ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

THEORETICAL PREDICTION OF PLASTIC INSTABILITY FOR ASYMMETRICAL MATERIALS

Shenghua Wu[1], Nannan Song[2], and F.M.Andrade Pires[2]

[1] Faculty of Engineering, University of Porto
Rua Dr. Roberto Frias 4200-465 Porto, Portugal
e-mail: {shenghua,fpires}@fe.up.pt

[2] INEGI, University of Porto
Rua Dr. Roberto Frias 4200-465 Porto, Portugal
snannan@inegi.up.pt

Keywords: Forming Limit Diagram (FLD), Plastic instability, Necking, Yield function.

Abstract. *The prediction of formability is one of the most important tasks in sheet metal process simulation. The forming limit diagram (FLD) is an important tool that is used to understand and characterize the formability of sheet metals. This method meets both manufacturer and user's requirements and is widely used in factory and research laboratories* [1, 2]. *However, experimental measurements and determination of the FLD is costly and time consuming. Therefore analytical and numerical predictions based on the theory of plastic instability allow determining the critical value under different loading paths and has already attracted significant attention for formability evaluation.*

In this contribution, a mathematical formulation is derived by using Swift and Hill analytical theory. The formulation is numerically implemented in a user developed FLD MATLAB script to predict diffuse and localized necking. This work also presents and discusses the influence of different yield surface and asymmetrical effect on the formability under different loading strain path.

1 INTRODUCTION

In sheet metal forming, necking and wrinkling are commonly observed failure. In order to describe the occurrence of necking and present the formability, a most realistic and general method was introduced at the beginning of 1960s by Keeler [1] and Goodwin [2] who proposed the concept of Forming Limit Diagram, which involved a wide range of forming limit strains of a homogeneous material at different strain-states (from uniaxial tension, to plane strain tension and biaxial tension). The forming limit strain was constructed with experiments on various proportional loading paths by measuring the distortions of small grids pre-marked on the sheet surface [3-5]. However, due to the time-consuming, costly and tedious experimental work involved in FLD measurement, a theoretical prediction of the FLD has become popular and essentially important in sheet metal forming and the related industries.

Since the 1950s, based on different failure criteria, a number of analytical/theoretical models for predicting FLDs have been developed [6]. According to the concept the limit strains can be calculated on the basis of certain plastic instability criteria. One of the first criteria made in order to evaluate diffuse instability and evaluate formability of sheet material was proposed by Swift in 1952 [7]. He estimates diffuse necking to occur for an isotropic material when the major principal strain reaches limit values. This swift criterion is limited to isotropic material. To overcome this limitation Moore and Wallace [8] expanded the criterion to take into account anisotropic behavior using Hills's 48 yield criterion. Hill in 1952 [9] found that the forming limits predicted by the diffuse plastic instability criterion underestimate the forming capacity for the left hand side of forming limit diagram. To overcome this underprediction, Hill proposed a localized plastic instability criterion, which states that a local instability can occur only after the formation of a diffuse instability, the necking direction is coincident with the direction of zero-elongation, and gave the final limited strain formulation for isotropic materials. This study led to the well-known zero extension assumption, i.e. that the localization band develops along the zero extension direction in a sheet metal. This analysis predicted that localized necking would not occur in a uniform sheet, subject to positive biaxial stretching for which no zero extension direction exists. Therefore, Hill's criterion is only applicable to the LHS of the FLD. J. Majak *et al* [10] and Dudzinski et al [11] proposed a simple algorithm for anisotropic yield criteria, however, this algorithm cannot capture the shape evolution of yield surface and strength differential (SD) effect for asymmetrical material.

The purpose of this contribution is to extend the algorithm proposed by Majak and develop new theories to capture the anisotropic and asymmetrical mechanical behavior simultaneously. This algorithm is used for the limited strain determination in connection with the Swifts instability condition for diffuse necking and the Hills instability condition for localized necking.

2 THEORETICAL FRAMEWORK

2.1 Yield function

To describe both the asymmetry between tension and compression and the incompressible plastic anisotropy observed in HCP metal sheets, Cazacu and Barlat [12] introduced a general and rigorous method which is based on the theory of representation of tensor functions. A 4th order linear transformation is operated on the stress deviator $\boldsymbol{S}$ to obtain the transformed stress $\boldsymbol{\Sigma}$, which can be defined as

$$\boldsymbol{\Sigma} = \boldsymbol{CS} \tag{1}$$

where $\boldsymbol{C}$ is a constant 4th order tensor, which includes 9 independent anisotropy coefficients, Let (x,y,z) be the reference frame associated with orthotropy. In the case of a sheet, x, y and

z represent the rolling, transverse, and the normal directions. Relative to the orthotropy axes (x,y,z), the tensor $\boldsymbol{C}$ is represent by

$$C=\begin{bmatrix} C_{11} & C_{11} & C_{11} & 0 & 0 & 0 \\ C_{11} & C_{11} & C_{11} & 0 & 0 & 0 \\ C_{11} & C_{11} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{11} \end{bmatrix} \tag{2}$$

Thus, the final orthotropic yield function can be given by

$$\bar{\sigma}=\left[\left(\left|\Sigma_1\right|-k\Sigma_1\right)^a+\left(\left|\Sigma_2\right|-k\Sigma_2\right)^a+\left(\left|\Sigma_3\right|-k\Sigma_3\right)^a\right]^{1/a} \tag{3}$$

where Σ_1, Σ_2, Σ_3 are the principal values of $\boldsymbol{\Sigma}$. The only restrictions imposed on the tensor $\boldsymbol{C}$ are: (i) to satisfy the major and minor symmetries and (ii) to be invariant with respect to the orthotropy group. Thus, for 3-D stress conditions the orthotropic criterion involves 9 independent anisotropy coefficients; it reduces to the isotropic criterion when $\boldsymbol{C}$ equal to the 4th order identify tensor. It is worth noting that although the transformed tensor is not deviatoric, the orthotropic criterion is insensitive to hydrostatic pressure and thus the condition of plastic incompressibility is satisfied. For $k\in[-1,1]$ and any integer $a\geq 1$, the anisotropic yield function is convex in the variables Σ_1, Σ_2, Σ_3 (principal transformed stresses).

For the sheet metal forming, the material is considered to behave as an orthotropic membrane under the plane stress conditions, thus the stress and strain component in the third direction are vanished, as

$$\sigma_{33}=0,\sigma_{13}=0,\sigma_{23}=0 \tag{4}$$

Thus the transformed stress can be rewritten as

$$\Sigma=\begin{bmatrix} \phi_1\sigma_{11}+\varphi_1\sigma_{22} \\ \phi_2\sigma_{11}+\varphi_2\sigma_{22} \\ \phi_3\sigma_{11}+\varphi_3\sigma_{22} \\ C_{44}\sigma_{12} \end{bmatrix} \tag{5}$$

In this situation, the principle stress can be rewritten as

$$\begin{aligned} \Sigma_1 &= \frac{(\phi_1+\phi_2)\sigma_{11}+(\varphi_1+\varphi_2)\sigma_{22}}{2}+\sqrt{\left[\frac{(\phi_1-\phi_2)\sigma_{11}+(\varphi_1-\varphi_2)\sigma_{22}}{2}\right]^2+C_{44}^2\sigma_{12}^2} \\ \Sigma_2 &= \frac{(\phi_1+\phi_2)\sigma_{11}+(\varphi_1+\varphi_2)\sigma_{22}}{2}-\sqrt{\left[\frac{(\phi_1-\phi_2)\sigma_{11}+(\varphi_1-\varphi_2)\sigma_{22}}{2}\right]^2+C_{44}^2\sigma_{12}^2} \\ \Sigma_3 &= \phi_3\sigma_{11}+\varphi_3\sigma_{22} \end{aligned} \tag{6}$$

where

$$\phi_1 = \frac{1}{3}\left(2C_{11} - C_{12} - C_{13}\right)$$
$$\phi_2 = \frac{1}{3}\left(2C_{12} - C_{22} - C_{23}\right)$$
$$\phi_3 = \frac{1}{3}\left(2C_{13} - C_{23} - C_{33}\right)$$
$$\varphi_1 = \frac{1}{3}\left(-C_{11} + 2C_{12} - C_{13}\right)$$
$$\varphi_2 = \frac{1}{3}\left(-C_{12} + 2C_{22} - C_{23}\right)$$
$$\varphi_3 = \frac{1}{3}\left(-C_{13} + 2C_{23} - C_{33}\right)$$

Since the effective stress $\bar{\sigma}$ is the first order homogeneous function in stresses, from the work equivalence principle it follows that the law of evolution for the effective plastic strain (associated with $\bar{\sigma}$) reduces to $d\bar{\varepsilon} = d\gamma$.

2.2 Plastic instability formulation

The first instability criterion was proposed by Considère in 1885 [13], who analyses the formation of the necking in tensile specimens. The geometrical construction is given by the relationship

$$F = \sigma A \tag{7}$$

where F, σ and A are the force, stress and cross-sectional area respectively, and by the total differential

$$dF = \sigma dA + d\sigma A \tag{8}$$

Under tensile conditions, the force-displacement curves presents a local maximum at the beginning of the neck formation, $dF = 0$, then equations yields

Swift used Considère's criterion to determinate the limit strains in biaxial tension. The sheet element was analyzed for two perpendicular directions. Therefore the diffuse necking should be occur when the force differentials at these two perpendicular directions are equals to zero, as

$$\begin{aligned} dF_1 &= 0 \\ dF_2 &= 0 \end{aligned} \tag{9}$$

After a mathematical manipulation by taking the plastic incompressibility into account, the diffuse constraints can be written as

$$\begin{aligned} d\sigma_1 &= \sigma_1 d\varepsilon_1 \\ d\sigma_2 &= \sigma_2 d\varepsilon_2 \end{aligned} \tag{10}$$

Combining with proportional loading constraints, the Swift's diffuse necking condition can be presented in tensor notations as

$$d\sigma_i = \sigma_1 A^{Swift} d\varepsilon_j \tag{11}$$

where the Swift's instability tensor can be calculated by

$$A_{ij}^{Swift}\left(\alpha\right) = \begin{bmatrix} 1 & 0 \\ \alpha & 0 \end{bmatrix}.$$

where α represents the stress ratio ($\alpha = \sigma_2 / \sigma_1$).

For analysis of the negative minor strain region of FLD, the localized necking develops along one direction, which is inclined with respect to the loading direction. Hill's localized necking condition is employed. According to Hill's instability condition the localized necking (through thickness neck) occurs when the rate of strain hardening is equal to the rate of geometric softening.

$$dT = d\sigma_1 t + \sigma_1 dt = 0 \tag{12}$$

where t represents the thickness of the sheet.

Taking the strain though thickness place of in-plane strain, the constraints of localized necking are written as following

$$\begin{aligned} d\sigma_1 &= \sigma_1\left(d\varepsilon_1 + d\varepsilon_2\right) \\ d\sigma_2 &= \alpha d\sigma_1 \end{aligned} \tag{13}$$

Similarly, we can written it as a compact form, the Hill's localized necking condition reads

$$d\sigma_i = \sigma_1 A_{ij}^{Hill} d\varepsilon_j \tag{14}$$

where A_{ij}^{Hill} is the Hill's instability tensor, which can be calculated by

$$A_{ij}^{Hill}\left(\alpha\right) = \begin{bmatrix} 1 & 1 \\ \alpha & \alpha \end{bmatrix} \tag{15}$$

After comparison with Swift instability tensor, it is clear seen that two instability condition has similar formulation, so here we can written it as a generic way by

$$d\sigma_i = \sigma_1 A^{instability} d\varepsilon_j \tag{16}$$

where $A^{instability}$ is taken equal with A_{ij}^{Hill} and A^{Swift}, respectively.

The mechanical response of the sheet metal will be described by a rigid-plastic model. Hence for the total strains and total strain increments are equal to the corresponding plastic strains and plastic strain increments, respectively. Given the strain history from the previous step, the effective strain increment at current step, strain path β (stress path α), stress, strain, strain increment in region (a) can be calculated. The main ingredient of the constitutive model is the yield function:

$$\Phi = \bar{\sigma}\left(\sigma_{11}, \sigma_{22}\right) - \sigma_Y\left(\bar{\varepsilon}\right) = 0 \tag{17}$$

where σ_Y represent equivalent stress and is calculated from the hardening law.

If the normality flow rule is assumed to hold for plastic flow of materials, according to the consistency condition $d\Phi = 0$, the differential yield function can be written as

$$d\Phi = \frac{\partial\bar{\sigma}}{\partial\bar{\varepsilon}} d\bar{\varepsilon} + \frac{\partial\bar{\sigma}}{\partial\sigma_i} d\sigma_i - \frac{\partial\sigma_Y}{\partial\bar{\varepsilon}} d\bar{\varepsilon} = 0 \tag{18}$$

By applying classical plasticity theory the instability criterion is derived in above subsection, the plastic anisotropy, associated plastic flow law and hardening parameters, the final plastic instability function should be written as

$$\frac{1}{\bar{\sigma}}\frac{\partial\bar{\sigma}}{\partial\bar{\varepsilon}} + \frac{1}{f}\frac{\partial\bar{\sigma}}{\partial\sigma_i} A_{ij}^{instability} \frac{\partial\bar{\sigma}}{\partial\sigma_j} - \frac{1}{\sigma_Y}\frac{\partial\sigma_Y}{\partial\bar{\varepsilon}} = 0 \tag{19}$$

where the function f is given by ratio $f = \bar{\sigma}/\sigma_1$.

Due to the equivalent stress is a homogeneous function of degree one with respect to the stress components, the first item of the above equation is only dependent on the equivalent plastic strain. After solving this equation by using iterative methods, the limit strains $\bar{\varepsilon}^*$ can be obtained.

3 RESULTS AND DISCUSSION

The material used in this contribution was high-purity (99%) titanium. The initial texture of the as received material has a basal texture, with the majority of the grains having their c-axis at 30^0 to the normal to the plane of the plate. It is considered that the materials'hardening is isotropic and governed by the equivalent plastic strain according to a power-law:

$$\sigma_Y = K\left(\varepsilon_0 + \bar{\varepsilon}\right)^n \tag{20}$$

where K, ε_0 and n are materials parameters.

All the material parameters can be seen in Table 1 and Table 2.

$\sigma_Y = K\left(\varepsilon_0 + \bar{\varepsilon}\right)^n$		
K	ε_0	n
413MPa	0.6445	1.0

Table 1: Materials parameters of pure titanium.

$\bar{\varepsilon}$	k	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}	C_{44}
0.02	-0.304	0.971	1.316	0.022	0.189	0.152	0.972
0.05	-0.313	0.989	1.243	0.089	0.193	0.173	0.909
0.1	-0.363	0.992	1.046	0.016	0.075	0.053	0.983
0.15	-0.419	0.996	0.915	-0.015	0.021	0.000	1.016
0.2	-0.472	0.998	0.849	-0.048	-0.012	-0.034	1.050
0.25	-0.518	0.998	0.815	-0.089	-0.041	-0.068	1.092
0.3	-0.554	0.998	0.797	-0.130	-0.068	-0.099	1.134
0.35	-0.635	1.000	0.772	-0.178	-0.097	-0.135	1.183

Table 2: Coefficients of Cazacu06 for pure titanium [14].

The yield surface evolution at different deformation level was shown in Figure1. From there, it is clear seen that the shape of yield surface changed when the material suffer different deformation. It leads to the identified anisotropic parameters in Cazacu06 model at different equivalent plastic deformations have different value, and equivalent stress value is not only dependent on the stress, but also the equivalent plastic strain. Therefore, it is necessary to take this effect into account when we use the analytical method to predict formability.

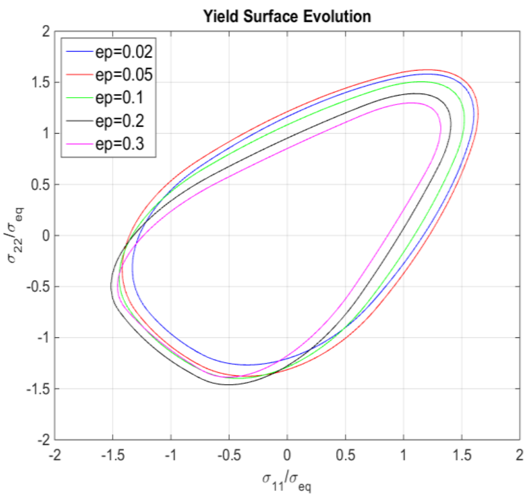

Figure 1: Yield surface evolution of pure titanium.

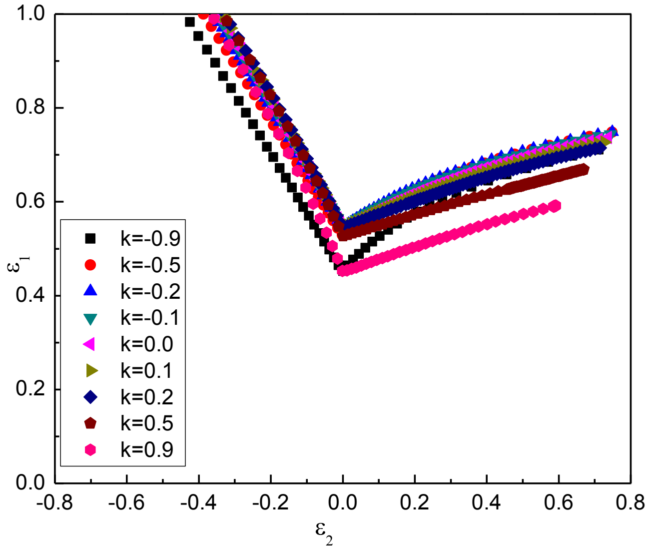

Figure 2: FLD for different *k* value.

To determine a complete FLD, Swift's and Hill's theories are used calculate the forming limit strains on the left and the right side, respectively, of the FLD. Before start to study yield surface evolution, we want to demonstrate the influence of material parameter *k* in Cazacu06 yield function on the FLD, which can be seen in Figure 2. By comparison of FLD at different *k* values, it can be seen that the coefficient k has great effect on the forming limited strain at the right hand side of FLD.

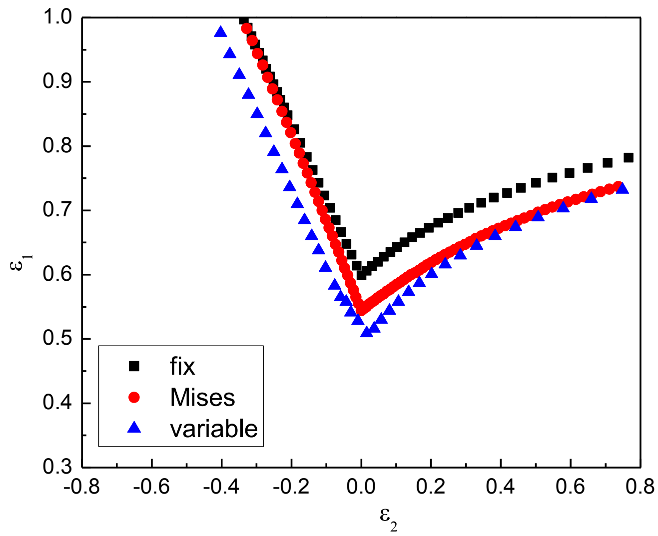

Figure 3: FLD for different yield surfaces.

Figure 3 displays the predicted FLD for von Mises yield criterion, Cazacu06 with fixed anisotropic coefficients (it means that the shape of the yield surface doesn't changes when the materials surfer plastic deformation), Cazacu06 with different anisotropic coefficients for different deformation level. It is clear seen that the Cazacu06 with fixed coefficient overestimated the formability of the materials, particularly for the right hand side of FLD.

4 CONCLUSIONS

In this paper, an extended Swift and Hill's plastic instability by considering the yield surface evolution was proposed and implemented into a unified instability analytical algorithm. The shape of the yield surface has a great influence on the forming limit strains. By considering the yield surface evolution, it can reduce the overestimation of limited strains.

REFERENCES

[1] S.P. Keeler, W.A Backofen, Plastic instability and fracture in sheet stretched over grid punches, *American Society of Metals Transactions* Quarterly 56 25-48, 1964.

[2] G.M. Goodwin, Application of strain analysis to sheet metal forming in the press shop, *Society of Automotive Engineers*, 1968.

[3] H.J. Kleemola, and J.O.Kumpulainen, Factors influencing the forming limit diagram: Part I – The experimental determination of the forming limits of sheet steel, *Journal of Materials Processing Technology*, 3, 289-302, 1980.

[4] H.J. Kleemola, and J.O.Kumpulainen, Factors influencing the forming limit diagram: Part II – Influence of sheet thickness, *Journal of Mechanical Working Technology*, 3(3-4), 303-311, 1980.

[5] ASTM Standard E2218-02, Standard Test Method for Determining Forming Limit Curves, 2002.

[6] D. Banabic, H.J. Bünge, K. Pöhlandt, A.E. Tekkaya, Formability of Metallic Materials I, Plastic Anisotropy, Formability Testing and Forming Limits, Springer Verlag, Heidelberg, pages 173-213, 2000.

[7] H.W. Swift, Plastic instability under plane stress, *Journal of the Mechanics and Physics of Solids*, 1,1:1-18,1952.

[8] G.G. Moore and J. F. Wallance, The effect of anisotropy on instability in sheet metal forming, *Journal of the Institute of Metals*, 9:33-38, 1965.

[9] R.Hill, On discontinuous plastic states, with special reference to localized necking in thin sheets, *Journal of the Mechanics and Physics of Solids*, 1:19-30, 1952.

[10] J. Majak, M.Pohlak, R.Kuttner, A simple algorithm for formability analysis, *Journal of Achievements in Materials and Manufacturing engineering*, v22 57-60, 2007.

[11] D. Dudzinski, A. Molinari, Perturbation analysis of thermoviscoplastic instabilities in biaxial loading, *International Journal of Solids and Structures*, 27(5): 601–628, 1991.

[12] O. Cazacu, B. Plunkett, Orthotropic yield criterion for hexagonal closed packed metals, International Journal of Plasticity. 22(7), 1171-1194, 2006.

[13] A. Considère, Annales des Ponts et Chaussées, 9: 574-775, 1885.

[14] B. R. Baudard, O. Cazacu, P. Flater, N. Chandola, J. L. Alves, Unusual plastic deformation and damage features in Titanium: Experimental test and constitutive modeling, *Journal of the Mechanics and Physics of Solids*, V 88, 100-122, 2016.