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MINIMAL MASS DESIGN OF STRENGTHENING TECHNIQUES FOR PLANAR AND CURVED MASONRY STRUCTURES

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Abstract. *We present a discrete element model of a masonry structure strengthened through the application of reinforcing elements designed to work in tension. We describe the reinforced masonry structure as a tensegrity network of masonry rods, mainly working in compression, and tension elements corresponding to fiber-reinforced composite reinforcements, which are assumed to behave as elastic-perfectly-plastic members. We optimize a background structure connecting each node of the discrete model of the structure with all the neighbors lying inside a sphere of prescribed radius, in order to determine a minimal mass resisting structure under the given loading conditions and prescribed yielding constraints. Fiber-reinforced composite reinforcements can be naturally replaced by any other reinforcements that are strong in tension (e.g., timber or steel beams/ties). Some numerical examples illustrate the potential of the proposed strategy in designing tensile reinforcements of a three-dimensional structure composed of a masonry vault and supporting walls.*

1 Introduction

It is known that old masonry often features nearly zero tensile strength [1, 2, 3]. Nowadays, it is a common practice to reinforce such structures by applying tensile reinforcements made of traditional materials, such as steel, or innovative high-strength materials [4, 5, 6]. Strips and/or meshes of materials like Fiber Reinforced Polymers (FRP) or Fabric Reinforced Cementitious Matrix (FRCM) composites are often bonded to masonry structures to improve their mechanical properties [7, 8, 9]. However, it is worth remarking that such strengthening techniques, when improperly used, may lead to an excessive over-strength of the reinforced structure, and reduced 'cracking-adaptation' capacity [10].

The modern *Discrete Element Modeling* (DEM) of masonry structures includes computer-assisted, funicular-network procedures [11], *Lumped Stress Models* [12, 13, 14, 15], and *Thrust Network Approaches* (TNA) [16, 17, 18]. A recent study [19] has presented a tensegrity approach to the 'minimal-mass' FRP-/FRCM reinforcement of masonry vaults and domes. Such a procedure employs tensegrity concepts to find an optimal resisting mechanism of the reinforced structure, under given loading conditions, in line with the 'Italian Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures' [21]. The latter indeed allows the designer to describe the response of the reinforced structure through simplified schemes, on assuming that tensile stresses are directly taken by the FRP reinforcements, and the stress level may be determined by adopting a distribution of stresses that satisfies the equilibrium conditions but not necessarily the strain compatibility (cf. Sect. 5.2.1 of [21]). The approach proposed in [19] describes the reinforced structure as a tensegrity network of masonry rods, working in compression, and tension elements corresponding to the FRP-/FRCM- reinforcements, which are assumed to behave as elastic-perfectly-plastic members. It optimizes a *background structure* connecting each node of a discrete model of the structure with all the neighbors lying inside a sphere of prescribed radius, in order to determine a minimal mass resisting structure under the given loading conditions and prescribed yielding constraints [22]. The FRP/FRCM reinforcements can be naturally replaced by any other reinforcements that are strong in tension (e.g., timber or steel beams/ties).

The present study generalizes the approach presented in [19, 20] to the case of 2D and 3D discrete models of masonry structures with arbitrary shape. Such an extension allows us to explore the potential of the tensegrity modeling of reinforced masonry structures in the design of non-invasive reinforcement patterns of systems formed by masonry walls, vaults and domes. We formulate a design procedure that seeks for an optimal and lightweight pattern of reinforcing elements giving rise to a minimal mass resisting mechanisms of the examined structure, under given loads and yielding constraints. Due to the safe theorem of the limit analysis of elastic-plastic bodies [23], the existence of such a mechanism ensures that the reinforced structure is safe under the examined loading conditions, on assuming elastic-perfectly-plastic response of all members. The input variables of the proposed procedure consist of a 3D point cloud defining the geometry of the structure to be reinforced, obtainable, e.g., through in-situ laser-scanning, together with the material densities and yielding strengths of masonry and reinforcing elements.

The remainder of the paper is structured as follows. Section 2 illustrates the adopted minimal mass modeling of a reinforced masonry structure under given yielding constraints and loading conditions. Next, Sect. 3 presents case studies dealing with the FRP-/FRCM-reinforcement of an independent cloister vault (Sect. 3.1) three-dimensional structural system formed by a cloister vault and supporting walls (Sect. 3.2). We conclude with final remarks and directions of future research in Sect. 4.

2 Minimal-mass reinforcement of a masonry structure

Let us generalize the optimization strategy presented in Sections 2 and 3 of Ref. [19] to the general case of an arbitrary masonry structure, whose geometry is described by three-dimensional set of n_n nodes with position vectors $\mathbf{n}_k$ ($k = 1,...,n_n$). Such nodes may be condensed over one or multiple structural surfaces, e.g, the intrados and the extrados surfaces of a planar wall or a vaulted structure.

We introduce a background structure (refer, e.g., to the example of Fig. 1) by connecting each node $\mathbf{n}_k$ with all the nodes $\mathbf{n}_j$ such that it results $|\mathbf{n}_k - \mathbf{n}_j| \leq r_k$ (*interacting neighbors*), through two elements working in parallel: a compression element (or *bar*) $\mathbf{b}_i = \mathbf{n}_k - \mathbf{n}_j$; and a tension element (or *string*) $\mathbf{s}_i = \mathbf{n}_k - \mathbf{n}_j$. Assuming that such a background structure is subject to a number m of static loading conditions, we write its equilibrium equations as follows

$$\mathbf{A}\mathbf{x}^{(j)} = \mathbf{w}^{(j)} \tag{1}$$

where j is the loading condition index ($j = 1,...,m$); $\mathbf{A}$ is the static (or equilibrium) matrix; $\mathbf{w}^{(j)}$ is the external load vector; and $\mathbf{x}^{(j)}$ is the vector collecting all the force densities in bars and strings (refer to [22] for the analytic expression of $\mathbf{A}$).

We now assume that bars and strings behave as as elastic-perfectly-plastic members, with yield strength σ_{b_i} in the generic bar (compressive yield strength), and yield strength σ_{s_i} in the generic string (tensile yield strength). We let A_{b_i} denote the cross-section area of $\mathbf{b}_i$, and let A_{s_i} denote the cross-section area of $\mathbf{s}_i$. The masses of such members are respectively given by $m_{b_i} = \rho_{b_i} A_{b_i} b_i$, and $m_{s_i} = \rho_{s_i} A_{s_i} s_i$, where ρ_{b_i} and ρ_{s_i} respectively denote the mass densities of $\mathbf{b}_i$ and $\mathbf{s}_i$; b_i denotes the length of $\mathbf{b}_i$ and s_i denotes the length of $\mathbf{s}_i$. Moreover, in correspondence with the j-th loading condition, we let $\lambda_{b_i}^{(j)}$ denote the force density carried by $\mathbf{b}_i$, and let $\gamma_{s_i}^{(j)}$ denote the force density carried by $\mathbf{s}_i$, such that $\lambda_{b_i}^{(j)} > 0$ when $\mathbf{b}_i$ is compressed, and $\gamma_{s_i}^{(j)} > 0$ when $\mathbf{s}_i$ is stretched. Yielding constraints impose that it results

$$\lambda_i^{(j)}\, b_i \leq \sigma_{b_i} A_{b_i}, \quad \gamma_i^{(j)}\, s_i \leq \sigma_{s_i} A_{s_i} \tag{2}$$

in correspondence with all the bars and strings, and all the loading conditions.

We seek for an optimized resisting mechanism of the examined structure through the following linear program [22, 19]

$$\begin{aligned} &\underset{\mathbf{x}^{(j)},\mathbf{y}}{\text{minimize}} \quad m = \mathbf{d}^T \mathbf{y} \\ &\text{subject to} \quad \begin{cases} \mathbf{A}\mathbf{x}^{(j)} = \mathbf{w}^{(j)} \\ \mathbf{C}\mathbf{x}^{(j)} \leq \mathbf{D}\mathbf{y} \\ \mathbf{x}^{(j)} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0} \end{cases}, \end{aligned} \tag{3}$$

where

$$\mathbf{y} = [A_{b_1} \ \cdots \ A_{b_{n_b}} \mid A_{s_1} \ \cdots \ A_{s_{n_s}}]^T \tag{4}$$

$$\mathbf{d}^T = [\rho_{b_i} b_i \ \cdots \rho_{b_{n_b}} b_{n_b} \mid \rho_{s_i} s_i \ \cdots \rho_{s_{n_s}} s_{n_s}] \tag{5}$$

$$\mathbf{C} = \begin{bmatrix} \mathrm{diag}(b_1, \cdots, b_{n_b}) & \underline{0} \\ \mathbf{0} & \mathrm{diag}(s_1, \cdots, s_{n_s}) \end{bmatrix} \tag{6}$$

$$\mathbf{D} = \begin{bmatrix} \mathrm{diag}(\sigma_{b_1}, \cdots, \sigma_{b_{n_b}}) & \mathbf{0} \\ \mathbf{0} & \mathrm{diag}(\sigma_{s_1}, \cdots, \sigma_{s_{n_s}}) \end{bmatrix} \tag{7}$$

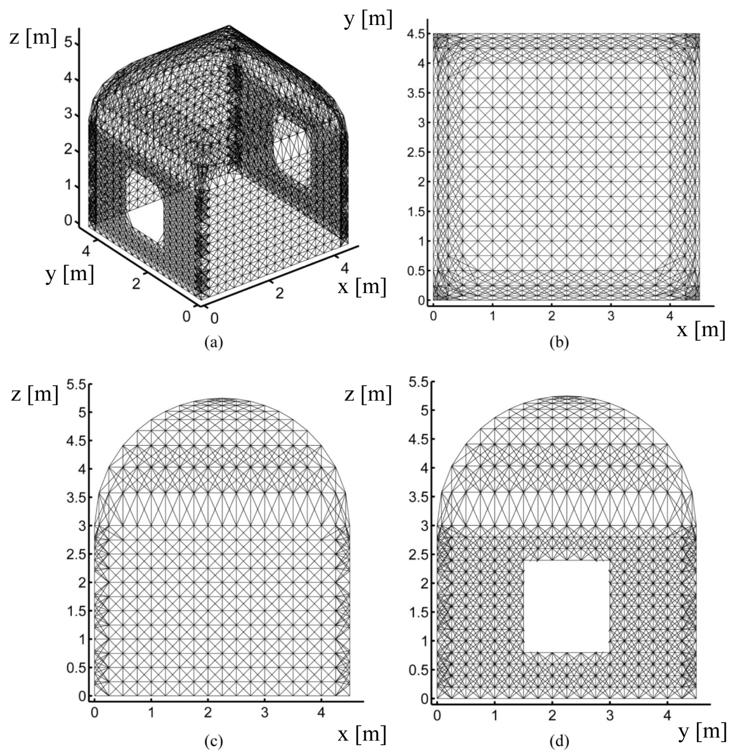

Figure 1: Background structure associated with a 3D point cloud describing the geometry of a cloister vault supported by perimeter walls (dimensions in meters): (a) 3d view; (b) top view; (c)-(d) side views. The connection distance r_k is equal to 0.75 m for nodes at the connection between walls and cloister, and 0.5 m for the reimaning nodes.

The solution to problem (3) provides minimal-mass configuration of the background structure; chooses whether a bar or a string connects each couple of interacting nodes; and returns bars and strings with zero cross-section areas in correspondence with the interacting nodes that do not need to be connected in the minimal mass configuration, under the given equilibrium (1) and yielding (2) constraints.

3 Numerical Results

The present section provides a collection of numerical applications of the minimal mass optimization procedure described in Sect. 2, which are aimed at designing optimal reinforcements of 3D structural complex formed by a cloister vault and supporting walls (Sects. 3.1, 3.2).

On assuming that each analyzed structure is formed by masonry struts with uniform compressive yield strength σ_b, and tension reinforcements with uniform yield strength σ_s, we employ the in-house software 'tensopt' [25] to numerically solve problem (3). We convene to mark the reinforcing elements by red lines and the masonry struts by solid black lines.

3.1 Cloister vault

We first examine a cloister vault made of 'Neapolitan' tufe brick masonry, which is largely diffused in the area of Naples, with 15.0 kN/m^3 self-weight, and 13 MPa compressive strength σ_b. We assume a tensile strength σ_b equal to 376.13 MPa, which corresponds to an average value of the bond strengths of the FRP and FRCM reinforcements of masonry structures analyzed in [7, 8], respectively (we employed formula (5.6) of [21] to estimate such a strength).

Fig. 2 shows the minimal mass FRP/FRCM reinforcements that we obtained for the present example ($t_f = 0.17$ mm). The geometry of the examined vault are illustrated in above figure, together with the corresponding background structure, which features 441 nodes and 4508 connections (see Figs. 2a-c). The optimal reinforcement of such a vault under vertical loading is mainly formed by parallel FRP/FRCM strips with 0.17 mm thickness and 82 mm maximum width near the crown (Figs. 2d-f). The above reinforcements are integrated with diagonal FRP/FRCM strips with about 140 mm maximum width near the intersections of the four vault segments, under combined vertical and seismic loading (Figs. 2g-l). The analyzed seismic loading consists of horizontal forces with magnitude equal to 0.35 of the magnitude of vertical forces in all nodes, which mimic the effects of a seismic excitation of the examined structure, through a conventional, static approach [26]. The compressed network include couples of diagonal arches near the corners, parallel-line arches, and diagonal struts over the vault segments (Figs. 2d-l).

3.2 3D system formed by planar and curved masonry structures

This second example is concerned with a 3D system composed of 4 orthogonal walls featuring 4.5 m horizontal length, 3.0 m height and 50 cm thickness, which support a cloister vault with 2.25 m central rise and 25 cm thickness (cf. Fig. 1). The two walls parallels to the y axis of a Cartesian frame with the z-axis placed along the vertical show 1.5 m $\times$ 1.6 m central openings. The background structure illustrated in Fig. 1 features 1385 nodes and 15378 potential connections. It is worth noting that in the present case we model both the perimeter walls and the vault as 2D membranes lying in the 3D Cartesian space.

We here assume σ_b = 1.21 MPa, σ_s = 112.5 MPa, $t_f = 0.17$ mm, and masonry selfweight equal to 15.0 kN/m^3 (tufe masonry). The optimal design reinforcement for the current example is illustrated in Figs. 3 and 4, under the action of pure vertical loading (structure selfweight), and the combined action of selfweight and seismic loading in the $+y$ direction, respectively. The optimal reinforcements under the action of pure vertical loading are mainly placed along the perimeter at the base of the cloister ($z \approx 3$ m); along horizontal lines over the two piers of the y-walls with openings; and along diagonal lines at the intersections of the vault segments (Fig. 3).

For what concerns the seismic loading condition (cf. Fig. 4), we observe that the optimal reinforcement strategy combines that corresponding to vertical loading with additional diagonal reinforcements over the two piers of the walls featuring central openings, and reinforcements aligned-with- or orthogonal-to-the junctions between the vaults segments, when moving towards the crown of the vault (cf. Figs. 4 and 3). Due to the adopted membrane modeling of

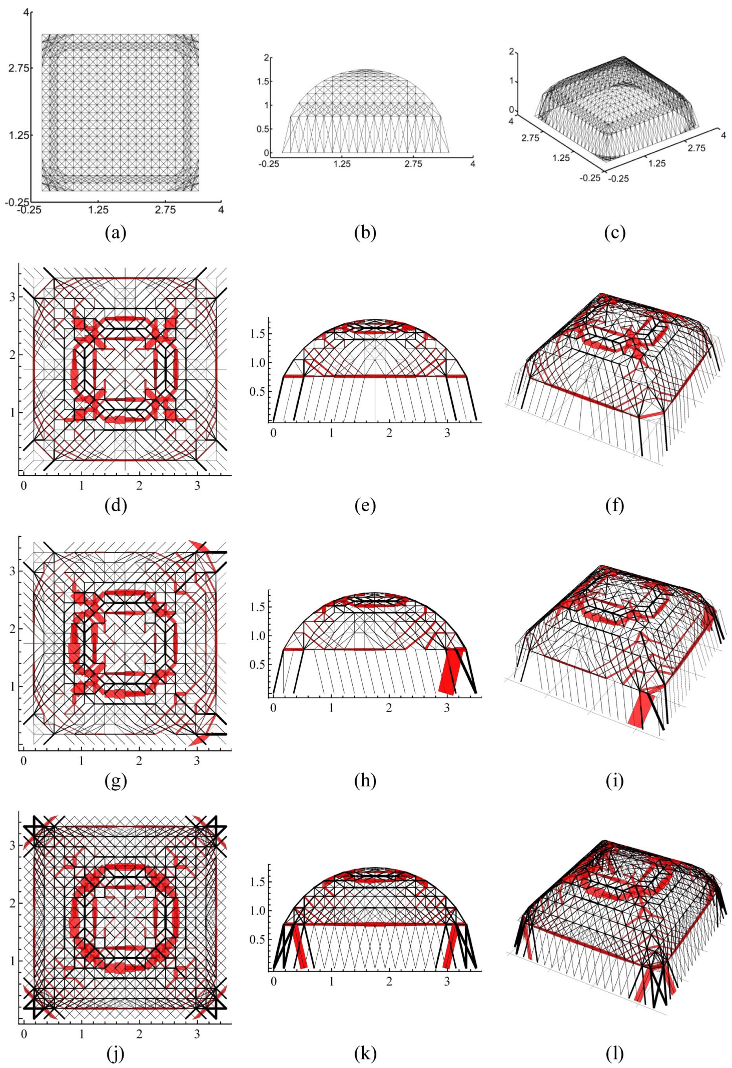

Figure 2: Top, side and 3D views of the optimal reinforcement patterns of a cloister vault with FRP/FRCM strips of thickness 0.17 mm (marked in red), under different loading conditions. The widths of the FRP/FRCM reinforcements are magnified by a factor 2 for visual clarity. (a)-(c): Background structure. (d)-(f): Vertical loading. (g)-(i): Seismic loading in the $+x$-direction. (j)-(l): Combined vertical loading and seismic loading in two perpendicular directions.

all the elements forming the current structure, only the two walls parallel to the direction of the seismic forces ($+y$-axis) are actually interested by the effects of such forces, among all the vault supports.

Comparing the results shown in Figs. 2 and 3-4, we realize that the presence of perimeter walls in the current model leads us to design different topologies of the reinforcing elements, as compared to those predicted by the modeling of the vault as an independent structure constrained by fixed spherical hinges at the base. This is mainly due to the fact that the perimeter walls do not carry forces orthogonal to their planes in the current model, and therefore cannot be exactly replaced by spherical hinges. It is worth noting that the current model predicts major reinforcements over the perimeter walls, and lighter reinforcements over the surface of the vault, as compared to that employed in the previous section.

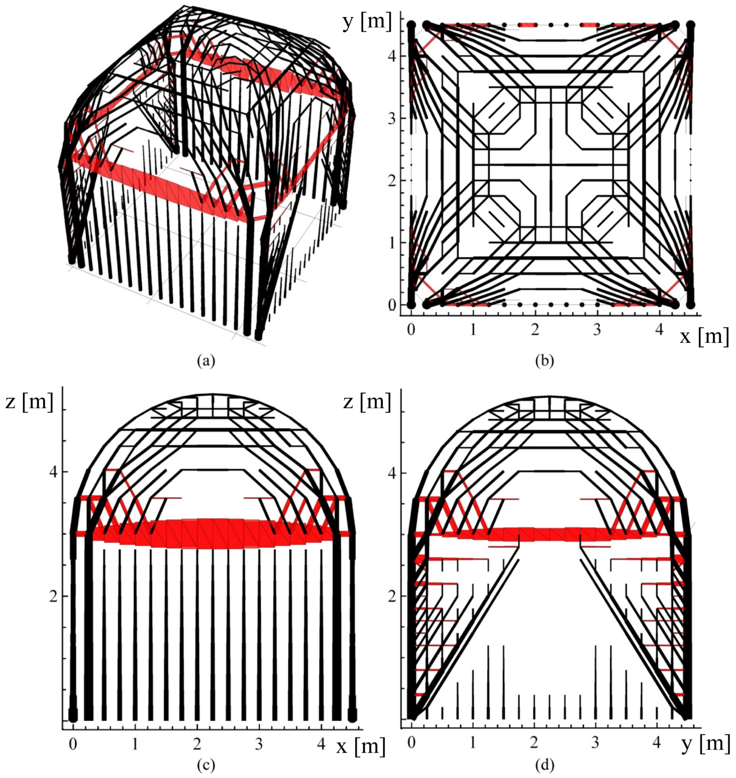

Figure 3: Optimal reinforcement patterns of the cloister vault supported by walls under vertical loading (reinforcements marked in red): (a) 3d view; (b) top view; (c) xz view; (d) yz view (α = 3.343, $V_f = 0.647 \times 10^{-3}$ m^3, $\mu_f = 0.567 \times 10^{-3}$). The widths of the reinforcements are magnified by a factor 2 for visual clarity.

4 Concluding remarks

We have presented an extension of the tensegrity approach formulated in Ref. [19] for the minimal mass reinforcement of masonry vaults and domes that do not react in tension. Such an extension allows us to analyze masonry structures of general shape and dimensions, including 2D walls, 3D walls, and structural complexes formed by an arbitrary combination of walls, vaults and domes.

The reinforcements analyzed in the present study consist of linear elements, such as, e.g., FRP-/FRCM-reinforcements, steel ties, timber beams, and any other reinforcements that are strong in tension. The adopted optimization approach allows us to design non-invasive reinforcement patterns, which can be able to preserve a sufficient crack-adaption capacity of the structure [10, 19, 27], under the respect of the equilibrium equations and material yield limits.

The given numerical results have highlighted that the proposed reinforcement design approach is able to handle both in-plane and out-of-plane loadings, walls with openings, and arbitrary support conditions of vaulted structures. It is worth remarking that the proposed strengthening approach matches the safe theorem of the limit analysis of elastic-plastic bodies [23, 10], and is in line with the recommendations of modern standards for the the design and construction of strengthening techniques for existing structures [21].

Future directions of the present study will be aimed at analyzing the minimal mass reinforcement of a variety of case-studies dealing with masonry structures of arbitrary geometry and complexity. Additional future research lines include the generalization of the proposed design approach to tensegrity materials and structures [28]-[34], and a wide campaign of exper-

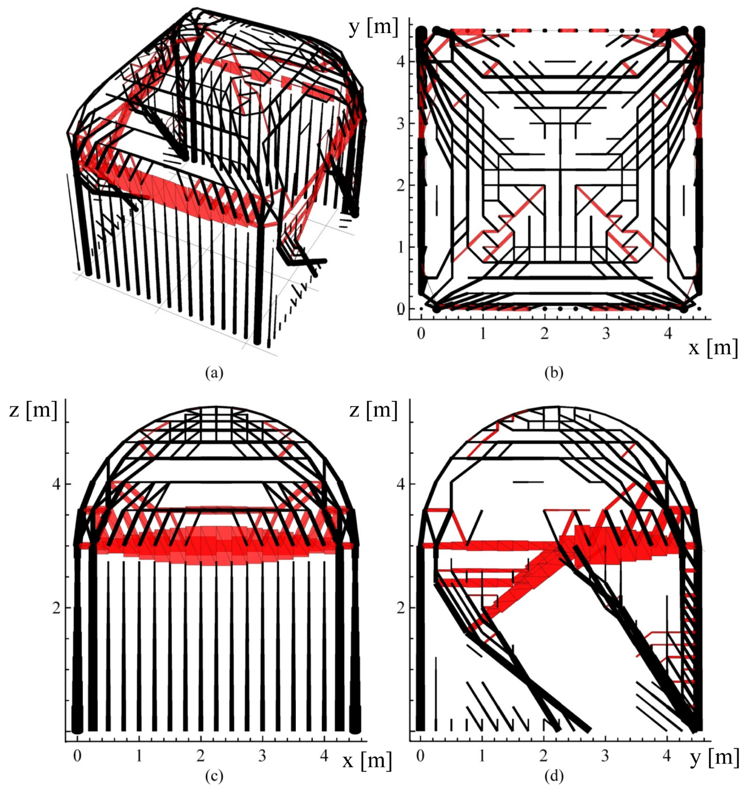

Figure 4: Optimal reinforcement patterns of the cloister vault supported by walls under combined vertical and seismic loadings in the +y-direction (reinforcements marked in red): (a) 3d view; (b) top view; (c) xz view; (d) yz view ($\alpha = 3.343$, $V_f = 1.010 \times 10^{-3}\ m^3$, $\mu_f = 0.807 \times 10^{-3}$). The widths of the reinforcements are magnified by a factor 2 for visual clarity.

imental validations of the design procedure presented in Sect. 2, through laboratory testing of real-scale and reduced-scale models under static and dynamic loading [35].

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