ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5-10 June 2016

SCALE EFFECT IN MICROFLOWS MODELLING WITH THE MICROPOLAR FLUID THEORY

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Keywords: Micropolar fluid, Squeeze flow, Hagen-Poiseuille flow.

Abstract. *The aim of this paper is to perform a comparative study of the scale effect on the applicability of the theory of micropolar fluids to microflows modelling. Flows which often appear in microfluidics systems: the squeezing flow between flat plates and the Hagen–Poiseuille flow are considered.*
For each flow the maximal geometrical dimension of the flow field for which the "micropolar ity" of the fluid affects flows characteristics is predicted and its value is calculated for water, blood and electro-rheological suspension. Results indicate that for the Hagen–Poiseuille flow the dimension is ten times greater than for the squeezing flow.

1. INTRODUCTION

Fluid flows in small scales have received increased attention over the last decade due to the fact that modelling and simulating micro- and nanoscale fluid systems is still a challenging task. In such scales the effect of interaction between the fluid molecules and the solid wall molecules is much more significant than in its bulk case described by classical (i.e. Cauchy type) continuum mechanics theories that are not capable of explaining the observed effects such as: microrotation of molecules, density fluctuations or the electrokinetic effect [1]. Recently, the most powerful, and perhaps the most widely used tools for studying such fluid systems are the molecular dynamic and the Monte Carlo methods. However, it is well known that both approaches are computationally expensive.

Other possibility to model flows in small scales is to employ a generalized continuum mechanics approach, in which more material constants allow to capture some microstructural features that are non-detectable by classical theories. The micropolar fluid theory was developed by Eringen in 1966 [2] in the frame of the generalized continuum mechanics. It takes into account the microrotation of molecules (microrotation – spinning motion of molecules that is independent on the rotation of the flow velocity field) and introduces additionally a couple stress tensor. In this theory, field equations are presentable in terms of two independent kinematic vector fields: the velocity and the microrotation vector, and equations involve six material coefficients. Microrotation vector differs from vorticity vector which is defined as rotation of velocity. The stress tensor is not symmetric.

Micropolar fluid flow equations are being reduced to their counterparts in the classical continuum mechanics (Cauchy), when the microrotation is omitted or when the characteristic linear dimension of the flow field is large enough [3]. It indicates that the geometrical size of the flow field plays a crucial role in the applicability of the micropolar fluid theory to microflows modelling. Till now, only the Hagen–Poiseuille flow in circular tube in this aspect was examined in detail [3].

This paper presents the analysis of the scale impact on the effective micropolar fluid theory microflows modelling. The Hagen–Poiseuille flow and the squeezing flow are considered. Flow characteristics are determined. Values of experimentally determined micropolar fluid constants [4] and values predicted on molecular dynamic simulations [5] are used in the calculations. Formulas for the maximal distance between plates for the squeezing flow and the maximal cross-sectional size of the channel for the Hagen–Poiseuille flow, for which the micropolar effects of the fluid affects flow characteristics are established and their values for some real fluids are calculated. Beyond this dimension, it pays off to carry on the calculations on the basis of the classical (Cauchy) hydrodynamics. The comparative study of obtained results for considered flows is performed. The results indicate that the scale effect depends on the type of the flow. The maximal geometrical dimension of the flow field, for which the fluid micropolar effects are negligible, is ten times greater for the squeeze flow than for the Hagen-Poiseuille flow. It indicates that for each type of the fluid flow its “own” limiting dimension value should be estimated for effective modelling by using the micropolar fluid theory.

2. MICROPOLAR FLUID FIELD EQUATIONS

The micropolar fluid theory (MFT) is referred as fluids with an asymmetric stress tensor or extended Navier–Stokes theory [6]. The coupling between the hydrodynamic flow degree of freedom which is described by the velocity vector, and the microscopic molecular spin angular velocity degree of freedom which is described by the microrotation vector is visible in field equations. In the most general form, the micropolar field equations which represent conservation of mass, conservation of impulse and angular momentum for incompressible and viscous fluid are as follows:

$$\mathrm{div}\boldsymbol{V} = 0 \tag{1}$$

$$\rho \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \rho \boldsymbol{f} - \mathrm{grad}\, p - (\mu + \kappa)\mathrm{rotrot}\, \boldsymbol{V} + \kappa \mathrm{rot}\boldsymbol{\omega} \tag{2}$$

$$\rho I \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = (\alpha + \beta + \gamma)\mathrm{graddiv}\boldsymbol{\omega} - \gamma \mathrm{rotrot}\boldsymbol{\omega} + \kappa \mathrm{rot}\boldsymbol{V} - 2\kappa\boldsymbol{\omega} + \rho \boldsymbol{g} \tag{3}$$

where ρ is density, $\boldsymbol{V} = (V_1, V_2, V_3)$, the velocity field, $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ the micro-rotation field, I the gyration parameter, $\boldsymbol{f} = (f_1, f_2, f_3)$, body forces per unit mass, p the hydrostatical pressure, μ the classical viscosity coefficient, κ, λ the vortex viscosity coefficients and α, β, γ are gyroviscosity coefficients satisfying the following inequalities:

$$3\alpha + \beta + \gamma \geq 0,\ 2\mu + \kappa \geq 0,\quad 3\lambda + 2\mu + \kappa \geq 0,\quad \gamma \geq \beta,\quad \kappa \geq 0,\quad \gamma \geq 0.$$

The constitutive equations for the stress tensor $\boldsymbol{T} = \{T_{ij}\}$, and the couple stress tensor $\boldsymbol{C} = \{C_{ij}\}$, are expressed by following equations, respectively as:

$$T_{ij} = (-p + \lambda V_{k,k})\delta_{ij} + \mu(V_{i,j} + V_{j,i}) + \kappa(V_{j,i} - \varepsilon_{ijk}\omega_k) \tag{4}$$

$$C_{ij} = \alpha\omega_{k,k}\delta_{ij} + \beta\omega_{i,j} + \gamma\omega_{j,i} \tag{5}$$

ε_{ijk} – the Levi-Civita tensor, δ_{ik} – the Kronecker delta.

The micropolar fluid flow equations arrive at the classical Navier–Stokes equations: (i) if transport coefficients: α,β,γ, and κ vanish (ii) if microrotation is ignored (iii) when the characteristic linear dimension of the flow field is large enough [3].

3. SIMPLE MICROPOLAR FLUID FLOWS

In this section the quantitatively analyze of a scale impact on the effective micropolar fluid microflows modelling is presented. Flows, for which analytical solutions are known, are considered. This way errors arising from numerical solutions are excluded. Two different flows are studied which represent two systems where fluid is confined in narrow channel: i) pressure driven planar Hagen–Poiseuille flow, ii) squeezing flow of narrow film between parallel plates. Such flows often appear in microfluidics systems. The influence of the molecular spin (microrotation) on the hydrodynamic quantities in dependence of characteristic linear flow dimension is analyzed. Exact, analytical solutions of Eqs (1-3) for the flows at hand are provided in the literature for different boundary conditions imposed on velocity and microrotation on the confining surfaces. In this paper, their simplest form is used; vanishing velocity and microrotation on the confined surfaces. Hydrodynamic quantities which characterize each of flows are expressed by analytical formulas. The results are compared with those obtained for classical (i.e. Cauchy type) continuum mechanics theory, in which mass transport is described by the Navier–Stokes equation and which omits microrotation vector.

3.1. Flow reduction in a planar Poiseuille flow

A fluid confined between parallel walls located at z = 0 with h distance between them is considered. The z direction is the only direction of confinement and the walls are infinite in extent in the x, y-plane. Pressure gradient drives the planar flow. The micropolar equations

(1-3) can be solved analytically with Dirichlet slip boundary conditions where $\boldsymbol{V}(h) = 0$, $\boldsymbol{V}(0) = 0$ and $\boldsymbol{\omega}(\mathrm{h}) = \mathbf{0}$, $\boldsymbol{\omega}(0) = \mathbf{0}$ It has been shown that the relative flow rate reduction due to spinning motion of molecules is given by [4]:

$$Q(h) = Q_m(h)/Q_{\mathrm{N}}(h) \tag{6}$$

where

$$Q_m(h) = Q_{\mathrm{N}}(h)\left\{1 + \frac{2\delta}{2+\delta}\frac{4}{(kh)^2}\left[1 - \frac{\cosh(kh)}{\sinh(kh)}\right]\right\}$$

$$k = \sqrt{\frac{(2\mu+\kappa)\kappa}{(\mu+\kappa)\gamma}}$$

$$\delta = \frac{\kappa(1-\alpha_0)}{\mu_{\mathrm{N}}}$$

Q_m and Q_n are the flow rates when the fluid spinning motion of molecules described by microrotation is included (in the frame of micropolar fluid theory) and when it is ignored (in the classical - i.e. Cauchy type continuum mechanics theory).

3.2. Squeezing flow of narrow film between parallel plates

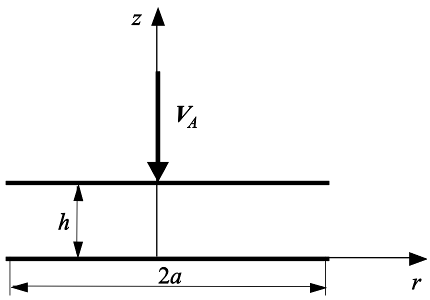

Fig. 1. Geometry of squeezing flow

A two-dimensional flow in fluid layer $0 < z < h$, $r < a$ between two parallel plates circular approaching each other symmetrically is considered. The plate at $z = 0$ is immobile, while the second plate moves with velocity $\boldsymbol{V_A}$.

The micropolar equations (1-3) can be solved analytically in Reynolds approach with Dirichlet slip boundary conditions where $\boldsymbol{V}(h) = \boldsymbol{V_A}$, $\boldsymbol{V}(\mathbf{0}) = \mathbf{0}$ and $\boldsymbol{\omega}(h) = \mathbf{0}$, $\boldsymbol{\omega}(0) = \mathbf{0}$ and the relative load capacity increase due to the microrotation is defined as [8]:

$$W(h)=W_m(h)/W_N(h) \tag{7}$$

where:

$$W_m(h) = \frac{\pi\mu V_A a^4}{8h^3\left[\frac{1}{12} + \frac{l^2}{h^2} - \frac{Nl}{2h}\coth\frac{mh}{2}\right]}$$

and

$$W_{\mathrm{N}}(h) = \frac{3\pi\mu_{\mathrm{N}} V_A a^4}{2h^3}$$

Symbols W_m and W_N denote the load capacity when the fluid microrotation is included and when it is ignored i.e. predicted in the frame of the micropolar fluid theory and in the frame of the classical - i.e. Cauchy type continuum mechanics theory respectively.

4. RESULTS AND DISCUSION

In this section values of hydrodynamic quantities: (i) $Q(h)$ – relative flow rate reduction for the Hagen–Poiseuille flow (6), (ii) $W(h)$ – relative load capacity increase (7) for squeezing flow are calculated as a function of distance h between walls confined the flow for water, blood and electrorheological suspensions, and plotted in Figures 1 and 2 respectively.

The values of micropolar viscosity coefficients used in calculations were found in literature. For blood, the values $\mu = 2.9 \cdot 10^{-3}$, $\kappa = 2.32 \cdot 10^{-4}$, $\gamma = 10^{-6}$ are listed in [7]. For water the values evaluated by Hansen et al. (2011) using equilibrium Molecular Dynamics are used: $\mu = 0.7 \cdot 10^{-3}$, $\kappa = 0.17 \cdot 10^{-3}$, $\gamma = 2.1 \cdot 10^{-21}$. For electro-rheological suspension, the viscosity coefficient values are given in reference to viscosity μ_N and reads: μ/μ_N = = $1.922 \cdot 10^{-3}$, $\kappa/\mu_{\mathrm{N}} = 5.845$, $\gamma/\mu_{\mathrm{N}} = 9.94 \cdot 10^{-10}$ [4].

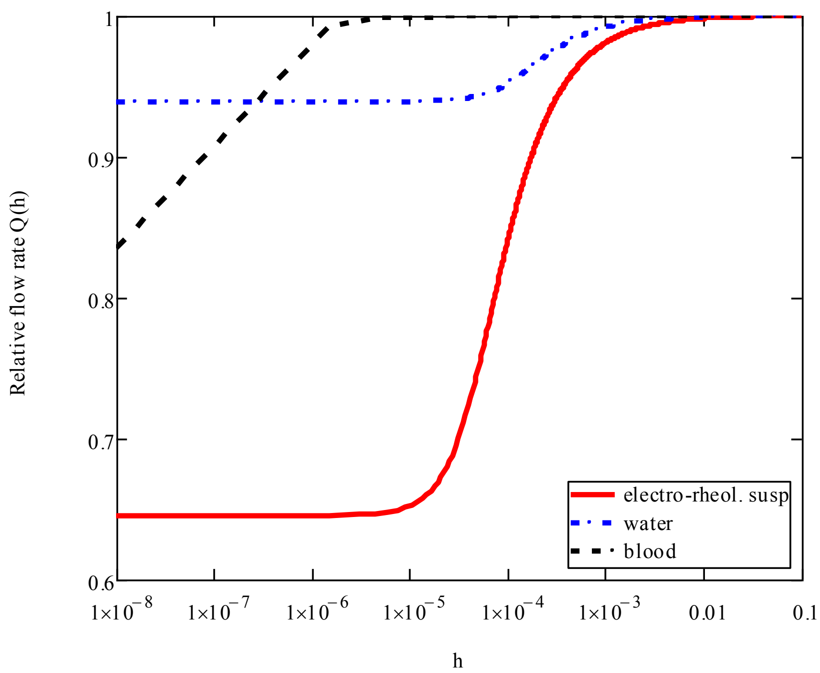

Fig. 2. Relative flow rate $Q(h)$ (6)versus distance h [m] between walls in Hagen–Poiseuille flow

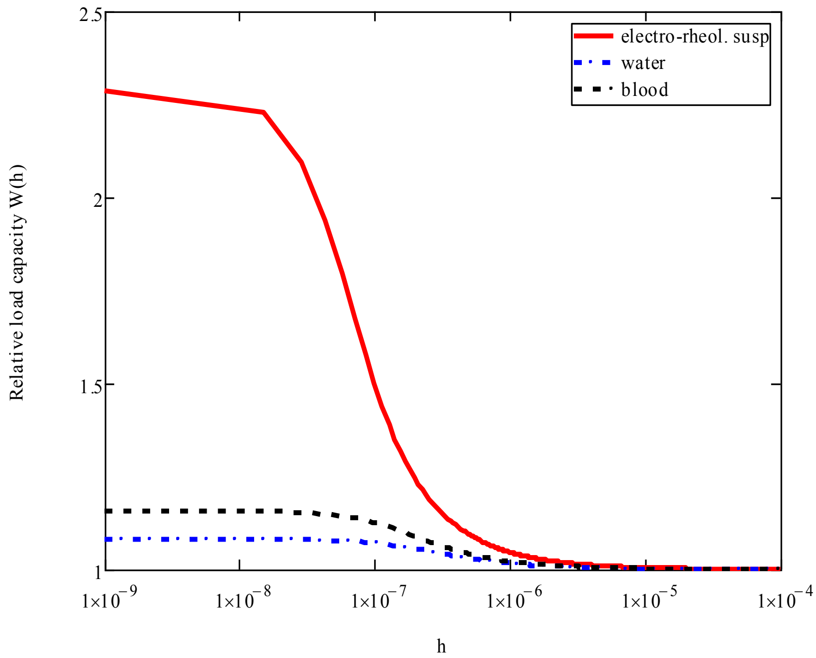

Fig. 3. Relative load capacity *W(h)* (7) versus distance *h* [m] between plates in squeezing flow

It can be seen from data presented in Figure 2 that there exists a flow reduction with microchannels height. Results presented in Figure 3 show that for every fluid, beginning from a certain distance between approaching plates, the load capacity calculated using the micropolar fluid model is bigger than the one calculated with the use of the classical model of the fluid. Comparing relative quantities plots for a given fluid, we observe that values of distances h when deviations started are not the same in two different flows. For Hagen-Poisuille the flow rate 1% reduction begins for h value greater than the value of h for which relative load capacity increment is equal to 1%.

What is more important, it can be observed that every fluid has its "own" value of the limiting distance h, starting from which the relative quantities plotted in Figs 2 and 3 begins to change. But comparing the limiting distance h for given fluid in two different flows we can observe that the distances differ.

To visualise the observed effect, the relative flow rate defined by formula (6) and the relative load capacity defined by formula (7) are plotted in Figs 4 and 5 in terms of the microstructural parameters L and N, *very* often used when compare various micropolar fluid properties. They are defined as follows:

$$N = \sqrt{\frac{\kappa}{2\mu_N + \kappa}}, \quad L = \frac{L_c}{l}, \quad l = \sqrt{\frac{\gamma}{4\mu_N}} \tag{7}$$

where $\mu_N = \mu + \kappa/2$.

Symbol L_c denotes characteristic length for given flow geometry, therefore for considered problems L_c should be replaced by h - a distance between walls.

To perform the data comparison, the parameter N values are assumed: N=0.3, N=0.6 and N=0.9.

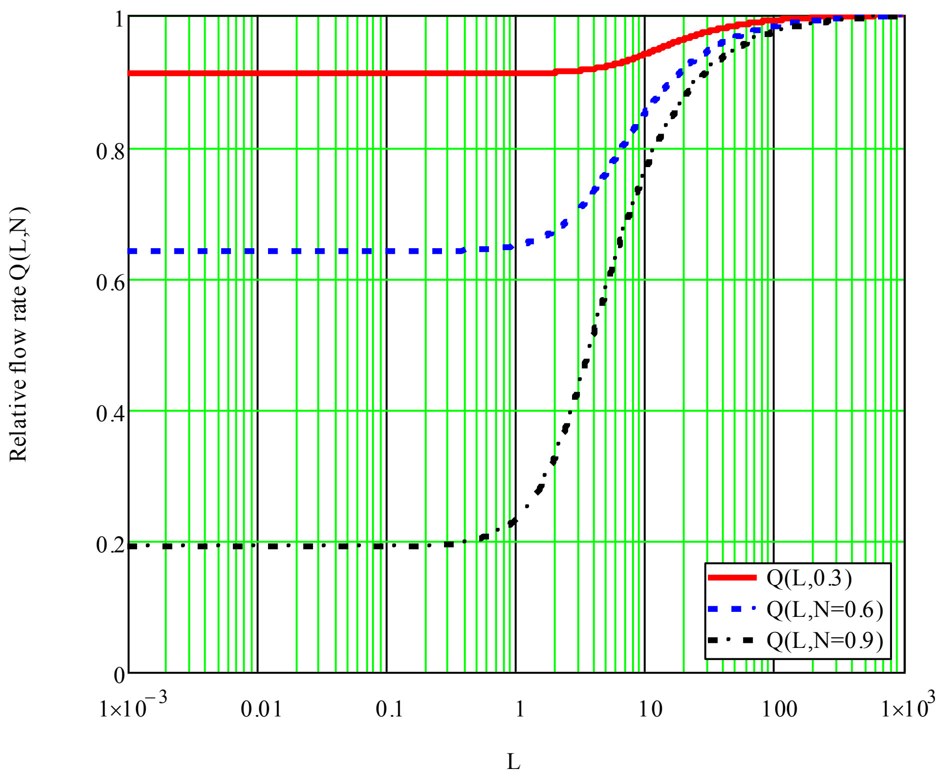

Fig. 4. Relative flow rate Q(L,N) in Hagen–Poiseuille flow versus parameter L

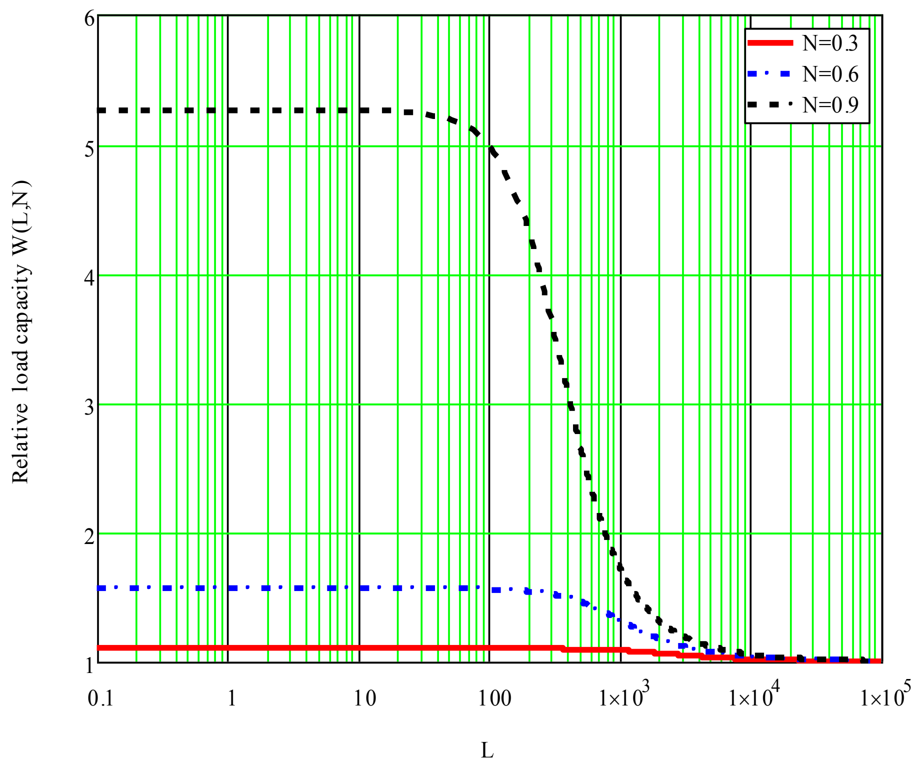

Fig. 5. Relative load capacity W(L,N) in squeezing flow versus parameter L

Comparing plots from Figures 4 and 5 we observe that only for L parameter values satisfying inequality: $L < 10000$ the relative load capacity increases. The flow reduction in the planar Poiseuille flow appears when only $L < 1000$. It means that values of geometrical dimensions of flow field for which the " micropolarity" affect characteristics are different for considered flows. Their values can be easily calculated by use formula (7) and are equal respectively: $h_{max}=10^4\ (\gamma/(4\mu_+2\kappa))^{1/2}$, $h_{max}=10^3\ (\gamma/(4\mu_+2\kappa))^{1/2}$.

5. CONCLUSIONS

- Results of the performed analysis prove that sufficient deviations of hydrodynamic quantities from classical results are important on the small scale which depends on rheological properties of the fluid. This means that the coupling between the molecular rotations must be taken into account to give a correct prediction of the flow characteristics on suitable small length scales.
- Length scale for which the fluid micropolar effects are negligible is ten times greater for Hagen–Poiseuille flow then for squeeze flow.
- Comparative study of obtained results for considered flows show that the scale effect depends on the type of the flow.

REFERENCES

[1] H. Bruus, Theoretical Microfluidics (2008) Oxford, ISBN 978-0-19-923509-4.

[2] A.C. Eringen, (1966), Theory of micropolar fluids, J. Math. Mech. 16, 1, 1-16.

[3] A. Kucaba-Pietal, (2004), Microchannels flow modelling with the micropolar fluid theory, Bulletin of Polish. Academy of Sciences. Techn. Note. 52 (3), s. 209-214, http://bulletin.pan.pl/(52-3)209.pdf.

[4] P. Prokhorenko, N.P. Migoun, M. Stadthaus, (1999), Theoretical Principles of Liquid Penetrant Testing, DVS Verlag, Berlin.

[5] J.S. Hansen, H. Bruus, B.D. Todd, P. Daivis, (2011), Rotational and spin viscosities of water: Application to nanofluidics Phys. Review E 84, 036311.

[6] G. Łukaszewicz, (1999), Micropolar Fluids. Theory and Application, Birkhouser, Berlin.

[7] I. Papautsky, J. Brazzle, T. Ameel, A.B. Frazier, (1999), Laminar fluid behavior in microchannels using micropolar fluid theory, Sensors and Actuators, A, vol. 73, 2, 101-108.

[8] J. Prakash, P. Sinha, (1976), Squeeze film theory for micropolar fluids, Trans. ASME, F98, 1, 139-144.