ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

MULTISCALE MODELING OF THERMO-ELASTIC PROPERTIES OF MICROCRACKED MATERIALS

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Keywords: Continua with microstructure, heterogeneous materials, non-classical thermal conduction, microtemperature, coarse-graining.

Abstract. *A general framework able to describe the mechanical behavior of thermo-elastic materials with microcracks is proposed. The main features of our model come from the definition of additional kinetics descriptors, both mechanical and thermal (multifield model). The model is thermodynamically consistent and turns out to be non-local, in that it retains memory of the fine material structure through internal lengths and dispersion properties. It is shown that the multifield model allows us to reveal the presence of the microcracks through a dispersive oscillating behavior and a temperature decrease of microcracks, circumventing well known problems related to standard wave propagation and heat conduction.*

1 Introduction.

The mechanical behavior of many materials of growing interest in materials science (such as composites, granular materials, alloys, liquid crystals) is often strongly influenced by an existing or emergent microstructure (such as microcracks, voids, defects, dislocations, phases in multiphase materials). Due to the different material properties of the microstructure, the macroscopic material turns out to be highly anisotropic and eventually inhomogeneous. Several approaches to the modeling of such materials can be found in the literature, and for a synthetic description the reader is referred to [30] and references therein.

A fine description of such materials often requires an excessive computational effort, otherwise, a gross description within the so-called *generalized continuum mechanics* can be more suitable, on extending the conventional mechanics in order to incorporate intrinsic microstructural effects in the mechanical behavior [19, 8, 10, 2]. In this contribution we fit into a general framework of this kind, able to describe the mechanical behavior of thermo-elastic materials with microcracks. In particular we propose a continuum model endowed with a mechanical and thermal structure defined for predicting the macroscopic behavior of a material with distributed microcracks (due to manufacturing defects or lack of cohesion). To fix ideas, we focus on porous metal-ceramic composites (MCC, CMC) such as tungsten or titanium/molybdenum carbides (WC/Co, TiC/Mo_2C), alumina/zirconia materials (Al_2O_3/ZrO_2).

Our model is connected to a discrete description of the matter, with the perspective to a proper constitutive identification. Besides the standard descriptors – displacement and temperature – additional terms are introduced in an enhanced principle of virtual powers formulation. These additional microstructural descriptors respectively represent: on the mechanical side, the mean jump of the displacement field due to the presence of microcracks; while on the thermal side, the nonstandard thermal state variable of the thermal displacement, firstly considered by Helmholtz, and an additional microstructural kinetic variable, whose time derivative represents the mean jump of the temperature field due to the presence of microcracks. This model is non-local, in that it retains memory of the fine material structure through internal lengths and dispersion properties, and can be considered as a *continuum with microstructure* according to the definition by Capriz [8].

The continuum here described, with just reference to the mechanical structure, has been proposed in [18] and developed in [31, 37, 30] with applications to particle/fiber reinforced polymer/ceramic matrix and masonry-like materials. Recently, the microstructure has been extended to the thermomechanical framework in [13], stemming from the ideas exposed in [11], where a simple and paradigmatic example of continua with micro-structure is exposed within the beams theory framework. An application of a two-temperatures model to rod theory can be also found in [1]. More in general, microtemperatures have been considered in [38] and the entropy balance postulated by Green and Naghdi [16], extensively discussed in [**?**, 6], is used to obtain a new theory of heat for materials with inner structure. A more exhaustive reference framework on this topic is reported in [13].

The PDEs governing equations obtained have been tested in a simple one–dimensional setting, where we have shown the potentiality of the theory laid down. The performed numerical tests show that the presence of microcracks has twofold effect: on one side it makes the displacement and the temperature field wave-like and oscillating, what reveals the presence of the microcracking itself; on the other side, it makes the final temperature lower than in the case without microcracks, as one would expect.

2 Discrete–to–scale–dependent continuum model: towards a constitutive identification

A model of generalized continuum, as the one we propose, may be computationally efficient provided that a proper physical meaning of the additional fields is provided and a constitutive identification for the internal and the external actions is performed [31, 37, 27, 33, 28, 3, 14, 20, 4, 12, 2, 36]. In [36, 29] it has been shown that the Principle of Virtual Powers (PVP) can be expedient to perform coarse-graining procedures in the framework of the homogenization techniques. These corpuscular-continuous approaches are based on the assumption that a selected microscopic level structure of matter can be described as discontinuous, and that the transition from the coarse scale is governed by an *a priori* map between the large set of degrees of freedom of discrete systems and the deformation fields of the continuum. In this sense the procedure we propose can be connected to homogenization techniques adopted to derive constitutive models with internal variables; nevertheless an important difference has to be pointed out: when dealing with these techniques an effective stiffness of the material is obtained, depending on the properties and the distribution of the microcracks (see for instance [7, 17, 23]); analogous results can be obtained for the effective thermal conductivity [24]. In our approach, both the classical mechanical and thermal properties are unaltered, while self-actions (both mechanical and thermal), related to additional material properties, are added. The results of the two different approaches can be compared in terms of macroscopic displacement [29] and temperature.

Here below a synthetic and revisited version of the developments in [13], is reported. For further details the reader is referred to the quoted article.

At the microscopic level, the material is supposed to be composed of two interacting superposed lattices; the former that we will call *macrolattice* and denote by $\mathcal{L}$, is constituted of rigid particles representing the matrix, connected in pairs by elastic and conductive links, which carry only axial forces; the second lattice that we will call *microlattice* and denote by $\mathcal{L}'$, is made of interacting slits of arbitrary shape with a predominant dimension; they are thought as a device able to transmit to the macrolattice additional forces and thermal interaction, due to crack opening displacement. The two lattices are linked together by elastic and conductive bonds; in a purely mechanical framework, this approach has already been used in [18].

Let $\mathcal{A}$ and $\mathcal{B}$ two interacting particles of the macrolattice, whose centers occupy the positions $\mathbf{a}$ and $\mathbf{b}$. Let us consider two slits $\mathcal{A}'$ and $\mathcal{B}'$, whose centers are individuated by the positions $\mathbf{a}'$ and $\mathbf{b}'$. We endow both lattices with two physical structures, the one mechanical the other thermal; for both the structures, we define the kinetic variables. Let $\mathbf{u}_i$ the relative displacement between two points $\mathbf{u}_i = \mathbf{u}^a - \mathbf{u}^b$ of $\mathcal{L}$. Let the crack opening displacement over $\mathcal{A}'$ be indicated by $\mathbf{d}^{a'}$, and let $\mathbf{d}_j = \mathbf{d}^{a'} - \mathbf{d}^{b'}$ be the relative displacement jump between two interacting slits $\mathcal{A}'$ and $\mathcal{B}'$. We denote by $\vartheta_i = \vartheta^a - \vartheta^b$ the difference in temperature between $\mathcal{A}$ and $\mathcal{B}$; the corresponding kinetic variable we consider is the difference in thermal displacement $\alpha_i = \alpha^a - \alpha^b$, defined as $\dot{\alpha}_i = \vartheta_i$. Let the jump of temperature between the two sides of the slit $\mathcal{A}'$ be $\theta^{a'}$, while the relative temperature jump between two interacting slits $\mathcal{A}'$ and $\mathcal{B}'$ be $\theta_j = \theta^{a'} - \theta^{b'}$; consistently, we define $\beta^{a'}$ and β_j, with $\dot{\beta}^{a'} = \theta^{a'}$ and $\dot{\beta}_j = \theta_j$.

The strain measures of the $\mathcal{L}$ is $\mathbf{u}_i$, while the strain measure for $\mathcal{L}'$ is $\mathbf{d}_j$; for each pair of interacting particle-slit, a further strain measure is assumed as the vector $\mathbf{u}_l = \mathbf{u}^a - (\mathbf{u}^{a'} + \gamma^{a'}\mathbf{d}^{a'})$, with γ a function of $(|\mathbf{a} - \mathbf{a}'|)$ and of the number of the particles interacting with $\mathcal{A}'$.

The mechanical interaction between two particles through the i-th bond are represented by the vectors $\mathbf{t}_i^a$ and $\mathbf{t}_i^b$, with $\mathbf{t}_i^a = -\mathbf{t}_i^b =: \mathbf{t}_i$; the corresponding thermal interaction is given by the scalar quantities h_i^a and h_i^b, with $h_i^a = -h_i^b =: h_i$. The vectors $\mathbf{z}_j^{a'}$ and $\mathbf{z}_j^{b'}$, with $\mathbf{z}_j^{a'} = -\mathbf{z}_j^{b'} =: \mathbf{z}_j$ indicate the interactions between two slits along the j-th direction; the corresponding

thermal interactions are given by the scalar quantities $h_j^{a'}$ and $h_j^{b'}$, with $h_j^{a'} = -h_j^{b'} =: h_j$. The interacting force between the l-th bond will be denoted by $\mathbf{m}_l^a = -\mathbf{m}_l^{a'} = -\mathbf{m}_l$. The force due to the displacement $\mathbf{d}^{a'}$ is represented by the vector $\mathbf{z}_0^{a'}$; the zero-th order thermal interaction corresponding to the temperature of the particle $\mathcal{A}$ is denoted by h_0^a, those corresponding to the temperature of the particles $\mathcal{A}'$ is denoted by $h_0^{a'}$. All the defined fields are considered time-dependent.

The dynamics of the two lattices is specified by an extended notion of the internal virtual power, defined over the collection of virtual velocities ('primal' variables)

$$(\delta\mathbf{u}_i, \delta\mathbf{d}^{a'}, \delta\mathbf{d}_j; \delta\alpha^a, \delta\alpha_i, \delta\beta^{a'}, \delta\beta_j,), \tag{1}$$

defined as follows:

$$\begin{aligned}\delta\pi[(\delta\mathbf{u}_i, \delta\mathbf{d}^{a'}, \delta\mathbf{d}_j; \delta\alpha^a, \delta\alpha_i, \delta\beta^{a'}, \delta\beta_j,)] = \int_{\mathcal{T}} \Big(\sum_a h_0^a \delta\alpha^a + \sum_i (\mathbf{t}_i \cdot \delta\mathbf{u}_i + h_i \delta\alpha_i) \\ + \sum_{a'} (\mathbf{z}_0^{a'} \cdot \delta\mathbf{d}^{a'} + h_0^{a'} \delta\beta^{a'}) + \sum_j (\mathbf{z}_j \cdot \delta\mathbf{d}_j + h_j \delta\beta_j) + \sum_l \mathbf{m}_l \cdot \delta\mathbf{u}_l \Big),\end{aligned} \tag{2}$$

where the summations are extended to all the a particles, a' slits, i bonds between two generic elements of $\mathcal{L}$, j bonds between two generic elements of $\mathcal{L}'$ and all the l links between an element of $\mathcal{L}$ and an element of $\mathcal{L}'$; $\mathcal{T}$ is the observation time interval.

Consistently with the molecular theory of elasticity [9], to account for short range interactions, it is sufficient to consider homogeneous deformations by assuming that the lattice descriptors are connected to continuum fields as:

$$\begin{aligned}\mathbf{u}^a &= \mathbf{u}(\mathbf{x}) + \nabla\mathbf{u}(\mathbf{x})(\mathbf{a} - \mathbf{x}), \\ \mathbf{d}^{a'} &= \mathbf{d}(\mathbf{x}) + \nabla\mathbf{d}(\mathbf{x})(\mathbf{a}' - \mathbf{x}), \\ \alpha^a &= \alpha(\mathbf{x}) + \nabla\alpha(\mathbf{x}) \cdot (\mathbf{a} - \mathbf{x}), \\ \beta^{a'} &= \beta(\mathbf{x}) + \nabla\beta(\mathbf{x}) \cdot (\mathbf{a}' - \mathbf{x}),\end{aligned} \tag{3}$$

where: $\mathbf{u}$ is the *mechanical displacement*, whose time derivative $\mathbf{v} := \dot{\mathbf{u}}$ is the *velocity*; $\mathbf{d}$ is the *mechanical micro-displacement* $\mathbf{d}$, whose time derivative is the *micro-velocity* denoted by $\mathbf{l} := \dot{\mathbf{d}}$; α is the *thermal displacement*, whose time derivative $\vartheta := \dot{\alpha}$ represents the *absolute temperature*; β is the *thermal micro-displacement*, whose time derivative is $\theta := \dot{\beta}$, that we call *micro-temperature*. The field $\mathbf{d}$ is the perturbation induced on the regular macroscopic displacement field by the mean displacement jump which it would have over the actual microcrack; it can be interpreted as the difference between the actual displacement field in the body with microcracks and the regular displacement field of the same body without microcracks. The origin of this interpretation is investigated in [18] and [30]. Analogously, β is the perturbation induced on the regular macroscopic temperature field by the mean temperature jump which it would have over the actual microcrack as introduced in [13].

Thus, the following strain measures of the lattice system can be defined:

$$\begin{aligned}&\mathbf{u}_i = \nabla\mathbf{u}(\mathbf{a} - \mathbf{b}), \quad \mathbf{d}^{a'} = \nabla\mathbf{d}(\mathbf{a}' - \mathbf{b}'), \quad \mathbf{u}_l = \nabla\mathbf{u}(\mathbf{a} - \mathbf{a}') - \gamma^{a'}\big(\mathbf{d}(\mathbf{x}) + \nabla\mathbf{d}(\mathbf{a}' - \mathbf{x})\big), \\ &\alpha_i = \nabla\alpha \cdot (\mathbf{a} - \mathbf{b}), \quad \beta_j = \nabla\beta \cdot (\mathbf{a}' - \mathbf{b}'),\end{aligned} \tag{4}$$

where, for sake of simplicity, we omitted the dependence on $\mathbf{x}$. The above assumptions correspond to the hypothesis of regularity on the influence of microcracks on the deformation of

the body and come out from a generalized Cauchy-Born relation used to by-pass the intrinsic limitation of local descriptions avoiding the resort to non-homogeneous deformations [32, 30]. Considering the maps (3) and after some algebra, the mean virtual power of the system can be written as:

$$\delta\Pi[(\delta\mathbf{u},\delta\mathbf{d},\delta\alpha,\delta\beta)] = \frac{1}{V}\delta\pi[(\delta\mathbf{u}_i,\delta\mathbf{d}^{a'}\delta\mathbf{d}_j;\delta\alpha^a,\delta\alpha_i,\delta\beta^{a'},\delta\beta_j,)]$$
$$= \int_{\mathcal{T}} \big(\mathbf{S}\cdot\nabla\delta\mathbf{u} + \mathbf{z}\cdot\delta\mathbf{d} + \mathbf{Z}\cdot\nabla\delta\mathbf{d} + h\delta\alpha + \bar{\mathbf{h}}\cdot\nabla\delta\alpha + k\delta\beta + \bar{\mathbf{k}}\cdot\nabla\delta\beta\big), \tag{5}$$

where:

$$\begin{aligned}
\mathbf{S} &= \frac{1}{V}\Big(\sum_i \mathbf{t}_i\otimes(\mathbf{a}-\mathbf{b}) + \sum_l \mathbf{m}_l\otimes(\mathbf{a}-\mathbf{a}')\Big),\\
\mathbf{z} &= \frac{1}{V}\Big(\sum_{a'} \mathbf{z}_0^{a'} - \sum_{a'}\gamma^{a'}\mathbf{m}_l\Big),\\
\mathbf{Z} &= \frac{1}{V}\left(\sum_{a'}\mathbf{z}_0^{a'}\otimes(\mathbf{a}'-\mathbf{x}) + \sum_j \mathbf{z}_j\otimes(\mathbf{a}'-\mathbf{b}') - \sum_l \gamma^{a'}\mathbf{m}_l\otimes(\mathbf{a}'-\mathbf{a})\right),\\
h &= \frac{1}{V}\sum_a h_0^a,\\
k &= \frac{1}{V}\sum_{a'} h_0^{a'},\\
\bar{\mathbf{h}} &= \frac{1}{V}\sum_i h_i(\mathbf{a}-\mathbf{b}),\\
\bar{\mathbf{k}} &= \frac{1}{V}\left(\sum_{a'} h_0^{a'}(\mathbf{a}'-\mathbf{x}) + \sum_j h_j(\mathbf{a}'-\mathbf{b}')\right),
\end{aligned} \tag{6}$$

are the interaction measures and V is the volume of a representative volume element (RVE), easily to be defined if we assume a periodic microstructure but also detectable for non periodic assemblies [34, 35]. Note that the fabric vectors $\mathbf{a}'-\mathbf{x}$, $\mathbf{a}'-\mathbf{b}'$ and $\mathbf{a}'-\mathbf{a}$ account for the geometry of the microstructure, while the directions of the forces $\mathbf{z}_0^{a'}$ and $\mathbf{z}_j$ account for the orientation of the slits. On assuming the equivalence of the mean virtual power of the lattice system $\delta\Pi$ (5) with the virtual power density of a corresponding multifield continuum:

$$\delta w[(\delta\mathbf{u},\delta\mathbf{d},\delta\alpha,\delta\beta)] = \int_{\mathcal{T}} \big(\mathbf{S}\cdot\nabla\delta\mathbf{u} + \mathbf{z}\cdot\delta\mathbf{d} + \mathbf{Z}\cdot\nabla\delta\mathbf{d} + h\delta\alpha + \bar{\mathbf{h}}\cdot\nabla\delta\alpha + k\delta\beta + \bar{\mathbf{k}}\cdot\nabla\delta\beta\big), \tag{7}$$

by means of the localization theorem, we can recognize that Equations (6) define the stress and the thermodynamic measures of the equivalent continuum. $\mathbf{S}$, $\mathbf{z}$, $\mathbf{Z}$, h, $\bar{\mathbf{h}}$, k, $\bar{\mathbf{k}}$ are measures of the *mechanical* and *thermal interactions* that a point of the continuum body exchanges with its adjacent part, accounting for both macro- and micro-description, dual to the macroscopic kinetic variables.

Then, by assuming constitutive response functions for the mechanical and thermal interactions and in the discrete model we can obtain the effective constitutive relations of the multifield

continuum in the form [13]:

$$\begin{aligned}
&\mathbf{S} = \mathbb{C}\mathbf{E} + \mathbb{D}\nabla\mathbf{d} + (\vartheta - \bar{\vartheta})\mathbf{M} + (\theta - \bar{\theta})\mathbf{\Gamma},\\
&\mathbf{z} = \mathbf{G}\mathbf{d},\\
&\mathbf{Z} = \mathbb{R}\nabla\mathbf{d} + \mathbb{D}\mathbf{E} + (\vartheta - \bar{\vartheta})\mathbf{N} + (\theta - \bar{\theta})\mathbf{\Lambda},\\
&\mathbf{h} = -\chi\nabla\alpha - \nu\nabla\beta,\\
&\mathbf{k} = -\lambda\nabla\beta - \nu\nabla\alpha,
\end{aligned} \tag{8}$$

where $\bar{\vartheta}$ and $\bar{\theta}$ are given reference temperatures and $\mathbb{C}$, $\mathbb{D}$, $\mathbf{M}$, $\mathbf{\Gamma}$, $\mathbf{G}$, $\mathbb{R}$, $\mathbf{N}$, $\mathbf{\Lambda}$ are the constitutive terms, mechanical and thermal. The quantities h and k in 6 have the role of dissipation (see [13] for details on this issue) and then, as it customarily happens in classical thermo-elasticity, we set:

$$h = \mathbf{q}\cdot\nabla\vartheta, \quad k = \widetilde{\mathbf{q}}\cdot\nabla\theta, \tag{9}$$

where $(\mathbf{q}, \widetilde{\mathbf{q}})$ are the *heat influxes*, that we set, as usually done [5, 22], proportional to the entropy influxes through the temperature as follows:

$$\mathbf{q} = \vartheta\mathbf{h}, \quad \widetilde{\mathbf{q}} = \theta\mathbf{k}. \tag{10}$$

3 Balance equations

The structure of the continuum body identified is encoded in the virtual power density formula (7), and it is perceived ad a continuum with local vector structure.

Following the approach described in [13], by requiring the equivalence of the virtual internal and external power of such a continuum (generalized Principle of Virtual Power as in [21]) we derive following local balance equations:

(i) *momenta balances*:

$$\begin{aligned}
&\dot{\mathbf{p}} = \operatorname{div}\mathbf{S} + \mathbf{b},\\
&\dot{\mathbf{r}} = \operatorname{div}\mathbf{Z} - \mathbf{z} + \mathbf{g};
\end{aligned} \tag{11}$$

(ii) *entropies balances*:

$$\begin{aligned}
&\dot{\eta} = -\operatorname{div}\mathbf{h} - h + b,\\
&\dot{\omega} = -\operatorname{div}\mathbf{k} - k + g,
\end{aligned} \tag{12}$$

in $\mathcal{P}\times\mathcal{T}$, $\mathcal{P}$ being an arbitrary part of the body, $\mathbf{h} := -\bar{\mathbf{h}}$ and $\mathbf{k} := -\bar{\mathbf{k}}$, together with the boundary conditions:

$$\mathbf{S}\mathbf{n} = \mathbf{s}, \quad \mathbf{Z}\mathbf{n} = \mathbf{y}, \quad \bar{\mathbf{h}}\cdot\mathbf{n} = s, \quad \bar{\mathbf{k}}\cdot\mathbf{n} = y, \tag{13}$$

on $\partial\mathcal{P}\times\mathcal{T}$ and the initial conditions:

$$\mathbf{p}(x,t_i) = \mathbf{p}_i(x), \quad \mathbf{r}(x,t_i) = \mathbf{r}_i(x), \quad \eta(x,t_i) = \eta_i(x), \quad \omega(x,t_i) = \omega_i(x) \tag{14}$$

for $x\in\mathcal{P}$, where the fields $\mathbf{b}$, $\mathbf{g}$, b and g account for both the macro and the micro external *at-a-distance interactions*, respectively mechanical and thermal; the fields $\mathbf{s}$, $\mathbf{y}$, s and y account for the *contact interactions*; $\mathbf{p}$ is interpreted as the *linear momentum*, $\mathbf{r}$ the *microscopic linear momentum*, η the *entropy*, ω the *micro-entropy*, being all these fields defined per unit referential volume. As Equations (12) reveal, $\mathbf{h}$ and $\mathbf{k}$ can be interpreted as the *entropy* and *micro-entropy influxes*, respectively; the fields h and k are measures of *internal dissipation*, as anticipated in

the previous section. The extended version of the PVL, whence (11) and (12) come out, is based upon the introduction of: (i) a space-time integral, responsible of the appearance of time derivatives, and (ii) those thermal kinetic variables already introduced in the discrete system.

On substituting (8), (9) in (11) and (12), we obtain a system of partial differential equations in the unknowns $\mathbf{u}$, $\mathbf{d}$, α and β. It is worth noting that if, in order to have a linear system of PDEs, we neglect the quadratic quantities [15] and, in order to have a system in terms of the temperature where more meaningful boundary conditions can be setted, by differenting with respect to time the heat equations (12) become:

$$\begin{aligned} c\ddot{\vartheta} &= \chi\Delta\vartheta + \nu\Delta\theta + \mathbf{M}\cdot\nabla\ddot{\mathbf{u}} + \mathbf{N}\cdot\nabla\ddot{\mathbf{d}}, \\ \xi\ddot{\theta} &= \lambda\Delta\theta + \nu\Delta\vartheta + \mathbf{\Gamma}\cdot\nabla\ddot{\mathbf{u}} + \mathbf{\Lambda}\cdot\nabla\ddot{\mathbf{d}}. \end{aligned} \tag{15}$$

4 Numerical simulations

In this Section we present some numerical results confining our attention to a one-dimensional body $\mathcal{C}$ – i.e. a bar – whose typical point x lies on a line of direction $\mathbf{e}$.

We focus the attention to a conductive bar without microcracks versus a conductive bar with microcracks, referring the reader to [37] for underlying the ability of the purely mechanical multifield model to account for the presence of distributed microcracks. In this simple setting, the momenta balances (11) and the heat equations (15) read:

$$\begin{aligned} &\varrho\ddot{u} - Cu'' - Dd'' - M\theta' - \Gamma\theta' = 0, \\ &\mu\ddot{d} - Du'' - Rd'' + Gd - N\vartheta' - \Lambda\theta' = 0, \\ &c\ddot{\vartheta} = \chi\vartheta'' + \nu\theta'' + M\ddot{u}' + N\ddot{d}', \\ &\xi\ddot{\theta} = \lambda\theta'' + \nu\vartheta'' + \Gamma\ddot{u}' + \Lambda\ddot{d}', \end{aligned} \tag{16}$$

where we have set $\mathbf{p} = \varrho\ddot{u}\mathbf{e}$ and $\mathbf{r} = \mu\ddot{d}\mathbf{e}$, with ϱ and μ mass densities per unit length [37].

In order to evaluate the effects of microcracks and microtemperature in heat conduction, we here assume the coupling terms between the purely mechanical fields and the others are null (i.e. $M = D = \Gamma = 0$). As to the initial and boundary conditions for the temperature fields, we set

$$\begin{aligned} &\vartheta(x,0) = 0, \quad \vartheta(a,t) = t^2, \quad \vartheta(b,t) = 0, \quad \dot{\vartheta}(x,0) = 0, \\ &\theta(x,0) = 0, \quad \theta(a,t) = \frac{t^2}{100}, \quad \theta(b,t) = 0, \quad \dot{\theta}(x,0) = 0. \end{aligned} \tag{17}$$

The choice to set for the micro-temperature at the point a a value of $1/100$ of the macro-temperature is suggested by the physical requirement that the effects of the mean jump of the temperature due to microcracks is much smaller than the mean temperature itself. We define as visible temperature the field $\vartheta(x,t) - \theta(x,t)$.

In Fig. 1 we plot the results of our model vs the temperature in a bar without cracks, satisfying the following heat equation, consistent with $(16)_4$

$$c\ddot{\vartheta} = \chi\vartheta''. \tag{18}$$

If we compare the results of (18) with the model with microtemperature, we note that the presence of the microcracks has a double effect: on one side it makes the temperature field wave-like and oscillating; on the other side, it makes the final temperature lower than in the case without microcracks, as one would expect. Moreover, it is possible to show that the wave

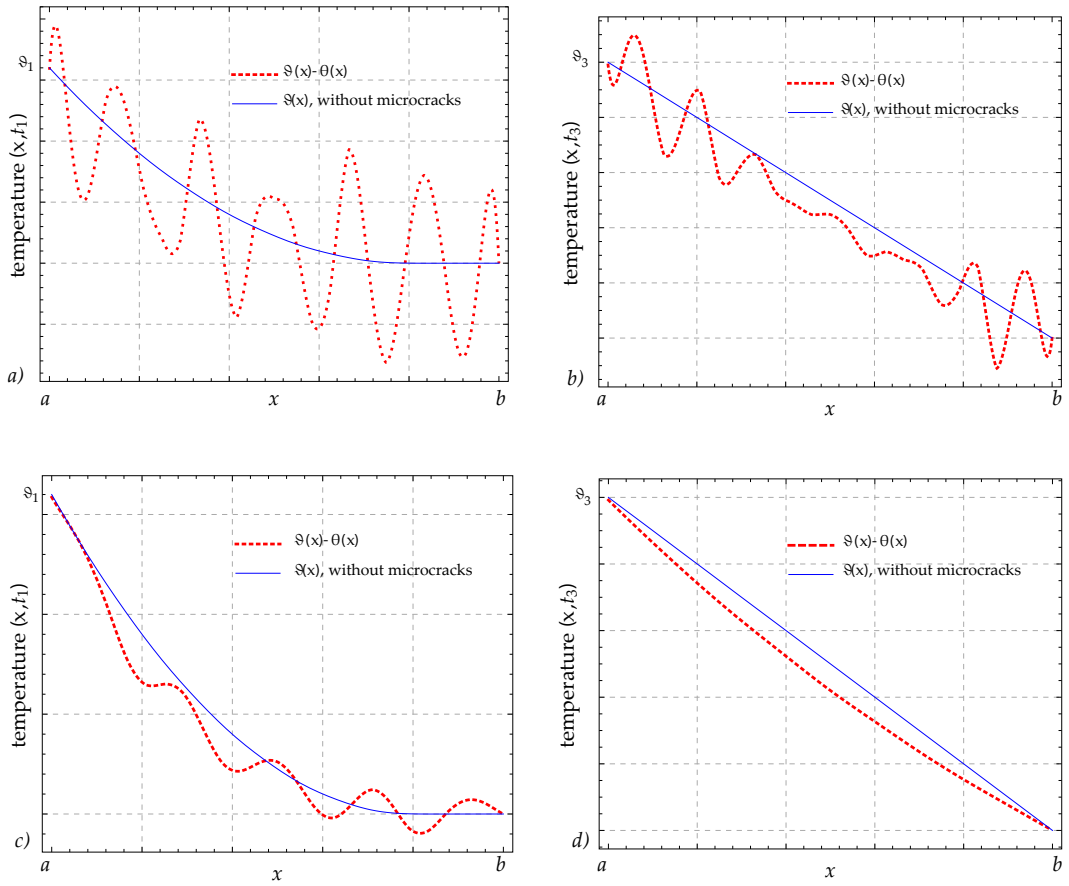

Figure 1: (a) and (b) Temperature in a bar without cracks (ϑ, continue blue line) and visible temperature in a bar with microcracks ($\vartheta - \theta$, dotted red curve), at two different time steps. (c) and (d) Same simulation with higher density of microcracks.

length is related to the microcracks density, what modifies the oscillatory behavior (see Fig. (1) (c)-(d)).

5 Final remarks

We have proposed a multiscale approach, within the framework of the generalized continuum thermo-mechanics, able to investigate the mechanical and thermal properties of a microcracked solid. The model is connected to a discrete description of the matter, with a perspective to a proper constitutive identification. Besides the standard descriptors – displacement and temperature – additional terms are introduced, able to take into account the presence of microcracks. The model proposed is non-local, in that it retains memory of the fine material structure through internal lengths and dispersion properties. The PDEs system obtained has been tested in a simple one-dimensional setting, where we have shown the potentiality of the theory laid down. Focusing in particular on the the thermal properties, we have shown that our model is able to accommodate a non-standard thermal propagation; our numerical tests show that the presence of microcracks has twofold effect: on one side it makes the temperature field wave-like and oscillating, what reveals the presence of the microcracking itself; on the other side, it makes the final temperature lower than in the case without microcracks, as one would expect.

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