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DAMAGE DETECTION OF A BRIDGE BY PARAMETRIC STATISTICAL MOMENT METHOD

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Abstract. The statistical moment-based damage detection method with approximate parametric solutions of the stationary second-order moments of the response is proposed for damage detection of a bridge. The method is based on an approximate explicit solution recently proposed by the authors, which allows to explicitly relate second-order moments of the nodal displacements and velocity to the structural stiffness. Application on a bridge is reported to check the consistency of the method and to investigate the influence of the entity of the damage and of the application point of the force on the identification procedure.

1 INTRODUCTION

Moment based approaches proved to be quite effective in the identification of structural damages and positions. Statistical measures of the response under stochastic excitation have succeeded numerically and experimentally both in the non-linear and linear settings.

Nonlinearity arising from a breathing crack affects higher order statistics which can be exploited to detect and locate stiffness discontinuities with high accuracy [1]-[5].

Statistical moments are suitable damage indices also if linear behavior is preserved when the damage arises. In fact they are sensitive to local structural damage but insensitive to measurement noise so that the statistical moment-based damage detection (SMBDD) method can be conveniently resorted to [6], [7]. The SMBDD method can be more effectively employed if approximate parametric solutions for nodal stationary statistical moment are available. The benefit of having explicit solutions is conspicuous when the least squares method is applied because the residual between the simulated response and the actual statistical moments is parameterized with respect to the quantities to be identified. This offers great convenience in applying Newton's method to search for the parameters of stiffness and damping inversely when the objective function is minimized. The parametric relationship between stationary second order moments proposed in [8] are resorted to.

The objective function is defined herein as the weighted sum of squared differences between the measured data from a simulated experiment and the corresponding analytical parametric values of nodal stationary second order displacements and velocity under Gaussian input. The weights can be chosen accordingly to the sensitivity of the related measure with respect to structural parameters. The procedure has already been applied to semi-rigid connections identification [9], and its robustness against measurement noise have been assessed [10].

In this paper the method is applied to damage identification in a frame grid modeling a span bridge. The simulated experiments allow different damage entity and position, whereas a low number of measurement point is foreseen. A single frame element is given a reduced stiffness modulus so to reproduce damage. The numerical investigation always gives back the correct identification for damage entity and position when the structure is loaded by a white noise load concentrated on a node.

2 SECOND ORDER MOMENT DYNAMIC EQUATION

Let us consider an n-DOF structural system whose dynamic behaviour is predicted by an FE linear model, characterized by the mass, damping and stiffness $(n \times n)$ matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, respectively. Moreover, let us assume that the structure is subjected to a zero mean stationary Gaussian white noise excitation $W(t)$ with intensity $q = 2\pi S_0$, S_0 being the constant power spectral density. The motion differential equations of the system are:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \boldsymbol{\tau} W(t) \tag{1}$$

where $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ are the n-vectors of nodal displacements, velocities and accelerations, respectively, and $\boldsymbol{\tau}$ is the influence forcing n-vector.

Let us introduce the modal coordinate transformation

$$\mathbf{u}(t) = \boldsymbol{\Phi}\mathbf{q}(t) \tag{2}$$

where $\mathbf{q}(t)$ is the m-vector of the modal coordinates, with $m \leq n$, and $\mathbf{\Phi}$ is the reduced modal matrix, of order $n \times m$, solution of the following eigenproblem:

$$\mathbf{K\Phi} = \mathbf{\Omega M \Phi} \tag{3}$$

normalized with respect to the mass matrix $\mathbf{M}$, so that $\mathbf{\Phi}^T \mathbf{M \Phi} = \mathbf{I}_m$, whereas $\mathbf{\Phi}^T \mathbf{K \Phi} = \mathbf{\Omega} = diag[\omega_j^2]$ is the diagonal matrix listing the squares of the first m natural frequencies of the predicted structure. Substituting Eq.(2) into Eq.(3), the latter becomes

$$\ddot{\mathbf{q}}(t) + \mathbf{\Xi}\dot{\mathbf{q}}(t) + \mathbf{\Omega q}(t) = \mathbf{p}W(t) \tag{4}$$

where $\mathbf{p} = \mathbf{\Phi}^T \boldsymbol{\tau}$ and $\mathbf{\Xi} = \mathbf{\Phi}^T \mathbf{C \Phi}$. Eq.(4) rules the dynamic behaviour of the predicted FE model in the modal space.

By using the 2m-dimensional vector state approach, Eq.(4) can be written in the following matrix form:

$$\dot{\mathbf{Z}}(t) + \mathbf{D}\,\mathbf{Z}(t) = \mathbf{v}W(t) \tag{5}$$

where $\mathbf{Z}(t) = [\mathbf{q}^T(t) \quad \dot{\mathbf{q}}^T(t)]^T$ is the 2m-vector of the modal state variables and

$$\mathbf{D} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_m \\ \mathbf{\Omega} & \mathbf{\Xi} \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} \mathbf{0} \\ \mathbf{p} \end{bmatrix} \tag{6}$$

being $\mathbf{I}_m$ the identity matrix of order m.

If the physical properties of the system, such as mass, damping and stiffness, are known, the response process is a Gaussian stationary one and its probabilistic characterization is determined once the stationary second order moments of the nodal displacements and velocities are evaluated. The latter are related to the second order moments of the modal coordinates. In fact, by introducing the *2n*-vector of the nodal state variables $\mathbf{Y}(t) = [\mathbf{u}^T(t) \quad \dot{\mathbf{u}}^T(t)]^T$, the following relation holds

$$\mathbf{m}_{\mathbf{Y},2}(t) = \mathbf{\Gamma}^{[2]} \mathbf{m}_{\mathbf{Z},2}(t) \tag{7}$$

In Eq.(7), $\mathbf{m}_{\mathbf{Y},2}(t) = \mathbf{m}_2[\mathbf{Y}(t)] = E[\mathbf{Y}(t) \otimes \mathbf{Y}(t)] = E[\mathbf{Y}^{[2]}(t)]$ is the $4n^2$-vector which collects all the second order moments of the displacements and velocities of the nodal coordinates, and $\mathbf{m}_{\mathbf{Z},2}(t) = \mathbf{m}_2[\mathbf{Z}(t)] = E[\mathbf{Z}(t) \otimes \mathbf{Z}(t)] = E[\mathbf{Z}^{[2]}(t)]$ is the $4m^2$-vector which collects the second order moments of the displacement and velocities of the m selected modal coordinates, where the symbol $\otimes$ means Kronecker product and the exponent in square brackets means Kronecker power [11],[12]. Furthermore,

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi} \end{bmatrix} \tag{8}$$

is a transformation matrix of order $2n \times 2m$, so that $\mathbf{Y}(t) = \mathbf{\Gamma Z}(t)$, and $\mathbf{\Gamma}^{[2]} = \mathbf{\Gamma} \otimes \mathbf{\Gamma}$.

The differential equations describing the time evolution of the vector $\mathbf{m}_{\mathbf{Z},2}(t)$ can be written as follows [13]

$$\dot{\mathbf{m}}_{\mathbf{Z},2}(t) + \mathbf{D}_2 \mathbf{m}_{\mathbf{Z},2}(t) = \mathbf{v}_2 q \tag{9}$$

where

$$\mathbf{D}_2 = \mathbf{D} \otimes \mathbf{I}_{2m} + \mathbf{I}_{2m} \otimes \mathbf{D}, \qquad \mathbf{v}_2 = \mathbf{v} \otimes \mathbf{v} = \mathbf{v}^{[2]} \tag{10}$$

Note that all the cross moments appear twice in the vector $\mathbf{m}_{\mathbf{Z},2}(t)$. Then a condensation can be employed in order to reduce the size of the problem by clearing away the repetitions. So operating the problem dimension reduces from $4m^2$ to $N = 2m^2 + m$.

The stationary response is given as solution of the algebraic equations obtained removing the time dependence from Eq.(9):

$$\mathbf{D}_2 \mathbf{m}_{\mathbf{Z},2} = \mathbf{v}_2 q \tag{11}$$

and the vector $\mathbf{m}_{\mathbf{Y},2}$ of the stationary second order moments of the nodal displacements and velocities can be evaluated by Eq.(7). From the previous equation it appears that the evaluation of the stationary second order moments of the modal coordinates and its velocities requires the inversion of the matrix $\mathbf{D}_2$.

3 STIFFNESS PARAMETERS VARIABILITY

Let us assume, in agreement with realistic models, the mass parameters exhibit negligible fluctuations and can be considered known. On the contrary, the stiffness of the elements commonly suffer from significant uncertainty, and their values may deviate from predicted ones. For example, this is the case of variation introduced by damage. The damage identification could be accomplished exploiting by an inverse parametric analysis aimed to detect its location and entity starting from the knowledge of statistical response measures at some points of the structure excited by a white noise, according to the SMBDD method [6].

In the following, the SMBDD method is improved according to an explicit parameterization of the stationary second order moments of the structural response, which are expressed as analytical functions of element stiffnesses. The Global stiffness matrix can deviate from the predicted value $\mathbf{K}$ and it can be written in the following form:

$$\tilde{\mathbf{K}}(\boldsymbol{\alpha}) = \mathbf{K} + \sum_{i=1}^{R_K} \alpha_i \mathbf{K}^{(i)} \tag{12}$$

where $\tilde{\mathbf{K}}(\boldsymbol{\alpha})$ is the updated global stiffness matrix, while $\mathbf{K}$ and $\mathbf{K}^{(i)}$ are known matrices determined by assembling the frame stiffness matrix and $\boldsymbol{\alpha} = [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_{R_K}]^T$ is the vector collecting the stiffness parameter fluctuations. In Eq.(12), $\mathbf{K}$ represents the predicted value of the stiffness matrix and the summation denotes its fluctuating components. Accordingly, the updated dynamic matrix $\tilde{\mathbf{D}}(\boldsymbol{\alpha})$ can be written as:

$$\tilde{\mathbf{D}}(\boldsymbol{\alpha}) = \mathbf{D} + \sum_{i=1}^{R_K} \alpha_i \mathbf{D}^{(i)} \tag{13}$$

where $\mathbf{D}$ is the predicted dynamic matrix and

$$\mathbf{D}^{(i)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Omega}^{(i)} & \mathbf{0} \end{bmatrix} \tag{14}$$

with

$$\boldsymbol{\Omega}^{(i)} = \boldsymbol{\Phi}^T \mathbf{K}^{(i)} \boldsymbol{\Phi} \tag{15}$$

$\boldsymbol{\Phi}$ being the modal matrix relative to the predicted structure.

3.1 Stationary Stochastic Response

The stationary second order moments of the modal response are solution of the following set of algebraic equations :

$$\tilde{\mathbf{D}}_2(\alpha)\tilde{\mathbf{m}}_{\mathbf{Z},2}(\alpha)=\mathbf{v}_2 q \tag{16}$$

where

$$\tilde{\mathbf{D}}_2(\alpha)=\mathbf{D}_2+\sum_{i=1}^{R}\alpha_i\mathbf{D}_2^{(i)} \tag{17}$$

being

$$\mathbf{D}_2^{(i)}=\mathbf{D}^{(i)}\otimes\mathbf{I}_{2m}+\mathbf{I}_{2m}\otimes\mathbf{D}^{(i)} \tag{18}$$

Eq.(16) is the parametric counterpart of Eq.(11), with unknowns given by the vector of the second order moments of the nodal response $\tilde{\mathbf{m}}_{\mathbf{Z},2}(\boldsymbol{\alpha})$.

Solving the system governed by Eq.(16) means to obtain the explicit relationships between the vector $\tilde{\mathbf{m}}_{\mathbf{Z},2}(\boldsymbol{\alpha})$ and the parameter vector $\boldsymbol{\alpha}$. This cannot be generally done by inverting $\tilde{\mathbf{D}}_2(\boldsymbol{\alpha})$, which is a parametric matrix of large dimension, so that different procedures solving this problem in an approximate way have been proposed in the literature. The approximate analytical solution presented by [8], briefly recovered in the next section, is adopted herein.

4 THE EXPLICIT ANALYTICAL SOLUTION

Let us consider the case in which only one fluctuating parameter α_i is present,so that Eq.(16) is rewritten as:

$$\tilde{\mathbf{D}}_2(\alpha_i)\tilde{\mathbf{m}}_{\mathbf{Z},2}(\alpha_i)=\mathbf{v}_2 q \tag{19}$$

The vector of the second order moments of the modal response can be written as follows:

$$\tilde{\mathbf{m}}_{\mathbf{Z},2}(\alpha_i)=\mathbf{m}_{\mathbf{Z},2}+\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i) \tag{20}$$

where $\mathbf{m}_{\mathbf{Z},2}$ is the vector of the stationary second order moments of the predicted structure (for $\alpha_i=0$), solution of Eq.(11). Moreover, the matrix $\tilde{\mathbf{D}}_2(\alpha_i)$ takes on the following simpler form:

$$\tilde{\mathbf{D}}_2(\alpha_i)=\mathbf{D}_2+\alpha_i\mathbf{D}_2^{(i)} \tag{21}$$

By substituting Eq.(20) and Eq.(21) into Eq.(19), one obtains:

$$(\mathbf{D}_2+a_i\mathbf{D}_2^{(i)})[\mathbf{m}_{\mathbf{Z},2}+\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)]=\mathbf{v}_2 q \tag{22}$$

Taking into account Eq.(11), the following equation for the unknown vector $\mathbf{m}_{\mathbf{Z},2}^{(i)}(\theta_i)$ is obtained:

$$[\mathbf{D}_2+\alpha_i\mathbf{D}_2^{(i)}]\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)=-\alpha_i\mathbf{D}_2^{(i)}\mathbf{m}_{\mathbf{Z},2} \tag{23}$$

The solution of Eq.(23) appears not straightforward at a first glance since the inverse of the parametric matrix $(\mathbf{D}_2+\alpha_i\mathbf{D}_2^{(i)})^{-1}$ is required. However, the expression of the vector $\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)$ as an explicit function of the parameter α_i can be readily obtained as shown in the next.

Let us consider the right and left eigenproblems related to the $(N\times N)$ matrix $\mathbf{A}_2^{(i)}=\mathbf{D}_2^{-1}\mathbf{D}_2^{(i)}$ (assuming that the condensation has been employed, i.e. $N=2m^2+m$):

$$\mathbf{A}_2^{(i)}\boldsymbol{\Psi}_R^{(i)}=\boldsymbol{\Psi}_R^{(i)}\boldsymbol{\Lambda}^{(i)} \qquad\qquad \boldsymbol{\Psi}_L^{(i)}\mathbf{A}_2^{(i)}=\boldsymbol{\Lambda}^{(i)}\boldsymbol{\Psi}_L^{(i)} \tag{24}$$

Since $\mathbf{A}_2^{(i)}$ is a non-defective matrix, the following ortonormalization condition can be applied:

$$\boldsymbol{\Psi}_L^{(i)}\boldsymbol{\Psi}_R^{(i)}=\mathbf{I}_N, \qquad\qquad \boldsymbol{\Psi}_L^{(i)}\mathbf{A}_2^{(i)}\boldsymbol{\Psi}_R^{(i)}=\boldsymbol{\Lambda}^{(i)} \tag{25}$$

The $p\times N$ matrix $\boldsymbol{\Psi}_L^{(i)}$ and the $N\times p$ matrix $\boldsymbol{\Psi}_R^{(i)}$ collect, respectively, the p significant left and right eigenvectors, i.e. the eigenvectors related to the nonzero eigenvalues $\lambda_j^{(i)}$ (with $j=1,2,\dots,p$) listed in the diagonal $p\times p$ matrix $\boldsymbol{\Lambda}^{(i)}$. By introducing the following coordinate transformation:

$$\mathbf{m}_{\mathbf{Z},2}^{(i)}(\theta_i)=\boldsymbol{\Psi}_R^{(i)}\mathbf{n}_{\mathbf{Z},2}^{(i)}(\theta_i) \tag{26}$$

into Eq.(23), pre-multiplying both sides by $\boldsymbol{\Psi}_L^{(i)}$ and taking into account Eqs.(25), Eq.(23) becomes:

$$(\mathbf{I}+\alpha_i\boldsymbol{\Lambda}^{(i)})\mathbf{n}_{\mathbf{Z},2}^{(i)}(\alpha_i)=-\alpha\theta_i\boldsymbol{\Psi}_L^{(i)}\mathbf{A}_2^{(i)}\mathbf{m}_{\mathbf{Z},2} \tag{27}$$

From the previous equation, it is easy to obtain the exact explicit relationship between the vector $\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)$ and the generic parameter α_i:

$$\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)=-\alpha_i\boldsymbol{\Psi}_R^{(i)}\left(\mathbf{I}+\alpha_i\boldsymbol{\Lambda}^{(i)}\right)^{-1}\boldsymbol{\Psi}_L^{(i)}\mathbf{A}_2^{(i)}\mathbf{m}_{\mathbf{Z},2} \tag{28}$$

If more parameters are present, as a first approximation the cross effects can be neglected and the superposition principle can be applied. Therefore, the solution in the modal coordinates is given by the sum of the single contributions, that is:

$$\tilde{\mathbf{m}}_{\mathbf{Z},2}(\boldsymbol{\alpha})=\mathbf{m}_{\mathbf{Z},2}+\sum_{i=1}^{R}\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i) \tag{29}$$

where R is the number of the fluctuating parameters and the generic $\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i)$ is given by Eq.(28). Taking into account Eq.(7), the solution in terms of nodal displacement and velocities is

$$\tilde{\mathbf{m}}_{\mathbf{Y},2}(\boldsymbol{\alpha})=\mathbf{m}_{\mathbf{Y},2}+\sum_{i=1}^{R}\boldsymbol{\Gamma}^{[2]}\mathbf{m}_{\mathbf{Z},2}^{(i)}(\alpha_i) \tag{30}$$

Eq.(30), which gives an approximate solution if $R\geq 2$ because the joints effects of the parameter fluctuations are neglected, is sufficiently accurate for solving the inverse problem arising in structural identification. Eventually, more accurate solutions can be obtained by introducing cross terms [8].

5 INVERSE MEASUREMENT PROBLEM

The approximate explicit solution produced by Eq.(30) can be exploited for element stiffness identification starting from the knowledge of experimental measurements of the stationary second order moments of the response at some points of the structure. Assume that some measurements $\hat{m}_{y_k,2} = E[\hat{y}_k^2]$ are available, where $\hat{y}_k$ is the measured displacement or velocity of the k-th degree of freedom of the structure (the cap over the variable means "experimental"). The corresponding parametric expressions $\tilde{m}_{y_k,2}(\alpha)$ is evaluated as components of the vector given by Eq.(30). The inverse problem consists in retrieving the parameters α from given displacement or velocity measurements. Then, the identification of the R parameters may be obtained using the least squares estimation approach, where the objective function

$$f(\alpha) = \sum_{k=1}^{N_R} \left[W_k \frac{\tilde{m}_{y_k,2}(\alpha) - \hat{m}_{y_k,2}}{\hat{m}_{y_k,2}} \right]^2 \tag{31}$$

is to be minimized and N_R is the number of measurements.

6 APPLICATION: IDENTIFICATION OF DAMAGE IN A BRIDGE

The proposed damage identification procedure has been applied to a span of a bridge, discretized by beam finite elements, simply supported on two piers at its end (see Fig.1 and 2). The simulated experiment has been performed in relation to the FE model depicted in Fig. 3, where a 18 element discretization has been used. The data of the longitudinal beams are: second moment of area $I = 0.9803\ m^4$, polar (torsional) second moment of area $J = 0.1765\ m^4$, total span $L = 34.25\ m$, mass density per unit length $\rho_{bl} = 2.55 \times 10^3\ kg/m$. The data of the transversal beams are: second moment of area $I = 0.4667\ m^4$, polar (torsional) second moment of area $J = 0.0211\ m^4$, total span $L = 8.50\ m$, mass density per unit length $\rho_{bt} = 3.50 \times 10^3\ kg/m$, mass density per unit area of the deck $\rho_d = 3.12 \times 10^3\ kg/m^2$.

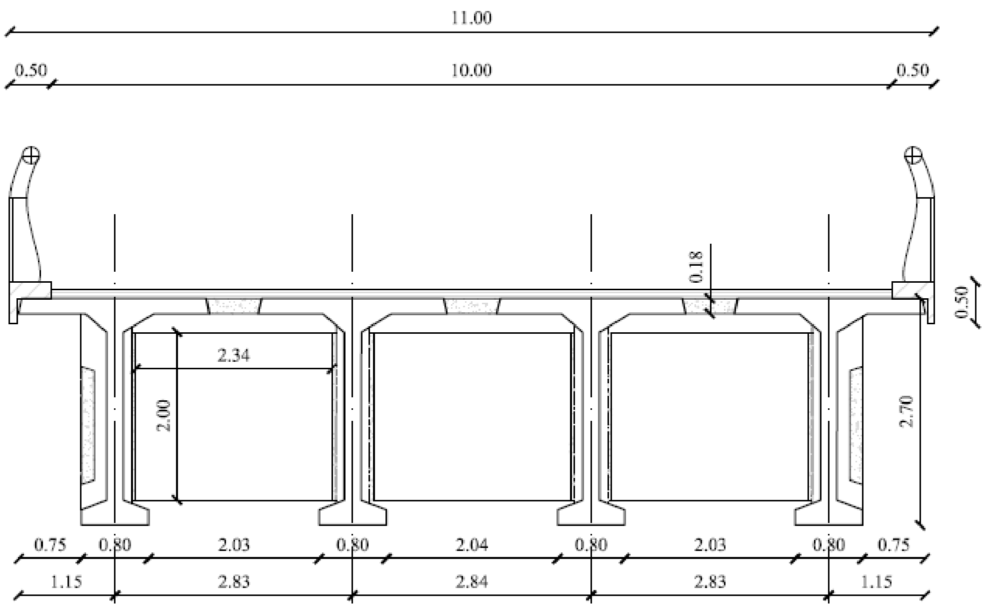

Figure 1: Cross Section of the bridge

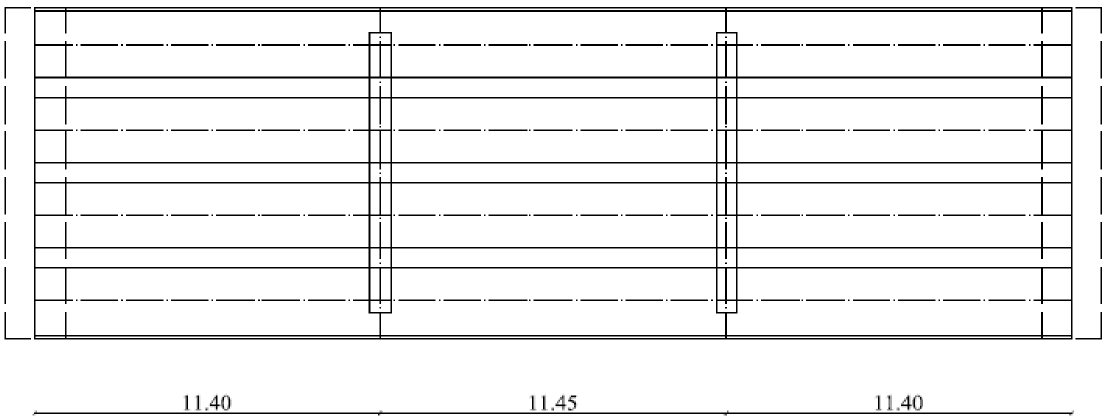

Figure 2: Schematic sketch of the bridge

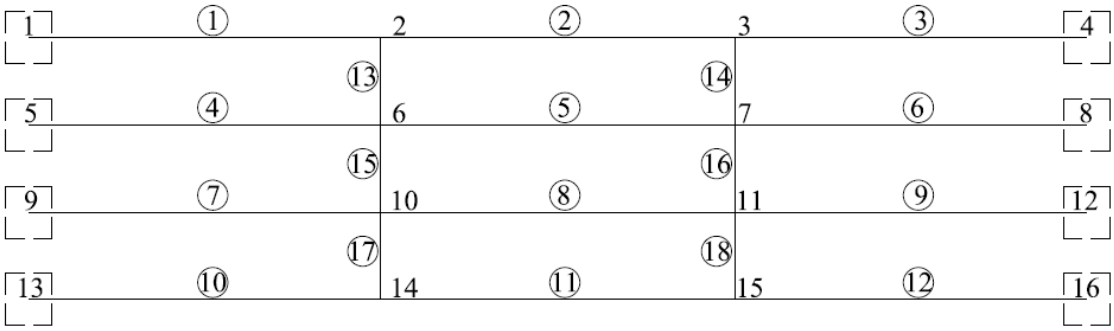

Figure 3: FE model of the bridge

Let us assume that the Young modulus of all the elements has the value $E = 3.46 \times 10^7\ kN/m^2$, except one with value $\hat{E}_i = E(1+\hat{\alpha}_i)$, where $\hat{\alpha}_i$ is a negative coefficient (with $-1 < \hat{\alpha}_i < 0$), representing the damaged zone. For simplicity, a damage due to a reduction of the Young modulus of a beam can be similarly taken into account. The bridge has been subjected to a vertical point force applied at a given node of the model, represented by a white noise. Of course more general loading could be treated. Suitable damping properties of the structure have been assumed in the cases above considered and a lumped mass matrix has been implemented.

The identification strategy relies upon the least squares estimation approach which is carried on for all possible damage scenarios by assuming a single value for the Young modulus at all the elements but the one where the damage is tentatively located. The real damage position is singled out by selecting the damage scenario with smaller residue (see Eq. (31)).

The numerical examples assume a limited number of measurements, namely $N_R = 4$, as given by second order stationary displacement at nodes 2, 3, 6 and 7.

The investigation starts from considering that the damage is located alternatively at the 10-th, 11-th and 17-th element of the simulated experimental model, with a value $\hat{\alpha}$ varying from -0.3 to -0.05.

Let us assume that the damage is present on the *j*-th element (with $j = 10,\ 11,\ 17$), with unknown value $\alpha_D^{(j)}$, and the remaining elements are supposed undamaged with value $\alpha_{ND}^{(j)}$

(with $j \neq 10, 11, 17$). Identification of the damage entity and position is determined in the element in which one has the smallest value of the residue in the minimization problem Eq.(31).

$\hat{\alpha}$ \ FE	10-th	Error %	11-th	Error %	17-th	Error %
–0.3	-0.3450	15.00	-0.3953	31.77	-0.3114	3.80
–0.2	-0.2194	9.70	-0.2316	15.83	-0.2042	2.13
–0.1	-0.1046	4.64	-0.1072	7.26	-0.1009	0.93
–0.05	-0.0511	2.27	-0.0517	3.51	-0.0502	0.44

Table 1: Stiffness reduction identified by the parametric model for damage located at the 10-th, 11-th and 17-th element of the beam, with $\hat{\alpha} = -0.3, -0.2, -0.1$ and -0.05, and point force at node 3.

$\hat{\alpha}$ \ FE	10-th	Error %	11-th	Error %	17-th	Error %
–0.3	-0.3462	15.40	-0.3373	12.45	-0.2949	1.68
–0.2	-0.2195	9.77	-0.2161	8.09	-0.1986	0.69
–0.1	-0.1046	4.66	-0.1038	3.87	-0.0998	0.19
–0.05	-0.0511	2.28	-0.0509	1.89	-0.0499	0.06

Table 2: Stiffness reduction identified by the parametric model for damage located at the 10-th, 11-th and 17-th element of the beam, with $\hat{\alpha} = -0.3, -0.2, -0.1$ and -0.05, and point force at node 10.

Element	Residue	α
1	6.0707E-07	0.00023
2	2.9979E-08	-0.00731
3	3.3123E-07	-0.00355
4	5.3909E-07	0.00167
5	5.1163E-07	-0.00302
6	1.5169E-07	-0.00555
7	4.7854E-07	0.01177
8	1.2607E-07	-0.02684
9	3.4168E-08	0.01527
10	1.6937E-07	0.01194
11	3.2713E-07	0.01237
12	6.0940E-07	-0.00023
13	4.6062E-08	-0.05273
14	1.9527E-07	0.01106
15	7.6440E-08	-0.07097
16	3.8792E-07	0.00221
17	8.2491E-10	-0.20426
18	5.0598E-07	0.00117

Table 3: Stiffness reduction identified by the parametric model for damage located at the 17-th element of the beam, with $\hat{\alpha} = -0.2$, and point force at node 3.

Element	Residue	α
1	3.4831E-06	0.00969
2	3.4501E-07	0.00948
3	7.1014E-06	0.00377
4	7.8148E-06	-0.00041
5	7.3201E-06	0.01713
6	7.8605E-06	-0.00310
7	7.7307E-06	-0.00903
8	3.4112E-07	-0.03551
9	1.4893E-06	-0.03172
10	2.3522E-06	-0.03710
11	3.6456E-07	-0.02275
12	2.1483E-06	-0.04871
13	5.2607E-06	0.08237
14	5.9452E-06	0.05694
15	5.2748E-06	-0.18271
16	6.9742E-06	-0.05382
17	2.0889E-10	-0.19862
18	7.0393E-06	-0.08502

Table 4: Stiffness reduction identified by the parametric model for damage located at the 17-th element of the beam, with $\hat{\alpha} = -0.2$, and point force at node 10.

Table 1 and 2 show the results of the identification procedure here proposed, for different location and entity of damage $\hat{\alpha}$ for point force applied at node 3 and at node 10, revealing as the accuracy of the method increase when the value of damages decrease. The method is capable to catch also small values of the damage, common in real cases. Table 3 and 4 show the results of the identification procedure here proposed at element 17-th for a point force applied at node 3 and at node 10, and entity of damage $\hat{\alpha} = -0.2$.

7 CONCLUSIONS

The statistical moment-based damage detection method has been applied to identify the stiffness of a grid structure modelling the span of a bridge. The numerical applications reveal that the method is effective when damage localization should be achieved with satisfactory accuracy. The simulated experiments are related to different scenarios where a single frame is given a reduced stiffness so to reproduce a fictitious damage. In all the cases considered, the proposed method, taking advantage of accurate approximate relationship between nodal second order statistical moments and element stiffnesses, is able to spot damage position and its entity.

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