is considered, in which the corneoscleral shell is modeled as two conjugated spherical segments. Meanwhile corneal shell is considered as a soft shell that has no resistance to flexural deformation and no "shrinkage" under the action of the tonometer. So deformation of the corneal shell is described by the relation $p_t = W/S$, where p_t – tonometric IOP, W – the weight of the tonometer, and S – the contact area of the corneal shell and the tonometer (a flat base stamp). This equation is applicable only for infinitely thin and soft shells.

Based on the assumption of balancing of the weight of applied tonometer and IOP the following relation can be written as $F = W = PS$. Considering that the area of contact between the tonometer and the corneal shell has the shape of a circle, the area of the contact zone is calculated by the formula $S = \frac{\pi d^2}{4}$. Modifying the formula to express it in terms of the contact zone diameter d, the following is obtained:

$$d = 2\sqrt{\frac{W}{\pi P}}, \tag{1}$$

This paper presents a three-dimensional finite element modeling of the contact problem of loading the corneal shell with a flat base stamp in the mathematical software package ANSYS. It investigates the change in the stress-deformed state of the corneal shell while being loaded with stamps of different weights. In particular, existance of possible "tear" of the surface of the flat stamp from the cornea inside the contact zone is checked. Also the relationship of thickness between corneal shell and its layers is explored while loading with flat stamp and varying IOP.

2 FINITE-ELEMENT MODEL

Corneal shell is modeled with a spherical segment of outer radius of 7.8 mm with variable thickness, which is divided into four layers: epithelium, Bowman's membrane (BM), corneal stroma, and Descemet's membrane (DM). In this model only the stroma has variable thickness, which varies, for example, from 0,535 mm in the center up to 1,135 mm at the edge, the other layers are set with constant thicknesses. The endothelium, the fifth layer of corneal shell, is not modeled due to its insignificant thickness. All layers are modeled as homogeneous; fibration is not taken into account.

According to [4] the cornea can be considered as a transversely isotropic shell. The elastic constants satisfy to the following system of inequality [5]:

$$|\nu_i^{'}| < \left(\frac{E_i^{'}}{E_i}\right)^{1/2}, \; -1 < \nu_i < 1 - 2\left(\nu_i^{'}\right)^2 \frac{E_i^{'}}{E_i}, \; \left(E_i > 0, E_i^{'} > 0\right), \; i = 1, ..., 4, \tag{2}$$

where E_i and $E_i^{'}$ - are the moduluses of elastisity for the surface of isotropy and in the direction of normal to the surface respectively; ν_i and $\nu_i^{'}$ - the Poisson's ratios; $G_i^{'}$ is the shear modulus for any plane normal to the surface of isotropy; Shear modulus for the surface isotropy G_i is given as:

$$G_i = \frac{E_i}{2\left(1 + \nu_i\right)} \tag{3}$$

Table 1: The thicknesses and elastic coefficients of the layers of the cornea

	Units	Epithelium	BM	Stroma	DM	Sclera
h_i	mm	0,043	0,012	0,535-1,135	0,01	0,6-1,2
E_i	MPa	0,06	0,6	0,3	0,9	5
$E_i^{'}$	MPa	0,003	0,03	0,015	0,045	0,5
G_i	MPa	0,0297	0,2970	0,1485	0,4455	2,4752
$G_i^{'}$	MPa	0,0010	0,0100	0,0050	0,0150	0,1668
ν_i		0,499	0,499	0,499	0,499	0,499
$\nu_i^{'}$		0,01	0,01	0,01	0,01	0,01

Table 1 specify the thickness of the layers of the corneal membrane, values of elastic moduli and Poisson's ratios in the tangential and thickness directions, which are used to solve the problem. It is assumed that the corneal tissue and scleral shell are close to incompressible, so ν_i is assumed to be 0,499. For the finite element model of the flat base stamp isotropic material with a coefficient of Young's modulus equal to $2 * 10^{11}$ Pa and Poisson's ratio $\nu = 0,3$ (steel) is used.

The IOP is set equal to 2000 Pa, which is normal, in the direction perpendicular to the inner surface. To reduce the influence of the boundary conditions on the investigated area of the corneal shell, a fixed support of the section of the sclera in the equatorial area of the eyeball is used as boundary conditions.

3 RESULTS

In this reseach a series of calculations is done, which correspond to the loading of the flat base stamp of different weights – 5, 7.5, 10, and 15 gram, simulating by so the process of measuring IOP by Maklakov's applanation tonometer using standard weights. Figure 1 shows

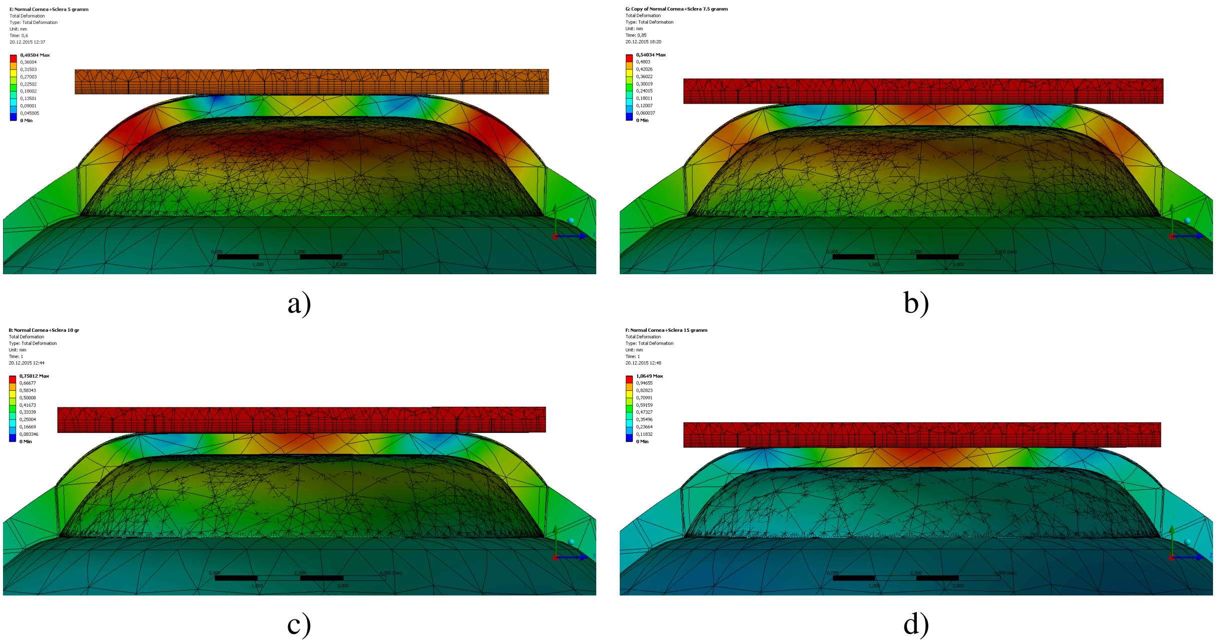

Figure 1: Deformation in the section YZ of cornea loaded by stamp with weight a) 5 g, b) 7.5 g, c) 10 g, d) 15 g

deformations in the cross-section YZ, and Figure 2 shows deformation from above the model.

While loading the stamp with weight of 5 g, there is a distinct "ring", which corresponds to the minimum deformations (close to zero by value) in the contact zone at the very edge. Moreover, these deformations become maximum (approx. 0.4) outside of the contact zone in the form of a ring. In case of lager weight of 7.5 g the diameter of the zone of contact is increased and another contact area of the maximum deformation appears in the center of the corneal shell. The subsequent increase in the weight of the stamp up to 10 g rises the deformations in the center and the diameter of the contact zone. This effect is enhanced by loading the heavier stamp with a mass of 15 g. Maximum values of deformations in the center of the contact zone are listed in table 3. In fact in figure 2 due to the ring of minimal deformation on the outer surface of the corneal shell, marked in blue, the diameter of the contact zone is increasing while raising the weight of the stamp.

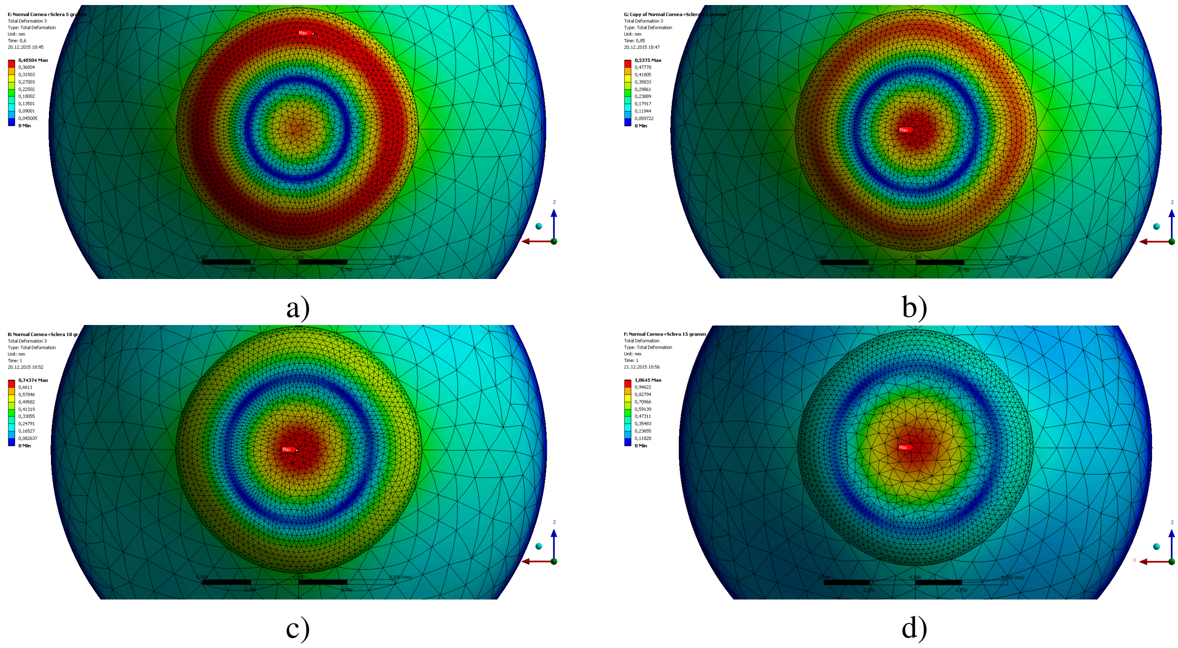

Figure 2: Deformation of cornea (top view) loaded by stamp with weight: a) 5 g, b) 7.5 g, c) 10 g, d) 15 g

Table 2: The diameters of the contact zone between the stamp and the epithelium of the cornea, mm

	the weight of the stamp, grams			
IOP, mm Hg	5	7,5	10	15
15	5,552	6,915	7,757	9,292
20	4,880	5,902	7,096	8,439
25	4,634	5,320	6,463	7,527
30	4,243	4,849	5,823	6,909

Approximating the data from table 2 we can obtain the following equation for change of the diameter of the contact zone d between the corneal epithelium of the shell and loaded flat base stamp depending on P:

$$d_5 = -0,0002P^3 + 0,013P^2 - 0,4144P + 9,3399 \tag{4}$$

Table 3: Maximum values of deformations in the center of the contact zone

	the weight of the stamp, grams			
IOP, mm Hg	5	7,5	10	15
10	0,468	0,748	0,957	-
15	0,350	0,540	0,750	1,065
20	0,222	0,367	0,566	0,858
25	0,128	0,280	0,409	0,667
30	0,110	0,232	0,312	0,501

$$d_{7.5} = 0,0007P^3 - 0,0397P^2 + 0,6309P + 4,0951 \tag{5}$$
$$d_{10} = 0,0005P^3 - 0,0362P^2 + 0,6551P + 4,2628 \tag{6}$$
$$d_{15} = 0,0016P^3 - 0,1153P^2 + 2,5427P - 9,2137 \tag{7}$$

where equations (4) – (7) correspond to the loading of the flat base stamp with weights of 5, 7.5, 10, and 15 g respectively. In these equations d – is measured in mm, and the IOP P – in mm Hg. In order to calculate the value of the IOP based on the diameter of the imprint inversed relationship is obtained:

$$P_5 = 4,5884d^2 - 56,682d + 188,15 \tag{8}$$
$$P_{7.5} = 2,182d^2 - 32,942d + 138,45 \tag{9}$$
$$P_{10} = 0,0865d^2 - 9,085d + 79,635 \tag{10}$$
$$P_{15} = -0,0036d^2 - 8,1348d + 86,393 \tag{11}$$

here the equations (8) – (11) correspond to the loading of the flat base stamp with weights of 5, 7.5, 10, and 15 g respectively. The units are the same d in mm, and P in mm Hg. P varies in the range from 15 to 30 mm Hg.

Based on received results of finite element modeling it is possible to obtain the diameter of the area of contact in dependence from the weight of loaded flat base stamp, while IOP is taken as constant. Approximating received data it is possible to obtain analytical relations between the diameter of the zone of contact d and weight of the load applied to W:

$$d_{15} = -0,0316W^2 + 0,9174W + 1,7695 \tag{12}$$
$$d_{20} = -0,0146W^2 + 0,6538W + 1,9338 \tag{13}$$
$$d_{25} = -0,0106W^2 + 0,5106W + 2,2751 \tag{14}$$
$$d_{30} = -0,0063W^2 + 0,3998W + 2,3477 \tag{15}$$

where equation (12) – (15) correspond to loading at a constant value of IOP of 15, 20, 25, and 30 mm Hg respectively. The units of d are mm and W is in grams. The weight values of the flat stamp applied range from 5 to 15 g.

The corneal thickness changes in the center of the shell when loaded with a flat base stamp. As shown on figure 3 the overall thickness of the corneal shell is reduced by 18.4% under normal IOP 2000 Pa as heavier flat stamp is used. Due to its small thickness Bowman's membrane is included in the thickness of the layer of epithelium in current analysis. But in the process of loading with the 5 g stamp their combined thickness was reduced from 0.055 mm to 0,0234 mm, which is more than two times, and during loading with the 15 g stamp thickness is reduced to 0,0112 mm, this is a reduction in the thickness of the epithelium of five times. So, about 30% of the reduction in the corneal thickness falls on the epithelium.

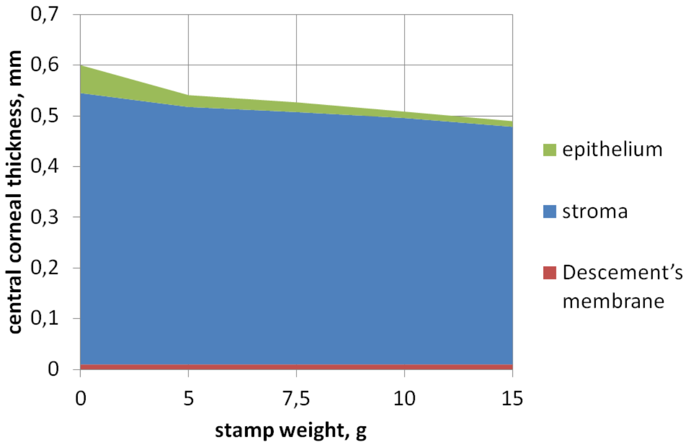

Figure 3: The change in the corneal thickness of the shell when loading by a stamp under normal IOP

4 CONCLUSION

There is no separation of the surfaces, in calculated range of IOP, in the contact zone of the surfaces of the stamp and of the corneal epithelium, when loading the stamp with a flat base on the corneal shell of the eye. Although the area of minimal deformations is close to zero (the ring marked in blue in figure 2), it is inside the contact zone.

As a result of loading simulation it is revealed that there is a five-times reduction in the thickness of the epithelium of the corneal shell, which is about a third of the total reduction in thickness of all the shell during loading.

Constructed finite element model of loading of the corneal shell with the flat base stamp allows to calculate the IOP depending on the weight of applied stamp and the thickness of the corneal shell of the eye in its center. Results obtained in the finite element simulation are in good agreement with clinical data.

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