ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

ON THE FREQUENCY CONTENT OF ERRORS ORIGINATED IN A TIME INTEGRATION COMPUTATIONAL COST REDUCTION TECHNIQUE

A. Soroushian[1], M. Hosseini[1], and S.M.H. Khalkhali[2]

[1] International Institute of Earthquake Engineering and Seismology (IIEES)
S. Lavasani (Farmaniyeh), North Dibaji, West Arghavan, No. 21, Tehran 19537, Iran
e-mail: {a.soroushian,Hosseini}@iiees.ac.ir, aramsoro@yahoo.com

[2] School of Engineering, Science and Research Branch of Islamic Azad University
Hessarak, S. Satari, Tehran 14778, Iran
e-mail: khalkhali_eng@yahoo.com

Keywords: Time Integration, Digitized Excitation, Computational Cost, Accuracy, Frequency Content.

Abstract. *The true behavior of structural systems, mainly essential in seismic analyses, is nonlinear and dynamic. In analysis of nonlinear dynamic behaviors of structural systems, a versatile tool is direct time integration. In spite of the versatility, the responses obtained from time integration are approximations and the computational costs are considerable. For analyses subjected to excitation available as digitized records, a technique to reduce the computational cost with negligible sacrifice of accuracy is proposed in 2008. The technique is successfully implemented in seismic analysis of frames, buildings structures, bridges, reservoirs, power stations, silos, etc., and undergone theoretical studies from different points of view. This is a complementary study that, with the aim of reducing the small errors because of the technique and reducing the computational costs even more, numerically investigates the frequency content of the errors originated in the computational cost reduction technique. As the conclusion, more errors are expectable in lower modes and/or modes with more contribution in the response. Further extensive study is essential.*

1 INTRODUCTION

Time integration is the most versatile approach to analyze the transient behaviors of semi-discretized structural systems. Nevertheless, the computational efforts are considerable, and the responses are approximations. In a review on the time integration process, after discretization of the mathematical models in space, by methods such as finite elements, we arrive at the semi-discretized models, typically stated below:

$$\begin{gathered}
\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{f}_{\text{int}}(t) = \mathbf{f}(t), \qquad\qquad 0 < t \le t_{end} \\
\text{Initial conditions:} \quad \left| \begin{array}{l} \mathbf{u}(t=0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_0 \\ \mathbf{f}_{\text{int}}(t=0) = \mathbf{f}_{\text{int}_0} \end{array} \right. \\
\text{Additional constraints: } \mathbf{Q}
\end{gathered} \tag{1}$$

where, t and t_{end} imply the time and the duration of the dynamic behavior, $\mathbf{M}$ is the mass matrix, $\mathbf{f}_{\text{int}}(t)$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation, $\mathbf{u}(\text{t})$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$ denote the vectors of displacement, velocity, and acceleration; $\mathbf{u}_0, \dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$, define the initial status of the model (regarding the essentiality of considering $\mathbf{f}_{\text{int}_0}$ in Eqs. (1), see [1, 2]), and $\mathbf{Q}$ implies nonlinearity restrictions, e.g. see [3, 4]. Time integration determines the status at discrete time instants $\Delta t, 2\Delta t, 3\Delta t$ …. (Δt stands for the integration step size), according to approximate formulations, introducing the integration method [5-8]. Because of the approximation, the step size Δt can not be large; however, because of the step-by-step nature, and the resulting high computational cost, the step size can also not be small. Considering this, and the unconditional stability of conventional time integration methods (see [6-9]), the integration step size recommended is basically obtained from [5, 7, 9-11]:

$$\Delta t = \begin{cases} \dfrac{T}{10} & \text{linear problem} \\ \dfrac{T}{100} & \text{nonlinear problem} \\ \dfrac{T}{1000} & \text{nonlinear problem involved} \end{cases} \tag{2}$$

where, T is the smallest period with considerable contribution in the response. In some practical cases, e.g. time integration analysis against earthquake records, where,

$$\mathbf{f}(t) = -\mathbf{M}\,\Gamma\,\ddot{\mathbf{u}}_g(t) \tag{3}$$

($\mathbf{M}$ represents the mass matrix, $\ddot{\mathbf{u}}_g(t)$ denotes the vector of ground motion at some different degrees of freedom (unless in multi-support excitations, $\ddot{\mathbf{u}}_g$ reduces to a function of time) and Γ is a matrix implying the effect of ground motion at different degree of freedom [12]), $\mathbf{f}(t)$ is available as a vector of digitized records, and Eq. (2) is being replaced with

$$\Delta t = \min({}_f\Delta t, \frac{T}{\chi}) \tag{4}$$

where, ${}_f\Delta t$ implies the step size, by which, $\mathbf{f}(t)$ is digitized, and in view of Eq. (2);

$$\chi = \begin{cases} 10 & \text{linear problem} \\ 100 & \text{nonlinear problem} \\ 1000 & \text{nonlinear problem involved in impact} \end{cases} \tag{5}$$

When, ${}_f\Delta t$ governs Eq. (4), some additional computational cost, e.g. about

$$\frac{T - \chi\, {}_f\Delta t}{\chi\, {}_f\Delta t} > 0 \tag{6}$$

for linear problems, is being dictated to the analyses, merely, in order to take into account the total information of $\mathbf{f}(t)$; see also [13].

In 2008, a technique is proposed for reducing the above-mentioned additional computational cost, while taking into account the total earthquake record, and not sacrificing considerable accuracy (compared to analyses with steps obtained from Eq. (4)) [13-16]. The past experiences were successful; see Table 1. Nevertheless, with the rapid scientific improvements, the value of χ, as well as the value of ${}_f\Delta t$ in Eqs. (4) and (5) is likely to be decreased;

System(s) analyzed	The cost reduction (%)	Source
SDOF system	75	[14]
2-DOF nonlinear system	49.27	[14]
Eight storey shear frame	80	[17]
Thirty-storey building	50	[18]
3-component earthquakes	66.7	[19]
Silo	77.65	[20, 21]
Water tank	66.7	[16, 22]
Building in pounding	12.7	[23]
Bridge with linear and nonlinear behaviors	45-80	[24-26]
Power stations	> 50	[27]
Regular residential buildings	50-87	[28, 29]
Bridges with pre-stressed elements, subjected to multi-support excitation and nonlinearities	30-70	[30]
Residential building with irregularities in height	50-80	[31]
Space Structures	>50	[32]
A telecommunication tower	>50	[33]
A cooling tower	>50	[34]

Table 1. Experiences on the computational cost reduction technique proposed in [14].

see also [35]. Accordingly, in view of Eq. (6), more enlargement of digitization step size would be desirable, and study towards improvement of the technique proposed in 2008 [14], especially for decreasing the even small inaccuracy induced by the technique, can cause considerable practical achievements. Considering this and in view of a recent research reported in [16], this paper presents an attempt to point out whether there exists any special distribution of the errors in different oscillatory modes. In the next section a brief theoretical discussion is presented. Numerical evidences, regarding the discussion in Section 2, are presented in Section 3, and the paper is concluded in Section 4.

2 BRIEF THEORETICAL DISCUSSION

In a review on the formulation of the technique proposed in 2008 [14] (for more detailed formulation, see [13, 15]), the replacement of the excitation $\mathbf{f}(t)$ with the excitation $\tilde{\mathbf{f}}(t)$, digitized at larger steps, is based on convergence and specifically on convergence of second order. In more detail, in view of the Eqs. (35), (38), and (39), in [14], also repeated below:

$$
\begin{aligned}
t_i = 0: &\quad \tilde{\mathbf{f}}_i = \mathbf{f}(t_i) \\
0 < t_i \le t_{end}: &\quad \tilde{\mathbf{f}}_i = \frac{1}{2}\mathbf{f}(t_i) + \frac{1}{4n'}\sum_{k=1}^{n'}\left[\mathbf{f}(t_{i+k/l}) + \mathbf{f}(t_{i-k/l})\right] \\
t_i = t_{end}: &\quad \tilde{\mathbf{f}}_i = \mathbf{f}(t_i)
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
t = \Delta t: &\quad n' = n-1 \\
\Delta t < t \le t_{end} - \Delta t: &\quad n' = \begin{cases} \dfrac{n}{2} & n = 2j,\ j \in \mathrm{Z}^+ \\ \dfrac{n-1}{2} & n = 2j+1,\ j \in \mathrm{Z}^+ \end{cases} \\
t = t_{end} - \Delta t: &\quad n' = n-1
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\Delta t = n_f \Delta t \le \min\left(h, \frac{T}{10}\right) \qquad \text{for linear system} \\
&\Delta t = n_f \Delta t \le \min\left(h, \frac{T}{100}\right) \qquad \text{for nonlinear system} \\
&\Delta t \le t_{end}
\end{aligned}
\tag{9}
$$

and the derivation of this formulation, there is no consideration about the dynamic nature of the phenomenon. Accordingly, it sounds reasonable to simply expect more contribution in the additional errors because of the technique, from the oscillatory modes contributing more in the dynamic behaviors, e.g. the first modes for many behaviors. This is also in consistence with Eq. (4), and also the notions of amplitude decay and period elongation [5-9], according to which, the higher modes, with lower values of T, are being time integrated more inaccurately and hence the additional errors because of the technique proposed in 2008 [14] are comparatively not considerable at the higher modes. Since, this claim is in agreement with a recent observations reported in [16], as a step towards profound study and clearing the distribution of the additional errors caused by the technique in different natural modes, some numerical examples are presented, in Section 3.

3 NUMERICAL EXAMPLES

3.1 2-D Frame

The 2D frame introduced in Fig. 1(a) and Table 2 is subjected to the ground acceleration displayed in Fig. 1(b) and studied considering linear behavior. The histories of top displacement and base shear reported in Fig. 2, together with Fig. 1(b), lead to

$$n = 2 \qquad (\equiv \quad \Delta t = 2_f \Delta t) \tag{10}$$

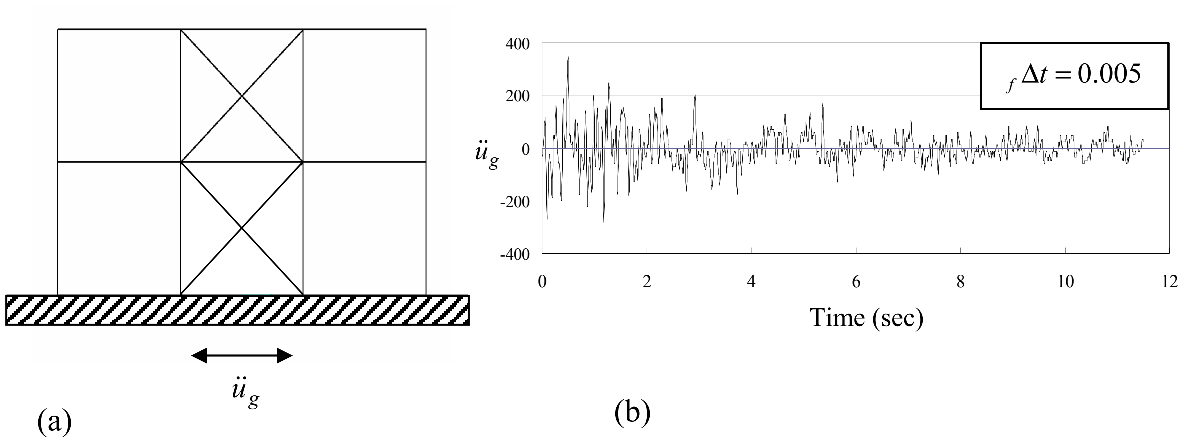


Figure 1: Structural system in the first example: (a) 2D frame (b) Earthquake record.

Members	**Profiles**
Columns	BOX 8x18x1.25
Beams	IPE 360
Braces	BOX 8x8x0.6

Table 2: Profiles of the structural members in Fig. 1(a).

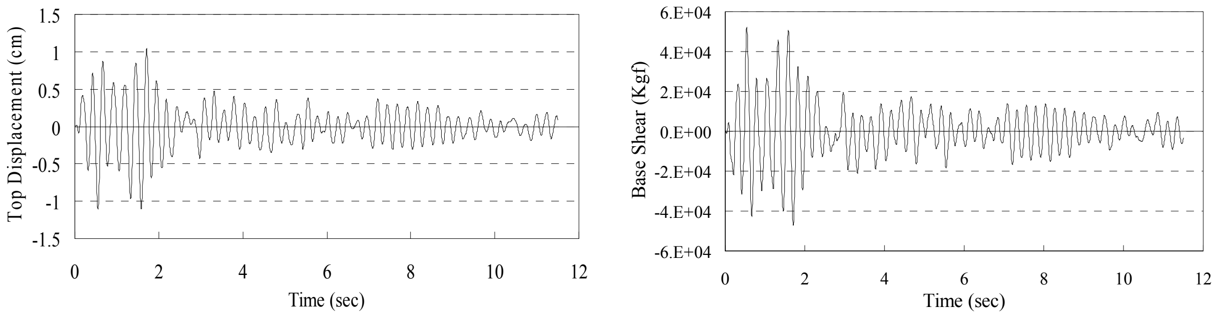


Figure 2: Responses for the structural system in Fig. 1.

in implementation of the technique proposed in [14]. In Figs. 3 and 4, the responses obtained from time integration analyses with the average acceleration method [5-7, 12, 36] and steps satisfying Eq. (4) are compared, with the responses obtained, when changing the step sizes to step sizes obtained from Eq. (4) after eliminating $_f\Delta t$, i.e.

$$\Delta t = n\,_f\Delta t \tag{11}$$

where, n is to be as stated in Eq. (10). As apparent in Fig. 3, the technique has successfully halved the computational cost, in the price of negligible loss of accuracy. Meanwhile, though hardly recognizable in Fig. 4, the errors because of the technique are more in lower oscillatory modes (higher values of T); in this example (and for many systems), these are the modes with more contribution in the response.

——— Ordinary analysis ($\Delta t = 0.005$ sec)

--------- Ordinary analysis after implementation of the technique ($\Delta t = 0.01$ sec)

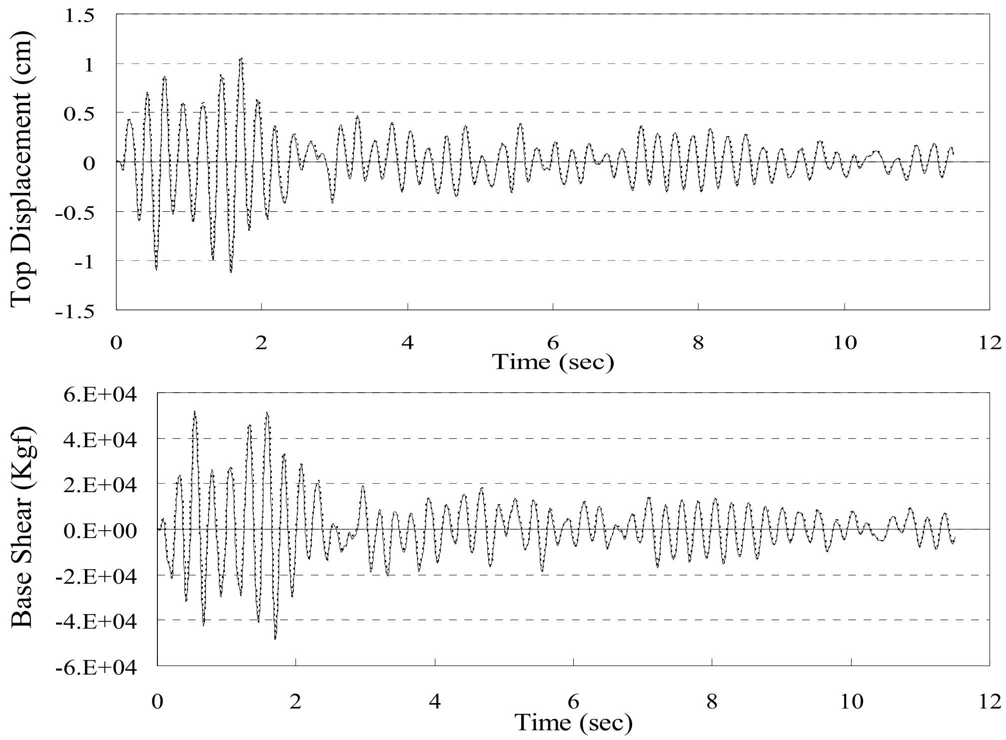


Figure 3: Responses computed for the structural system in Fig. 1 by the average acceleration method.

——— Ordinary analysis ($\Delta t = 0.005$ sec)

--------- Ordinary analysis after implementation of the technique ($\Delta t = 0.01$ sec)

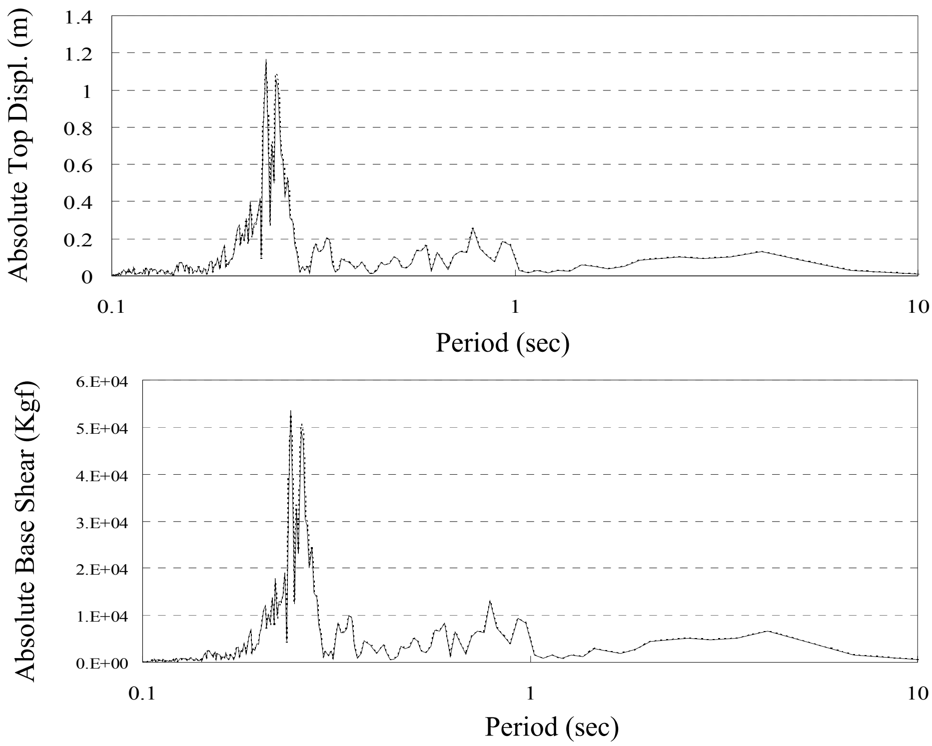


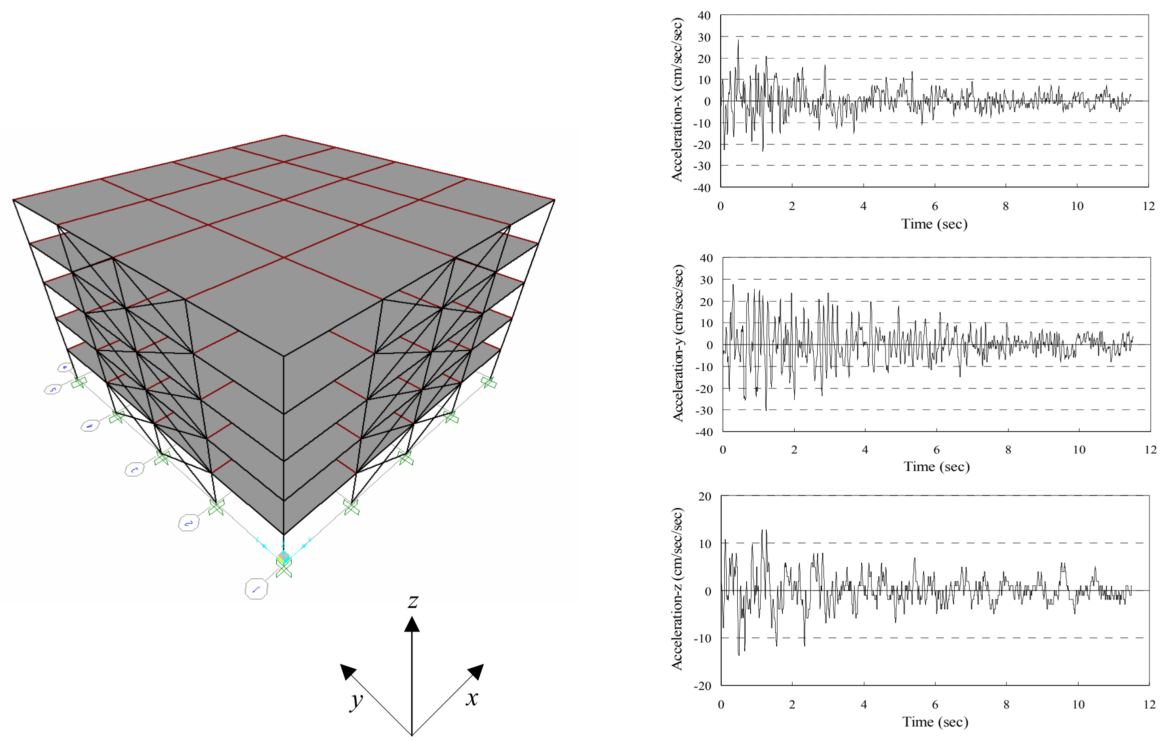
Figure 4: Frequency content of the response displayed in Fig. 3.

3.2 3-D Frame

The study reported in Section 3.1 is repeated for the 3-D frame introduced in Fig. 5(a) and Table 2, against the earthquake records in Fig. 5(b) applied in the main directions ($_f\Delta t = 0.005$ sec). The structural members are briefly introduced in Table 3; the structural system is dual (moment resisting frame plus bracings) with two-sided slabs at floors; the floors height are 3 meters and the vertical loads are distributed uniformly on the floors, while the earthquake records are applied in the main directions, and, in separate studies, the analysis methods are the HHT $(\gamma = 0.6, \beta = 0.3025, \alpha = -0.3)$ and the average acceleration [36, 37]. The almost exact responses are as displayed in Fig. 6. In view of Figs. 5(b) and 6 (also see [24]),

$$n = 5 \tag{12}$$

is an appropriate selection in implementation of the technique proposed in 2008 [14]. The resulting responses are as reported in Figs. 7 and 8, respectively, in time and frequency domains, once again, implying the good performance of the technique and the more errors of the technique in lower modes.



(a) (b)

Figure 5: Structural system in the second example: (a) 3D frame, (b) Ground motion.

Members	**Profiles**
Beams	IPE 270, IPE 300, IPE 360, IPE 400
Columns	BOX 20x20x1.25, BOX 22x22x1.25, BOX 22x22x1.6, BOX 24x24x1.75, BOX 26x26x2.22
Braces	BOX 12x12x1, BOX 14x14x1, BOX 9x9x1

Table 3: Profiles of the structural members in Fig. 5(a).

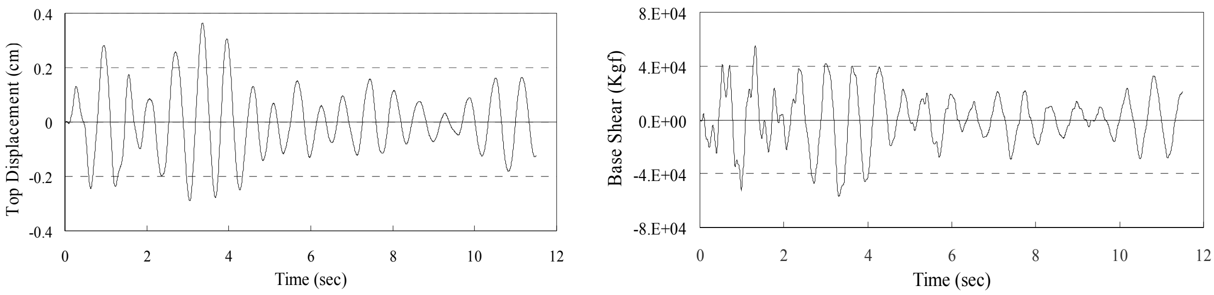


Figure 6: Top displacement and base shear histories for the second example.

—— Ordinary analysis ($\Delta t = 0.005$ sec)
------- Ordinary analysis after implementation of the technique ($\Delta t = 0.025$ sec)

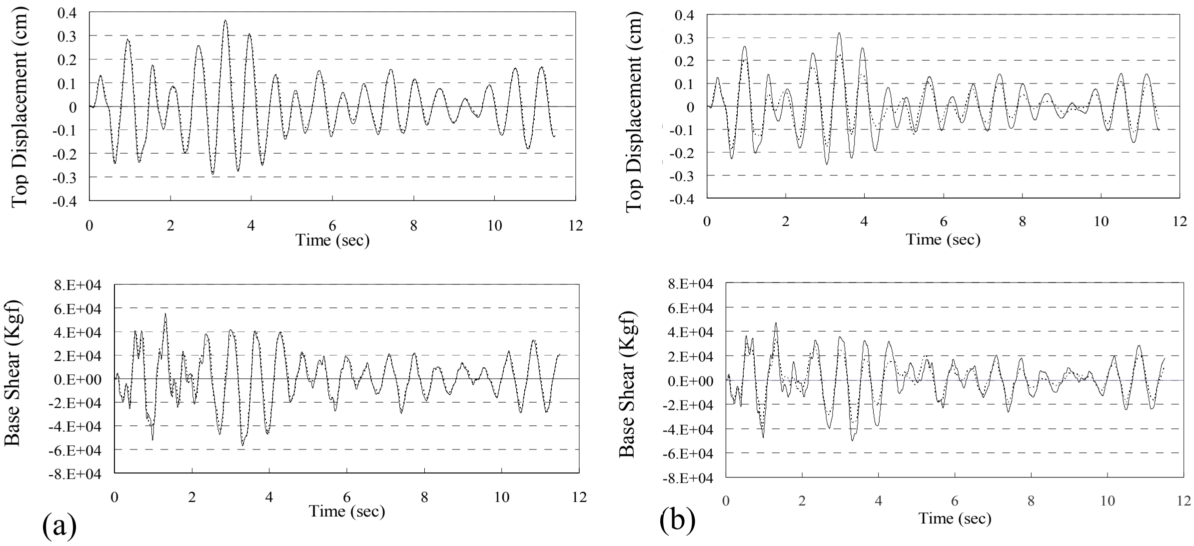


Figure 7: Time histories of the responses computed for the structural system introduced in Fig. 5 and Table 3 by: (a) Average acceleration, (b) HHT.

—— Ordinary analysis ($\Delta t = 0.005$ sec)
------- Ordinary analysis after implementation of the technique ($\Delta t = 0.025$ sec)

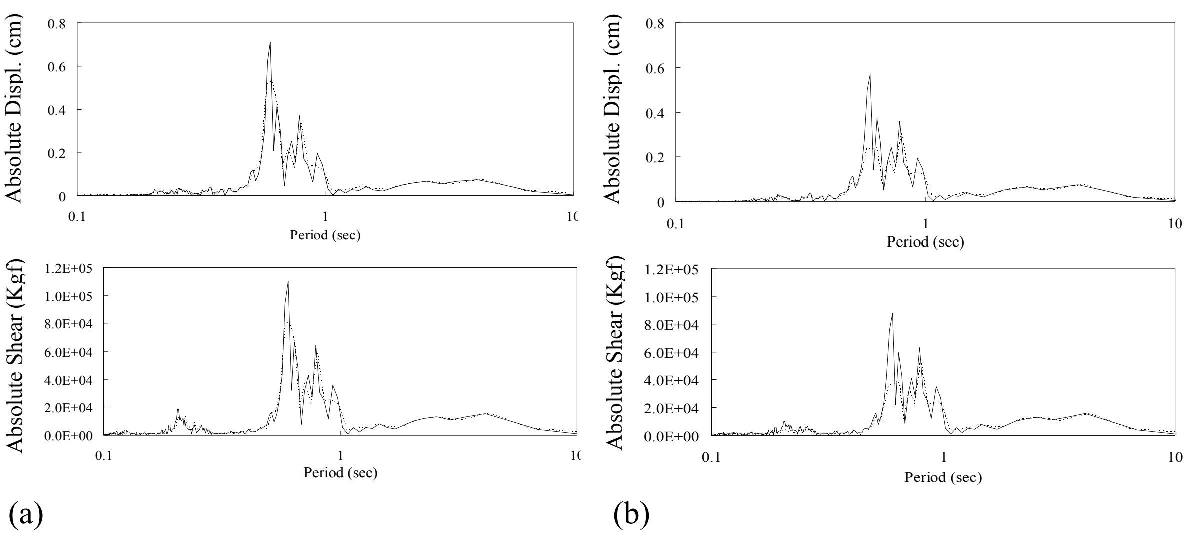


Figure 8: Frequency content of the response reported in Fig 7: (a) Average acceleration, (b) HHT.

The study is repeated taking into account the nonlinear behavior of the joints as well as the P-Δ effect. Merely the P-Δ actually affected the behavior and that only slightly. Accordingly, considering the difference between the linear and nonlinear cases in Eq. (2),

$$n = 1.25 \tag{13}$$

is an appropriate selection in the implementation of the technique, proposed in [14], in analysis of the system introduced in Fig. 5 and Table 3 (see also [38]). The results reported in Figs. 9 and 10, once again, are evidences for the claim in Section 2.

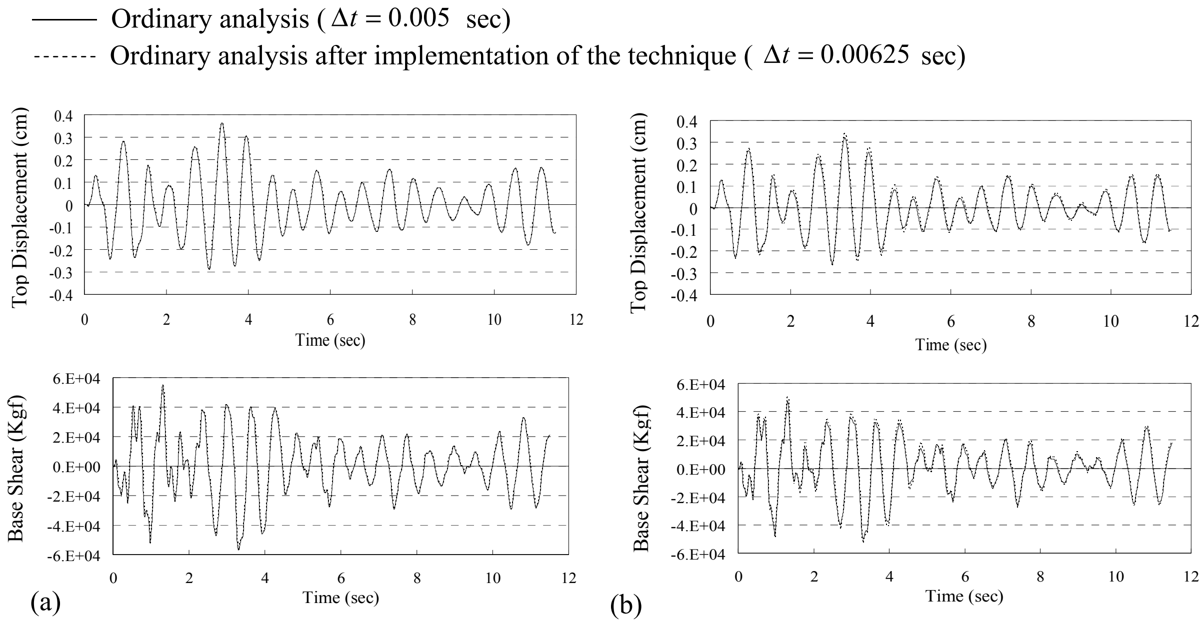


Figure 9: Time history of the response computed for the nonlinear case of the structural system introduced in Fig. 5 and Table 2 by: (a) Average acceleration, (b) HHT.

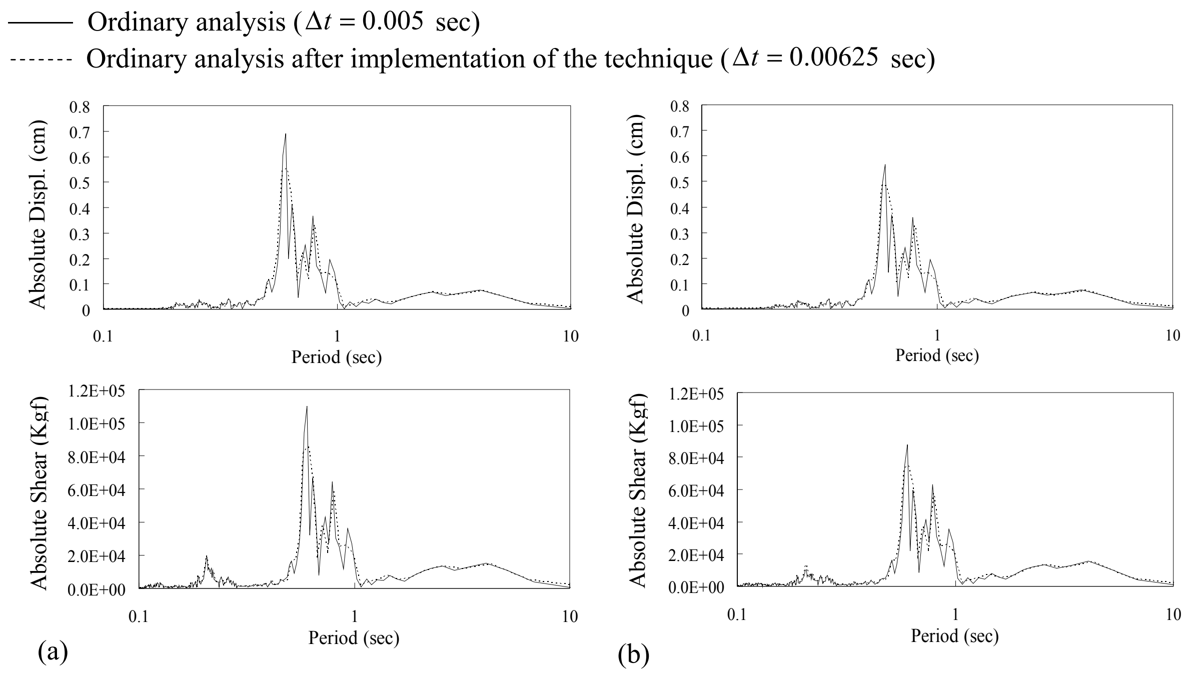


Figure 10: Frequency content of the response reported in Fig 9: (a) Average acceleration, (b) HHT.

4 CONCLUSIONS

The performance of a time integration computational cost reduction technique in frequency domain is studied via brief theoretical discussion and three examples. Accordingly, it sounds reasonable to expect more errors either in lower modes or in modes with high contribution in the response; this is not restricted to a specific time integration method and to linear behaviors. Towards more efficiency for the technique proposed in [14], further study is essential and highly recommended.

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