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EFFECT OF SOIL SPATIAL VARIABILITY ON THE DYNAMIC BEHAVIOR OF A SLOPE

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Abstract. *The analysis of earth slopes situated in seismic areas has been extensively investigated in literature using deterministic approaches where average values of the soil properties were used. In this paper, a probabilistic dynamic approach is presented for the slope stability analysis. In this approach, the effect of the soil spatial variability on the dynamic responses was investigated. The soil shear modulus G was modeled as an anisotropic non-Gaussian random field. The deterministic model was based on numerical simulations using the dynamic option of the finite difference code FLAC^{2D}. Notice that when dealing with dynamic problems, the deterministic model becomes very time-consuming. For this reason, one needs to keep to a minimum the number of calls to the deterministic model when performing the probabilistic analyses, the probabilistic method generally employed when considering spatially varying soil properties being the Monte Carlo Simulation (MCS) methodology. This method is known to be not suitable for the computation of the small failure probabilities encountered in practice because it becomes very time-expensive in such cases due to the large number of simulations required. In order to overcome the shortcoming related to the excessive number of calls of the deterministic model when performing Monte Carlo simulations, Echard et al. (2011) proposed an Active learning reliability method (called AK-MCS) combining Kriging and Monte Carlo Simulation. The method was shown to be very efficient as the obtained probability of failure is very accurate needing only a small number of calls to the deterministic model. The probabilistic dynamic analyses presented in this paper were performed using AK-MCS methodology by Echard et al. (2011). The failure probability was computed for the point located on the toe of the slope. For a given realization of the random field, failure is considered to be achieved at the toe if the maximal acceleration A_{max} at this point, as computed bu FLAC^{2D} software, exceeds a prescribed threshold value. Some probabilistic results corresponding to different values of the maximum threshold value are presented and discussed.*

1 INTRODUCTION

The response of a geotechnical structure subjected to a seismic loading has been extensively investigated in literature using deterministic approaches where average values of the soil properties (shear modulus, angle of internal friction, cohesion, etc.) were used. Notice however that the spatial variability of the soil properties may affect the behavior of geotechnical structures. Consequently, reliable responses of a geotechnical system cannot be predicted using a deterministic approach; a probabilistic technique seems to be necessary.

The probabilistic techniques enable the rigorous propagation of the different uncertainties from the input parameters to the system responses. It should be emphasized here that few authors have worked on the analysis of the seismic responses using probabilistic approaches [1-5]. This is because of the significant computation time required per simulation especially when using finite element/finite difference dynamic models.

In this paper, the effect of the soil spatial variability on the seismic response of a geotechnical structure is investigated. The case of an elastic slope subjected to a seismic loading was considered. The effect of the soil spatial variability on the maximal acceleration at the top and/or at the toe of the slope was considered. The objective is the computation of the probability P_f of exceeding a tolerable maximum acceleration A_{max}. The soil shear modulus was considered as a two-dimensional lognormally distributed random field where it is assumed to vary in both vertical and horizontal directions. The deterministic model was based on numerical simulations using the dynamic option of the FLAC^{2D} software. Notice that when dealing with numerical dynamic problems, the deterministic model becomes very time-consuming. For this reason, one needs to keep to a minimum the number of calls to the deterministic model when performing the probabilistic analyses, the probabilistic method generally employed when considering spatially varying soil properties being the Monte Carlo Simulation (MCS) methodology. This method is known to be not suitable for the computation of the small failure probabilities encountered in practice because it becomes very time-expensive in such cases due to the large number of simulations required.

In order to overcome the shortcoming related to the excessive number of calls of the deterministic model when performing Monte Carlo simulations, [6] proposed an Active learning reliability method (called AK-MCS) combining Kriging and Monte Carlo Simulation. The method was shown to be very efficient as the obtained probability of failure is very accurate needing only a small number of calls to the deterministic model. The basic idea of the present approach is to construct a kriging metamodel which is a surrogate (or a substitute) of the real numerical model based only on a few realizations and then, to obtain an estimation of the failure probability and the corresponding coefficient of variation based on this metamodel. The choice of the realizations used in the construction of the metamodel is made in such a manner to focus on the configurations that are close to the limit state surface and that have sufficiently high density of probability so that their contribution on the failure probability is significant. This leads to an accurate value of the failure probability.

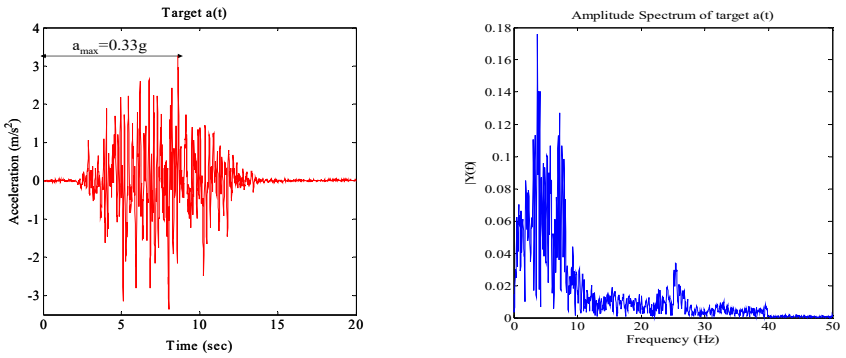
The paper is organized as follows: one first presents the deterministic numerical modeling of the dynamic problem and the corresponding results. Then, the probabilistic analyses and the corresponding probabilistic results are presented and discussed. The paper ends by a conclusion of the main findings.

2 DETERMINISTIC ANALYSIS

In this section, the deterministic numerical model is firstly presented. It is followed by the corresponding deterministic results.

2.1 Numerical modeling

The deterministic model is based on numerical simulations using the dynamic option of the finite difference code FLAC^{2D}. The input seismic signal used in this work is the synthetic signal of Nice for which the corresponding accelerogram is presented in Figure 1(a). This signal is used because it is representative of the French design spectrum [7]. It has a maximum acceleration equal to 0.33g. Its corresponding Fourier amplitude spectrum is shown in Figure 1(b).



(a) (b)

Figure 1. (a) Accelerogram of the synthetic signal of Nice and (b) the corresponding Fourier amplitude spectrum

In the finite difference dynamic analysis by FLAC^{2D}, numerical distortions may occur during the propagation of the seismic waves if the elements size of the mesh is not convenient. The size Δl of an element of the mesh should respect the following condition [8]:

$$\Delta l \leq \frac{V_s}{10 * f_{\max}} \tag{1}$$

where V_s is the shear wave velocity, and f_{max} is the maximum frequency of the incident seismic signal [9]. The shear wave velocity V_s in Equation (1) can be calculated using the values of the soil shear modulus G and the soil density ρ as follows:

$$V_s = \sqrt{\frac{G}{\rho}} \tag{2}$$

Even though an elasto-plastic model would be more realistic to model the soil behavior especially for the cases of medium and high earthquake ground motions GMs, an elastic model was used in this work. The aim is to investigate the effect of the soil spatial variability using a simple model with a reasonable computation time. Concerning the boundary conditions, FLAC^{2D} offers the option of applying absorbing boundary conditions of type "quiet boundaries" or "free field". These boundary conditions absorb the energy of the wave approaching these limits and thus avoid the reflection of these waves. In this paper, the boundary conditions applied to the vertical boundaries are of type "free field". This type of boundary conditions is suitable for vertical surfaces, the boundary conditions of type "quiet boundaries" being generally convenient in the case of horizontal surfaces. Finally, it should be mentioned that in the natural dynamic systems, the internal friction may lead to partial dissipation of the energy of vibration. The software FLAC^{2D} provides a damping of type "Rayleigh damping"

(among other types of damping) which is based on two parameters (i) the natural frequency of the system and (ii) the damping ratio (defined as a percentage of the critical damping).

2.2 Deterministic results

For the dynamic analyses, the values of the shear modulus, bulk modulus and density of the soil were as follows: G=100MPa, K=250MPa and ρ=1800 kg/m^3. In order to avoid the numerical distortion that may occur during the propagation of the seismic waves in the model, the elements size must satisfy Equation (1). By using Equation (2), the shear wave velocity was found to be equal to 235.7m/s. From Figure 1(b), one can see that the maximal frequency f_{max} is equal to 40Hz. Thus, the maximum size of the different elements must be less than or equal to 0.59m. In the studied model, the size Δl of the different elements was taken equal to 0.5m. Concerning the boundary conditions, boundary conditions of type "free field" were applied along the vertical boundaries of the model. The lower horizontal boundary was subjected to the seismic load (i.e. the synthetic accelerogram of Nice). Concerning the damping effect, an undamped system was considered to reduce the number of involved parameters. The effect of damping will be the subject of future work.

The slope geometry considered in the analysis is 10m in height and 45° in inclination angle (see Figure 2). In order to obtain accurate results of the maximum acceleration at the top and at the toe of the slope, one need to position correctly the vertical boundary conditions. For this purpose, the width of the slope L was initially considered to be equal to 60m and was successively increased by 20m (i.e. 10m from each side) until one obtains convergence of the maximum acceleration for the profile presented in figure 2 in red color. Figure 3 presents the maximum acceleration profiles for L=60m, 80m, 100m, 120m, 140m and 160m. From this figure, one can deduce that for L $\geq$ 140m, all the presented profiles coincide. For this reason, a width L=140m was selected for the probabilistic analysis. The corresponding computation time is equal to 50 minutes.

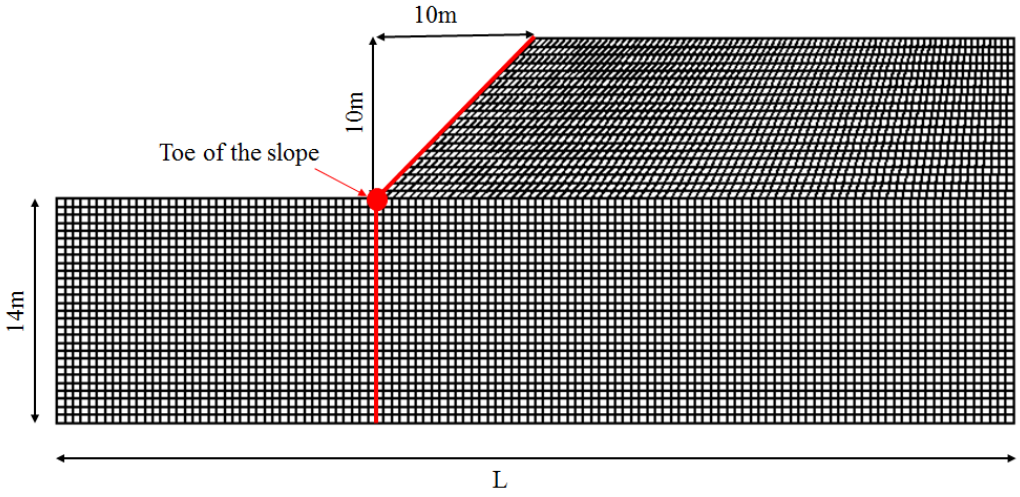


Figure 2. Slope geometry

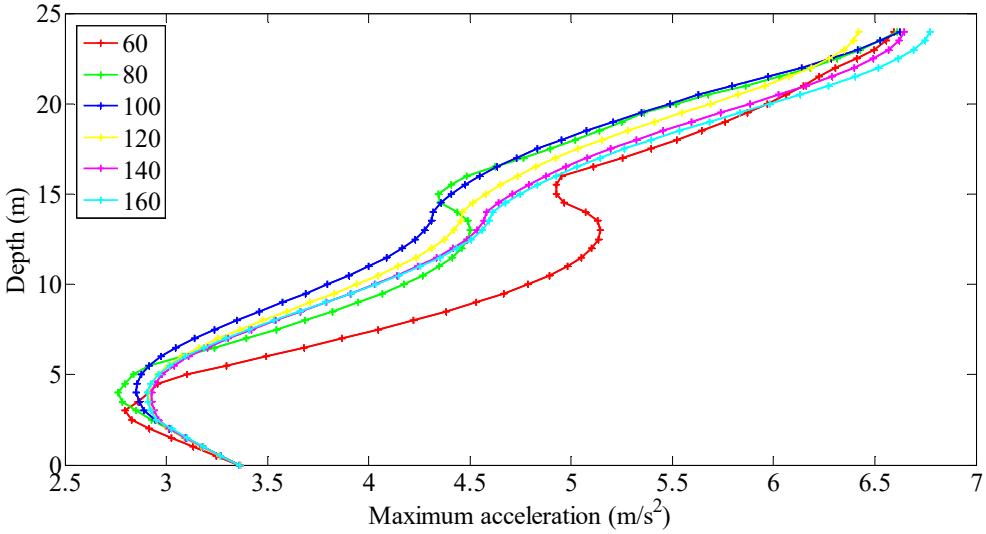


Figure 3. Maximum acceleration profiles for L= 60m, 80m, 100m, 120m, 140m and 160m

3 PROBABILISTIC ANALYSIS

The aim of this section is to present the probabilistic dynamic analysis. The impact of the soil spatial variability on the dynamic response was investigated. It should be noted here that the dynamic system response considered in this section involves the maximum acceleration (A_{max}) at the toe of the slope. The objective of the probabilistic analysis is the computation of the probability P_f of exceeding a tolerable maximum acceleration A_{max}. The soil shear modulus G was modeled as a two-dimensional log-normal random field. The EOLE methodology [10] was used to discretize the shear modulus random field (i.e. to obtain realizations of the soil shear modulus that respect the correlation structure of this field). For the two-dimensional random field used in this paper, the shear modulus was allowed to vary in both the horizontal and vertical directions. The deterministic model was based on numerical simulations using the finite difference code FLAC^{2D}. As for the probabilistic method used in this paper, the active learning reliability method combining kriging and Monte Carlo Simulation (i.e. AK-MCS) was employed. This method combines both the classical crude Monte Carlo Simulation (MCS) methodology and the Kriging meta-model technique. The performance function used to calculate the probability P_f of exceeding a tolerable maximum acceleration A_{max} was defined as follow:

$$\Gamma = \frac{A_{\max}}{A_{\max}\left(toe\right)} - 1 \tag{3}$$

where A_{max} is a prescribed tolerable maximum acceleration and A_{max}(toe) is the maximum acceleration at the toe of the slope as computed by FLAC^{2D} software due to the applied seismic loading.

In this section, the EOLE method of discretization of random fields is firstly presented. It is followed by a brief presentation of the crude Monte Carlo method, the kriging metamodeling technique and the combined use of the kriging and the Monte Carlo Simulation methodology (called AK-MCS) used for the probabilistic analysis.

3.1 Discretization of a non-isotropic log-normal random field

Consider a 2D non-isotropic log-normal random field Z^{LN} described by: (i) a log-normal marginal cumulative distribution function F_G, and (ii) a square exponential autocorrelation function ρ_Z^{LN} [(x, y), (x', y')] which gives the values of the correlation between two arbitrary points (x, y) and (x', y'). Notice that this function is given as follows:

$$\rho_z^{LN}[(x,\ y),\ (x\ ',\ y\ ')] = \exp\left(-\left(\frac{|x - x\ '|}{a_x} \right)^2 - \left(\frac{|y - y\ '|}{a_y} \right)^2 \right) \tag{4}$$

where a_x and a_y are the autocorrelation distances along x and y respectively. The EOLE method proposed by [10] to discretize a random field is used herein. In this method, one should first define a stochastic grid composed of s grid points (or nodes) and determine the log-normal autocorrelation matrix $\Sigma_{\chi;\chi}^{LN}$ which gives the correlation between each grid point of the stochastic mesh and the other grid points of this mesh using Equation (4). The number of grid points within the stochastic mesh is determined in such a manner that in each direction (x or y), there are five grid points within each autocorraltion distance. The log-normal autocorrelation matrix $\Sigma_{\chi;\chi}^{LN}$ should then be transformed into the Gaussian space using the Nataf transformation [11]. As a result, one obtains a Gaussian autocorrelation matrix $\Sigma_{\chi;\chi}^{G}$ that can be used to discretize the Gaussian random field Z as follows:

$$\tilde{Z}(x\ ,\ y\ ) \cong \mu_{\ln Z} + \sigma_{\ln Z} \sum_{j=1}^{N} \frac{\xi_j}{\sqrt{\lambda_j}} .\phi_j .\Sigma_{Z(x\ ,y\ );\chi} \tag{5}$$

where μ_{lnZ} and σ_{lnZ} are the mean and standard deviation values of the underlying normal distribution (i.e. ln(Z)); (λ_j, ϕ_j) are the eigenvalues and eigenvectors of the Gaussian autocorrelation matrix $\Sigma_{\chi;\chi}^{G}$; $\Sigma_{Z(x\ ,y\ );\chi}$ is the correlation vector between the value of the field at an arbitrary point (x, y) and its values at the different grid points; ξ_j (j=1, ..., N) is a vector of standard normal random variables; and N is the number of terms (expansion order) retained in EOLE method. This number N is obtained (i) by sorting the eigenvalues λ_j (j=1, …, s) in a descending order and (ii) by choosing the number N of eigenmodes that leads to a variance of the error which is smaller than a prescribed tolerance ε ($\varepsilon \approx 10\%$ in this paper). Notice that the variance of the error for EOLE is given by [10] as follows:

$$Var\left[\ Z(x,\ y) - \tilde{Z}(x,\ y)\ \right] = \sigma_{\ln Z}^2 \left\{ 1 - \sum_{j=1}^{N} \frac{1}{\lambda_j} \left(\left(\phi_j \right)^T \Sigma_{Z(x,y);\chi} \right)^2 \right\} \tag{6}$$

where $Z(x\ ,\ y\ )$ and $\tilde{Z}(x\ ,\ y\ )$ are respectively the exact and the approximate values of the random field at a given point (x, y) and $\left(\phi_j \right)^T$ is the transpose of the eigenvector ϕ_j. Once the Gaussian random field is obtained, it should be transformed into the log-normal space by exponentiating the approximated Gaussian random field $\tilde{Z}(x\ ,\ y\ )$ given by Equation (5). Notice finally that Eq. (5) or rather its equivalence for the log-normal random field will be used in this paper to compute the value of the random field at the center of the different elements of the numerical dynamic model to assign values of G to the different elements of this numerical model.

3.2 Monte Carlo method

The Monte Carlo simulation method consists in generating *K* samples which respect the joint probability density function $f_X(X)$ of the *M* random variables ($X_1, ..., X_M$) gathered in a vector *X*. For each sample, the system response is calculated. Thus; for the K samples, one obtains *K* values of the system response gathered in a vector $\Gamma = \left\{\Gamma\left(X^{(1)}\right),...,\Gamma\left(X^{(K)}\right)\right\}$ which may be used to determine the failure probability for a prescribed threshold of this system response. A very large number of samples (i.e. realizations) is required to obtain a rigorous value of the failure probability especially when computing small failure probabilities. It should be noted herein that the random variables considered in the present paper are the standard normal variables ξ_j ($j=1, ..., N$) that appear in Eq. (5) for the computation of a given realization of the shear modulus random field.

3.3 Kriging metamodeling technique

The metamodeling technique aims at replacing the response (or the performance function in this paper) of a computationally-expensive mechanical model by a metamodel (i.e. a simple analytical equation). The Kriging metamodeling technique is based on the idea that the performance function $\Gamma(\mathbf{x})$ is seen as a realization of a stochastic field $\delta(\mathbf{x})$ [12]. The first step of kriging consists of defining this stochastic field with its parameters according to an Experimental Design (ED) of length *k*. This stochastic field is given by:

$$\delta(\mathbf{x}) = F(\mathbf{x}, \beta) + Z(\mathbf{x}) \tag{7}$$

Where:

- $F(\mathbf{x}, \beta)$ is the deterministic part which gives an approximation of the mean value of the performance function. It corresponds to a regression model that can be written as:

$$F(\mathbf{x}, \beta) = \sum_{j=1}^{p} \beta_j f_j(\mathbf{x}^{(i)}) \qquad i=1, ..., k \tag{8}$$

  Where $\beta^T = (\beta_j, j = 1, ..., p)$ is the vector of coefficients to be determined, $f^T = (f_j, j = 1, . . . , p)$ is a collection of regression functions (or regressors) and k is the number of observations. The number *p* of regression functions is assumed to be less than or equal to the number *k* of observations so that the problem is not underdetermined (i.e. it does not lack equations to compute the unknowns). In this paper, ordinary kriging is selected which means that $F(\mathbf{x}, \beta)$ is a scalar to be determined (i.e. $F(\mathbf{x}, \beta) = \beta$).

- $Z(\mathbf{x})$ represents the fluctuation around the mean value. It is given by a Gaussian random process (or a random field) with zero mean and covariance between two points of space $\mathbf{x}$ and $\mathbf{x}'$ defined by:

$$COV\left(Z(\mathbf{x}), Z(\mathbf{x}')\right) = \sigma_Z^2 \rho_\theta(\mathbf{x}, \mathbf{x}') \tag{9}$$

  Where σ_Z^2 is the process variance and ρ_θ the correlation function defined by its set of parameters θ. Several models exist to define the correlation function. However, in this paper, the anisotropic squared-exponential function is selected. It is given by:

$$\rho_\theta(\mathbf{x},\mathbf{x}') = \prod_{i=1}^{N} exp\left(-\left(\theta_i\left(x_i - x_i'\right)\right)^2\right) \tag{10}$$

Where N is the number of random variables, x_i and x_i' are the i[th] coordinates of the points $\mathbf{x}$ and $\mathbf{x}'$ and θ_i is a scalar which is equal to the inverse of the correlation length in the i[th] direction.

As mentioned above, in order to completely define the kriging model, one has to compute the following parameters: β, σ_Z and the correlation function parameters θ_i. Given an initial Experimental Design (ED) with k samples, $\mathbf{x} = \left\{\mathbf{x}^{(1)},...,\mathbf{x}^{(k)}\right\}$, for which the response values have been computed and stored in $\mathbf{\Gamma} = \left\{\Gamma\left(\mathbf{x}^{(1)}\right),...,\Gamma\left(\mathbf{x}^{(k)}\right)\right\}$, the scalars β and σ_Z^2 are estimated according to [12] by:

$$\hat{\beta} = (\mathbf{1}^T \Sigma_{\Gamma;\Gamma}^{-1} \mathbf{1})^{-1} \mathbf{1}^T \Sigma_{\Gamma;\Gamma}^{-1} \mathbf{\Gamma} \tag{11}$$

$$\hat{\sigma}_Z^2 = \frac{1}{k}(\mathbf{\Gamma} - \hat{\beta}\mathbf{1})^T \Sigma_{\Gamma;\Gamma}^{-1} (\mathbf{\Gamma} - \hat{\beta}\mathbf{1}) \tag{12}$$

Where $\mathbf{1}$ is the vector of length k filled with $\mathbf{1}$, $\Sigma_{\Gamma;\Gamma}^{-1}$ is the inverse of the correlation matrix $\Sigma_{\Gamma;\Gamma}$. It should be mentioned that a row i of the correlation matrix gives the values of the correlation between the value of the response at the sampled point $\Gamma\left(\mathbf{x}^{(i)}\right)$ and all the values of the response at the sampled points $\mathbf{\Gamma} = \left\{\Gamma\left(\mathbf{x}^{(1)}\right),...,\Gamma\left(\mathbf{x}^{(k)}\right)\right\}$ as follows:

$$\left(\Sigma_{\Gamma;\Gamma}\right)_{i,j} = \rho_\theta\left(\mathbf{x}_i, \mathbf{x}_j\right) \qquad i=1,...,k \text{ and } j=1,...,k \tag{13}$$

However, as $\hat{\beta}$ and $\hat{\sigma}_\Gamma^2$ in Eqs. (11) and (12) depend on the correlation parameters θ_i through the matrix $\Sigma_{\Gamma;\Gamma}$, it is first required to obtain them using maximum likelihood estimation:

$$\theta = \arg\min_\theta \left(\det \Sigma_{\Gamma;\Gamma}\right)^{\frac{1}{k}} \hat{\sigma}_Z^2 \tag{14}$$

The next step consists of predicting the response of $\Gamma(\mathbf{x})$ in a given unknown point $\mathbf{x}$. At such a point, the Best Linear Unbiased Predictor (BLUP) $\hat{\Gamma}(\mathbf{x})$ of $\delta(\mathbf{x})$ is computed as [12]:

$$\hat{\Gamma}(\mathbf{x}) = \beta + \sum_{\Gamma(\mathbf{x});\Gamma} \sum_{\Gamma;\Gamma}^{-1} (\mathbf{\Gamma} - \beta\mathbf{1}) \tag{15}$$

Where $\Sigma_{\Gamma(\mathbf{x});\Gamma}$ is the correlation vector whose elements provide the correlation between the value of the response at the unsampled point $\Gamma(\mathbf{x})$ and the values of the response at the sampled points gathered in the vector $\mathbf{\Gamma} = \left\{\Gamma\left(\mathbf{x}^{(1)}\right),...,\Gamma\left(\mathbf{x}^{(k)}\right)\right\}$ as follows:

$$\left(\Sigma_{\Gamma(\mathbf{x});\Gamma}\right)_i = \rho_\theta\left(\mathbf{x}_i, \mathbf{x}\right) \qquad i=1,...,k \tag{16}$$

The kriging variance is defined as the minimum of the mean squared error between $\Gamma(\mathbf{x})$ and $\hat{\Gamma}(\mathbf{x})$. It can be expressed as the following analytical function:

$$\sigma_{\hat{\Gamma}}^2(\mathbf{x}) = \sigma_Z^2\left(\mathbf{1} + u(\mathbf{x})^t\left(\mathbf{1}^t \Sigma_{\Gamma;\Gamma}^{-1} \mathbf{1}\right) u(\mathbf{x}) - \Sigma_{\Gamma(\mathbf{x});\Gamma}^T \Sigma_{\Gamma;\Gamma}^{-1} \Sigma_{\Gamma(\mathbf{x});\Gamma}\right) \tag{17}$$

Where:

$$u\left(\mathbf{x}\right)=\mathbf{1}^{t}\Sigma_{\Gamma;\Gamma}^{-1}\Sigma_{\Gamma(\mathbf{x});\Gamma}-\mathbf{1} \tag{18}$$

Kriging is an exact interpolation method. The prediction of $\hat{\Gamma}\left(\mathbf{x}^{(i)}\right)$ in a point $\mathbf{x}^{(i)}$ of the design of experiments is exact (i.e. $\hat{\Gamma}\left(\mathbf{x}^{(i)}\right)=\Gamma\left(\mathbf{x}^{(i)}\right)$). Therefore, the kriging variance is null in these points and it becomes important in unexplored areas. These properties are interesting in reliability studies and metamodels as the Kriging variance represents a good index to improve a design of experiments.

3.4 AK-MCS methodology

The active learning reliability method called AK-MCS combines both the Kriging metamodeling technique and Monte Carlo Simulation. The iterative procedure suggested by [6] for computing the probability of failure P_f using this approach can be described as follows:

1. Generate a population S of n_{MC} (say 1,000,000) realizations of the spatially varying soil shear modulus using Monte Carlo Simulation. It should be emphasized here that the computation of the performance function for these realizations (as defined in equation 3 based on the computationally-expensive FLAC^{2D} model) is not required at this stage.
2. Randomly select from the S population a small number of realizations (called initial Design of Experiments DoE) containing N_1 realizations (say 20). For those N_1 realizations, one should evaluate the performance function given by equation (3) based on the computationally-expensive FLAC^{2D} model. The required initial DoE may be greater than 20 for a great number of random variables (i.e. for a spatially varying soil property with small values of the autocorrelation distances). This is because for high dimensional problems (i.e. when a large number of random variables is needed to discretize the shear modulus random field), the kriging model needs more points (i.e. realizations) in order to compute correlation parameters θ_i given by equation (15).
3. Compute the kriging model according to the small initial DoE containing the N_1 realizations. This kriging model is given by equation (16). It should be mentioned here that this stage was performed using the Matlab toolbox DACE [7]. In this paper, ordinary kriging model was used (i.e. the regression model is considered to be constant) and a square exponential correlation function was adopted in the analysis.
4. By using DACE toolbox, compute (for the whole population S containing the n_{MC} realizations of the random field) both the kriging predictor values $\hat{\Gamma}$ of the performance function (based on equation 16) and their corresponding kriging variance values $\sigma_{\hat{\Gamma}}^2$ (based on equation 18). From the obtained values of the kriging predictors $\hat{\Gamma}$, obtain an estimation of the probability of failure $\hat{P}_f$ by counting the number of negative predictors $N_{\hat{\Gamma}\leq 0}$ and dividing it by the total number of samples in S as follows:

$$\hat{P}_f=\frac{N_{\hat{\Gamma}\leq 0}}{n_{MC}} \tag{19}$$

Also compute the coefficient of variation of $\hat{P}_f$ as follows:

$$C\hat{O}V(\hat{P}_f)=\sqrt{\frac{1-\hat{P}_f}{\hat{P}_f n_{MC}}} \tag{20}$$

5. Identify the best next realization in S for which one will compute the performance function using FLAC^{2D}. This is performed by evaluating a learning function U for each realization in S. The learning function U is given by:

$$U\left(\mathbf{x}^{(i)}\right)=\frac{\left|\hat{\Gamma}\left(\mathbf{x}^{(i)}\right)\right|}{\sigma_{\hat{\Gamma}}\left(\mathbf{x}^{(i)}\right)} \qquad i=1,...,n_{MC} \tag{21}$$

The best next candidate realization is the one with minimum value of U. If this minimum value of U is smaller than 2, the performance function based on FLAC^{2D} is evaluated for the best candidate and the initial DoE is updated. Thus one should go back to step 3 and evaluate a new kriging model based on the updated DoE. Steps 3, 4 and 5 are repeated until the minimum value of U become larger than 2. At this stage the learning stops and the metamodel is considered accurate enough based on the n_{MC} realizations.

6. When the learning stops, one must compute the estimated values of both the probability of failure $\hat{P}_f$ and the coefficient of variation $C\hat{O}V\left(\hat{P}_f\right)$. The obtained value of $\hat{P}_f$ is considered to be accurate if $C\hat{O}V\left(\hat{P}_f\right)\leq 5\%$. If the estimated coefficient of variation is larger than 5%, one must increase the population S in step 1 and repeat the procedure.

It should be emphasized herein that a small initial DoE is chosen within the present approach (see step 2) in order to keep to a minimum the number of calls to the deterministic model. This initial DoE is successively increased by a single realization at each time (see step 5). The chosen realization is the one that is improving the most the metamodel because equation (22) searches for the realization that has a small kriging predictor (i.e. a realization that is close to the limit state surface) and/or a high kriging variance (i.e. a high uncertainty in the sign of its performance function value). Notice that the realizations with high uncertainties in the sign of their performance function values (positive or negative) are those that are close to the limit state surface. Finally, notice that the stopping criterion min(U)>2 corresponds to a maximal probability of making a mistake on the sign of the performance function of $\Phi(-2)=0.023$. This means that the stopping criterion is relevant making use of the realizations with a small uncertainty in making a mistake in their performance functions' signs, the probability of making a mistake in the signs of their performance function values being negligible.

3.5 Probabilistic dynamic results

The aim of this section is to present the probabilistic dynamic results of a soil slope exhibiting a spatially varying shear modulus. The objective is the computation of the probability P_f of exceeding a tolerable maximum acceleration A_{max}. The shear modulus G was modeled by a random field and it was assumed to follow a log-normal probability density function. The mean value and the coefficient of variation of G were respectively μ_G=100MPa and COV_G=25%. A square exponential autocorrelation function was used in this study to represent the correlation structure of the random field. The random field was discretized using EOLE method. Although an isotropic random field is often assumed in literature [13, 14], the vertical autocorrelation length tends to be shorter than the horizontal one due to the geological soil formation process for most natural soil deposits [15]. A common ratio of about 1 to 10 for these autocorrelation lengths can be used [16]. Notice that in this paper, only one configuration of the autocorrelation distances (a_x=50m and a_y=5m) was considered. An extension to this work may consider other configurations of a_x and a_y. The present study was performed in order to explore some interesting features related to the values of P_f for different values of the tolerable maximum acceleration A_{max}.

3.6 AK-MCS results for different values of the tolerable acceleration

In this section, the results obtained based on AK-MCS methodology are presented and discussed for the case where a_x=50m and a_y=5m. It should be mentioned here that a population S containing a total number of simulations n_{MC} equal to one million was selected. Then, a small initial design of experiment (DoE) of N_1=20 realizations among the whole population S was randomly selected. The performance function given by equation (3) was evaluated for this DoE. Thus, the computationally-expensive dynamic model based on finite difference code FLAC^{2D} was only called 20 times before evaluating the kriging model. The Kriging model was thus built based on this DoE. A simple analytical equation is then obtained and the performance function can be evaluated for the whole population S with no time: Both the kriging predictor and the kriging variance for the whole S population were evaluated based on equations (15) and (17) of the kriging model using the DACE Matlab toolbox [17]. The probability of failure $\hat{P}_f$ and the corresponding coefficient of variation $C\hat{O}V(\hat{P}_f)$ were computed using equations (19) and (20). Notice finally that several points (realizations) were added based on the evaluation of the learning function U given by equation (21) until reaching convergence.

Table 1 present the evolution of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ as a function of the number of added points (realizations) for four different values of the tolerable maximum acceleration A_{max} (A_{max}=15, 25, 30 and 35m/s^2). From this table, one may conclude that for small value of A_{max} (A_{max}=15m/s^2) one obtains a large value of $\hat{P}_f$. Figure 4 presents the variation of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ as function of the number of added points using the AK-MCS algorithm. This figure shows that the values of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ stabilized when the algorithm has added 100 extra points to the initial DoE containing 20 points. In the present paper extra points were added to show the efficiently of the algorithm and to demonstrate that no further modifications on the obtained values of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ may occur.

A_{max}(m/s^2)	$\hat{P}_f$	$C\hat{O}V(\hat{P}_f)$ (%)	Number of added points
15	170 x10^{-3}	0.21%	248
25	50 x10^{-3}	0.41%	185
30	12 x10^{-3}	0.1%	270
35	2.5x10^{-3}	0.1%	248

Table 1: Variation of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ together with the number of added points for different values of A_{max}.

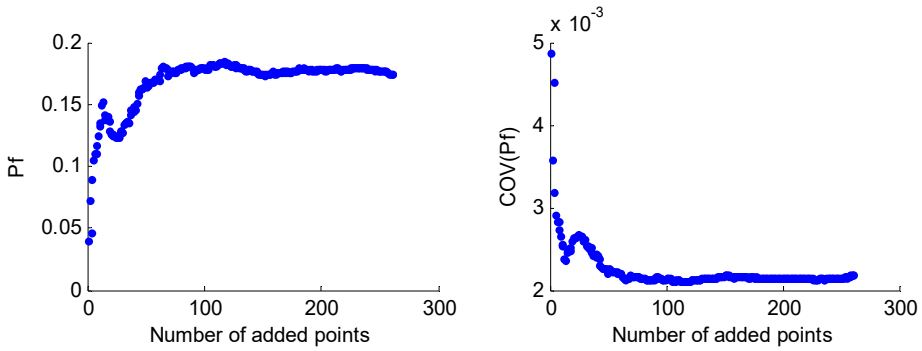


Figure 4. Variation of $\hat{P}_f$ and $C\hat{O}V(\hat{P}_f)$ as function of the number of added points for A_{max} =15m/s^2

4 CONCLUSION

The probabilistic dynamic analyses presented in this paper were performed using AK-MCS methodology. In a first stage (step 1), a significant number of realizations (about 1 million) of the spatially varying soil medium were generated by MCS using EOLE method. In a second stage (step 2), an initial design of experiments (DoE) based on a random selection of some realizations among the MCS population was determined and employed to compute the Kriging model. In a third stage (steps 3-5), a kriging model is constructed and a learning function U was employed to choose the best next realization to be used for the computation of the system response from the original deterministic model and to update the kriging model with this new response value. Notice that steps 3-5 were repeated several times until reaching the adopted stopping condition. At the end, the surrogate kriging model was considered to be enough accurate for the estimation of the failure probability and the corresponding value of the coefficient of variation, for the prescribed number of simulations (step 6). The probabilistic results have shown that for small value of the tolerable maximum acceleration A_{max} one obtains a large value of $\widehat{P}_f$. One the other hand, one can notice that number of realizations required by the AK-MCS algorithm (to obtain accurate values of $\hat{P}_f$ and $\widehat{COV}(\hat{P}_f)$) is relatively small. This is very interesting for computationally expensive dynamic problems.

REFERENCES

[1] Koutsourelakis, S., Prevost, J.H., and Deodatis, G. Risk assessement of an interacting structure-soil system due to liquefaction. *Earthquake Engineering and Structural Dynamic*, 31: 851-879, 2002.

[2] Nour, A., Slimania, A., Laouamia, N., and Afraa, H. Finite element model for the probabilistic seismic response of heterogeneous soil profile. *Soil Dynamics and Earthquake Engineering*, 23: 331-348, 2003.

[3] Popescu, R., Prevost, J.H., and Deodatis, G. 3D effects in seismic liquefaction of stochastically variable soil deposits. *Géotechnique*, 55(1): 21-31, 2005.

[4] Lopez-Caballero, F., and Modaressi Farahmand-Razavi,A. Assessment of variability and uncertainties effects on the seismic response of a liquefiable soil profile. *Soil Dynamics and Earthquake Engineering*, 30(7): 600-613, 2010.

[5] Al-bittar (2015)

[6] Echard B, Gayton N, Lemaire M. AK-MCS: an active learning reliability method combining Kriging and Monte Carlo simulation. Structural Safety 2011;33(2):145–54.

[7] Grange, S. Risque sismique: stratégie de modélisation pour simuler la réponse des structures en béton et leurs interactions avec le sol. *Ph.D thesis*, INPG, 2008.

[8] Itasca. FLAC 4.0 Manuals. – Minnesota, ITASCA Consulting Group, Inc 2000.

[9] Kuhlemeyer, R. L., and Lysmer, J. Finite element method accuracy for wave propagation problems. *Journal of Soil Mechanics & Foundations*, 99: 421-427, 1973.

[10] Li, C.C., and Der Kiureghian, A. Optimal discretization of random fields. *Journal of Engineering Mechanics*, 119(6): 1136-1154, 1993.

[11] Nataf, A. Détermination des distributions de probabilités dont les marges sont données. *CR Acad Sci*, 225: 42-3, 1962.

[12] Echard phd

[13] Fenton, G.A., and Griffiths, D.V. (2005). "Three-dimensional probabilistic foundation settlement." Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 131(2): 232-239.

[14] Fenton, G.A., Zhou, H., Jaksa, M.B., and Griffiths, D.V. (2003). "Reliability analysis of a strip footing designed against settlement." Applications of statistics and probability in civil engineering, Der Kiureghian, Madanat & Pestana (eds) © 2003 Millpress, Rotterdam.

[15] Cho, S.E., and Park, H.C. (2010). "Effect of spatial variability of cross-correlated soil properties on bearing capacity of strip footing." *International Journal for Numerical and Analytical Methods in Geomechanics,* **34:** 1-26.

[16] Baecher, G.B., and Christian, J.T. (2003). "Reliability and statistics in geotechnical engineering." *John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichewter, West Sussex PO19 8SQ, England,* 205-242.

[17] Lophaven SN, Nielsen HB, Sondergaard J. DACE, a matlab Kriging toolbox, version 2.0. Tech. Rep. IMM-TR-2002-12; Technical University of Denmark; 2002a. <http://www2.imm.dtu.dk/hbn/dace/>.