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LIMIT ANALYSIS LOCUS OF HIGH STRENGTH STEEL PLATES UNDER NON-QUADRATIC YIELD CRITERIA

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Abstract. *High strength steel sheets, widely used in industrial applications, exhibit a significant anisotropy of plastic response between their rolling and transverse direction. Hills yield criterion is the most popular in sheet metal forming FE simulations, because its parameters are easily determined. In the literature, several other criteria are proposed that can better describe the anisotropic behavior of specific materials. In limit analysis the material elastic properties are irrelevant. A formulation, based on the static theorem, requires only the equations of equilibrium and a respective yield criterion to describe the safety margins of a structure. Limit analysis implements standard FEM data into mathematical optimization packages to yield the respective safety factor. Depending on the yield criterion type, a large scale nonlinear mathematical programming problem must be solved. Quadratic yield criteria, like Hill, lead to Second Order Cone Programming problems. Non-quadratic yield criteria, on the other hand, lead to problems that can be treated only by general non-linear mathematical programming algorithms. In the present study, a polynomial criterion proposed by Yoshida et al. is used to construct the limit load locus of high strength steel structures. Results are compared to the von Mises and Hill criteria.*

1 INTRODUCTION

High strength steel sheets due to their crystallographic structure, chemical properties and forming process exhibit a significant anisotropy in their mechanical properties between rolling and transverse direction. Defining the safety margins of structures using such materials is crucial for their design.

Limit analysis allows the direct determination of the load bearing capacity of structures subjected to monotonically increasing loads, requiring only limited input data [1-5]. Standard FEM data is appropriately combined with optimization techniques to yield the respective safety factor. For common engineering structures usually a large scale mathematical programming problem has to be solved. Depending on the selected yield criterion a linear quadratic or general nonlinear optimization problem arises.

Within the framework of the direct methods of plasticity, anisotropic materials have been studied using the kinematic theorem [1] (upper bound approach). For example, general anisotropic structure limit analysis is presented in [6, 7], while in [8] limit analysis of orthotropic composite laminates is studied using the linear matching method. In [9] yield stresses for the different directions are incorporated in a single ellipsoidal yield surface used in 2D and 3D problems.

In the present work, limit analysis of anisotropic structures based on the static theorem is considered. More precisely, we use as basis the mathematical dual of an upper bound problem. To this goal, several yield criteria from the literature are compared. Together with the popular quadratic criteria von Mises [10] and Hill [11], a highly flexible "user-friendly" nonlinear criterion proposed by Yoshida [12] is used.

The plan of this paper is as follows. First limit analysis is presented as a nonlinear programming problem. Then, the yield criteria used are discussed. Finally, the paper closes with a numerical example and some concluding remarks.

2 LIMIT ANALYSIS AS A NONLINEAR PROGRAMMING PROBLEM

Let us consider a structure Ω made of an anisotropic elastic-plastic material. Let Ω be discretized using NE finite elements with NU free degrees of freedom and NG numerical integration points (Gauss points) for the whole structure. Limit analysis using the static theorem leads to the following non linear programming problem:

$$\begin{aligned} &P_{LA} && \max a \\ &s.t.: && \mathbf{H}_j\, \mathbf{s}_j = a\, \phi^{(ext)} \\ & && \mathbf{s}_j \in \mathcal{F}_j, \quad for\; j = 1, \ldots, NG \end{aligned} \tag{1}$$

Unknowns are the total elastoplastic stresses $\mathbf{s}_j$ and the load multiplier (safety factor) a. $\phi^{(ext)}$ is a vector of size NU that contains all the nodal loads applied to the structure. $\mathbf{H}_j$ is the equilibrium matrix depending on the discretization and the boundary conditions. $\mathcal{F}_j$ is the local yield criterion that has to be satisfied at every stress checking point. Depending on the type of $\mathcal{F}_j$ a different optimization problem has to be solved. Quadratic yield criteria lead to second order cone programming problems, for which several very efficient optimization packages exits (i.e. MOSEK [13]). Non-linear, non-quadratic yield criteria lead to problems that can be treated only by general non-linear mathematical programming algorithms as IPOPT [14].

The safety factor depends only on the yield criterion, discretization and boundary conditions. Elastic properties (i.e. Young's modulus or elastic stresses) do not affect the solution of the limit analysis problem P_{LA}. Obviously, anisotropy is taken into account only through appropriate yield criteria.

3 ANISOTROPIC YIELD CRITERIA

Yield criteria are central to the limit analysis problem. In this section the different criteria used will be shortly described for the plane stress case. The von Mises criterion [10] reads:

$$\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\,\sigma_{22} + 3\,\sigma_{12}^2 \leq \sigma_y^2 \tag{2}$$

σ_y is the uniaxial tensile yield stress. Hill proposed a number of criteria for the anisotropic behavior of metals. Hill-48 [11] criterion is the generalization of von Mises and one of the most popular in elastoplastic analyses of anisotropic structures. For plane stress case:

$$A_1\,{\sigma_{11}}^2\ - A_2\,\sigma_{11}\,\sigma_{22}\ +\ A_3\,{\sigma_{22}}^2\ + 3\,A_4\,{\sigma_{12}}^2\ = 1 \tag{3}$$

where $A_{1,\ldots,4}$ are the anisotropic parameters. They can be determined using experimental results like $r-$values r_0, r_{45} and r_{90} for the three tension axis directions from rolling direction of a sheet, as follows:

$$A_1 = 1, \quad A_2 = \frac{2r_0}{1+r_0}, \quad A_3 = \frac{r_0(1+r_{90})}{r_{90}(1+r_0)}, \quad A_4 = \frac{(r_0+r_{90})(1+2r_{45})}{3r_{90}(1+r_0)} \tag{4}$$

Hill criterion is always convex, quadratic and can be written as euclidean length constraint. Another interesting criterion is the "user-friendly" one, for high strength steels proposed by Yoshida et al [12]. This criterion is a sixth-order-polynomial with a high flexibility of describing anisotropic behavior of steel sheets using 16 coefficients. It is always convex and in good agreement with experimental results. For plane stress it reads:

$$\begin{aligned} &C_1\sigma_{xx}^6 - 3C_2\sigma_{xx}^5\sigma_{yy} + 6C_3\sigma_{xx}^4\sigma_{yy}^2 - 7C_4\sigma_{xx}^3\sigma_{yy}^3 + 6C_5\sigma_{xx}^2\sigma_{yy}^4 - 3C_6\sigma_{xx}\sigma_{yy}^5 \\ &+C_7\sigma_{yy}^6\ + 9\left(C_8\sigma_{xx}^4 - 2C_9\sigma_{xx}^3\sigma_{yy} + 3C_{10}\sigma_{xx}^2\sigma_{yy}^2 - 2C_{11}\sigma_{xx}\sigma_{yy}^3 + C_12\sigma_{yy}^4\right)\sigma_{xy}^2 \\ &+27\left(C_{13}\sigma_{xx}^2 - C_{14}\sigma_{xx}\sigma_{yy} + C_{15}\sigma_{yy}^2\right)\sigma_{xy}^4 + 27C_{16}\sigma_{xy}^6 = \sigma_y^6 \end{aligned} \tag{5}$$

Parameters $C_{1,\ldots,16}$ are given as functions of some anisotropic coefficients described in detail in the Appendix of reference [12]. If the anisotropy parameters for Hill or Yoshida criterion are set equal to ones, the criteria become equivalent to von Mises.

4 NUMERICAL EXAMPLE

The numerical example concerns the construction of the limit analysis locus for a square plate with a central hole with $d/L = 0.20$ (Fig. 1) subjected to loads P_1 and P_2. This example has been widely studied in the literature [1-4]. 880 CST elements were used to model the structure; due to symmetry only one quarter of the plate was examined. Material is high strength steel $HSS590$ with yield stress equal to $590MPa$. X is considered as the rolling direction.

Three yield criteria are used, namely von Mises, Hill and the one proposed by Yoshida in their original nonlinear form. Geometrical nonlinearities and damage were not considered in the present work. Open source optimization package IPOPT [14] was used to solve the arising nonlinear programming problems.

To construct the locus a series of limit analysis problems have to be solved. Let us consider the two basic load cases P_1 and P_2. Parameter $\theta \in [0^o, 90^o]$ is used to variate the load pattern. For $\theta = 0^o$ only P_1 load is applied, while for $\theta = 90^o$ only P_2 is applied.

$$\mathcal{V}_\theta = \{P : P = P_1\,cos(\theta) + P_2\,sin(\theta),\ \ P_1 = P_2 = 100\,MPa\} \tag{6}$$

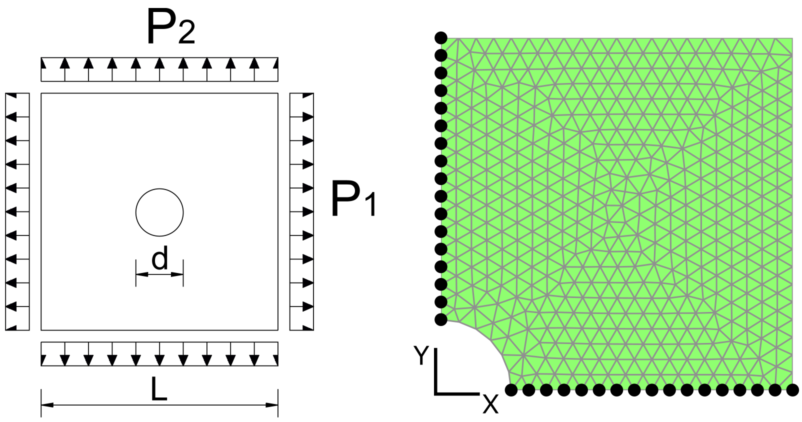

Figure 1: Square plate with central hole and FEM model.

For the von Mises yield criterion the only required parameter is the tensile yield stress. While for the Hill yield criterion anisotropy parameters $A_{1,\dots,4}$ have to be defined.

For our case:

$$r_0 = 0.43, \quad r_{45} = 1.41, \quad r_{90} = 0.61$$

$$A_1 = 1.0000, \quad A_2 = 0.6014, \quad A_3 = 0.7936, \quad A_4 = 1.5181$$

For the Yoshida yield criterion the 16 anisotropy parameters as described in [12] for $HSS590$, are presented in Table 1.

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1.0000	0.6014	0.4841	0.3982	0.4375	0.5752	0.7591	1.0350
C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
0.6789	0.6664	0.7540	0.9423	1.1601	1.0615	1.2470	1.7732

Table 1: Yoshida yield criterion anisotropy parameters.

Figure 2 depicts the three different loci constructed for the three yield criteria. Values on the figure are normalized using the uniaxial tensile yield stress. Significant differences of the safety factor (not always conservative) exist. Table 2 includes some of the limit analysis results for different values of parameter θ. Fourth and sixth column depict the difference compared to the results obtained using von Mises yield criterion.

Angle θ	von Mises	Hill	Diff. (%)	Yoshida	Diff. (%)
0^o	0.813	0.808	-0.54	0.810	-0.33
15^o	0.917	0.874	-4.67	0.881	-3.87
30^o	1.062	0.982	-7.57	1.005	-5.40
45^o	1.272	1.145	-9.99	1.222	-3.92
60^o	1.062	1.097	+3.26	1.094	+3.03
75^o	0.917	0.986	+7.57	0.942	+2.81
90^o	0.813	0.906	+11.61	0.849	+4.53

Table 2: Limit analysis results for different θ.

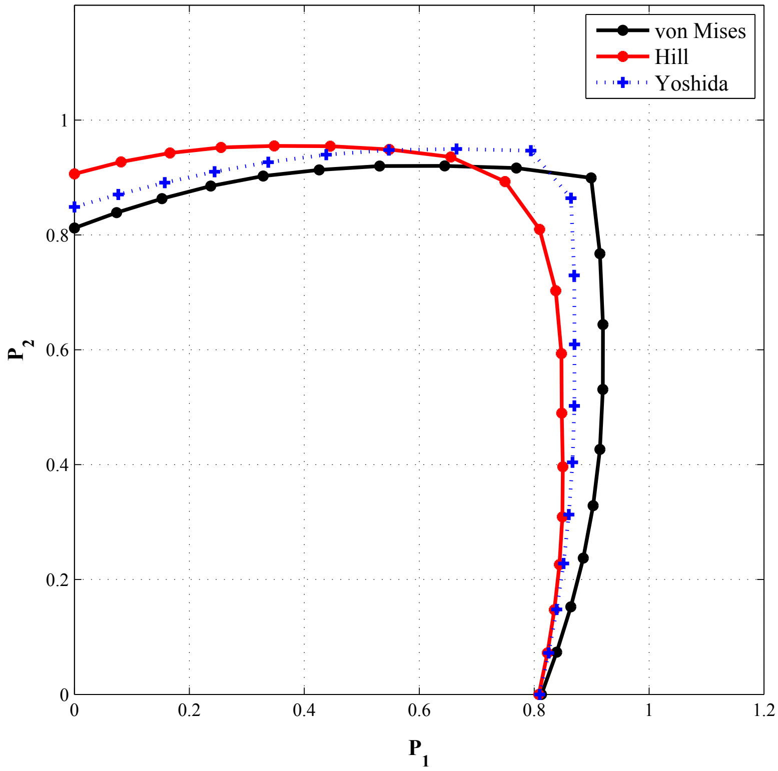

Figure 2: Limit analysis locus.

5 CONCLUSIONS

In the present work limit analysis is applied to high strength steel structures, exhibiting strong plastic anisotropy. Limit analysis requires only the equations of equilibrium and a respective yield criterion to describe the safety margins of a structure, since the material elastic properties are irrelevant to the collapse load analysis. So, in limit analysis anisotropy is described using only an adequate non-linear yield criterion like for example the popular Hill criterion, a generalization of von Mises, with parameters that are easily determined. Moreover, a polynomial yield criterion, with high flexibility of describing the anisotropic behaviour proposed by Yoshida et al was also used to illustrate the differences in the elastoplastic behaviour of high strength steels. Significant variations of the safety factor were observed for the numerical example studied, depending on the yield criterion used.

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