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ON THE USE OF S-PARAMETER TRANSFORMATIONS TO IMPROVE SURROGATE MODEL BEHAVIOUR OF MULTIPORT NETWORKS

Petrie Meyer

Dept. Electrical and Electronic Engineering, Stellenbosch University
Private Bag X1, Matieland, 7602, South Africa
e-mail: pmeyer@sun.ac.za

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Abstract. *This paper will show how the utilization of the Generalized Multi-Mode Scattering Parameter Transformation for multiport S-parameters can result in the reduction of the number of S-parameters needed to model a multiport network, and how correct choices can result in S-parameters which exhibit improved numerical behaviour in a given parameter space. An example of a two-port, six-line transmission line with the conductors spaced as pairs is used to show the difference between sets of common-differential mode excitations, and sets of single-ended excitations will be shown.*

1 INTRODUCTION

Multiport networks pose particular problems to surrogate modelling algorithms, as an N-port network is typically described by an NxN Scattering matrix, or N^2 frequency and parameter dependent variables. In general, the parameter set can be reduced using symmetry, reciprocity, and if applicable, the passive lossless condition, but the number of separate models required always grow quite rapidly with N.

To exploit symmetry to obtain a reduced S-parameter set, the network has to be excited by correct combinations of port excitations which enforce the symmetry conditions. In addition, different sets of excitations often result in sets of S-parameters which exhibit varying behaviour as a function of the parameter space, and a specific excitation can result in very well-behaved S-parameters, while a different excitation can result in (for instance) resonant spikes.

To find these more optimum sets of excitations is quite difficult, as the full response of the network has to be simulated for each full set of excitations. While commercial EM-solvers can all calculate combined responses from any full set of excitations, they can do this only for uncoupled ports. For coupled ports, e.g. in a multi-conductor transmission line, each different set of excitations requires a new full 3D EM analysis, which quickly becomes prohibitively expensive.

Recently, the author developed a transformation technique which allows for the transformation of multiport S-parameters between different sets of excitations, using only one set of EM-analyses [1, 2]. The technique works for coupled and uncoupled ports, and is very simple to implement. It has been implemented on multiport antennas and transmission lines. This paper will show how the utilization of the transformation technique on multiport S-parameters can result in the reduction of the number of S-parameters needed to model a multiport network, and how correct choices can result in S-parameters which exhibit improved numerical behaviour in a given parameter space. An example of a six-line transmission line with the conductors spaced as pairs will be used to show difference between sets of common-differential mode excitations, and sets of single-ended excitations.

2 S-PARAMETER TRANSFORMATIONS

The basis of equivalent port representations is the creation of new sets of port voltages and currents which consist of linear combinations of the port voltages and currents of another port description. A generalized procedure to do this was presented in [1], together with a number of examples showing the implementation for classical circuit and antenna problems. Here, a brief mathematical overview is given, and the reader is referred to [1] for details.

Consider an N-port network excited in two different ways to form two equivalent N-port networks A and B, as shown in Fig. 1. Each network is described by an S-matrix $\mathbf{S}^{\mathrm{A,B}}$, with port voltages $\left(V_{\mathrm{n}}^{\mathrm{A,B}}\right)$, port currents $\left(I_{\mathrm{n}}^{\mathrm{A,B}}\right)$, and port impedances $\left(Z_{\mathrm{n}}^{\mathrm{A,B}}\right)$ indicated for both networks. In addition, the incident $\left(a_{\mathrm{n}}^{\mathrm{A,B}}\right)$ and reflected $\left(b_{\mathrm{n}}^{\mathrm{A,B}}\right)$ waves are also shown at each port. For the case where the port voltages and port currents of network B are linear combinations of the voltages and currents of network A, the technique establishes a transformation from $\mathbf{S}^{\mathrm{A}}$ to $\mathbf{S}^{\mathrm{B}}$ for an arbitrary set of port impedances (e.g. where network A consists of N single-ended ports, each terminated by the same reference impedance, and network B of sets of differential ports and possible single-ended ports, each terminated by a different impedance).

For the completely general case, the port voltages and currents of network B can be expressed

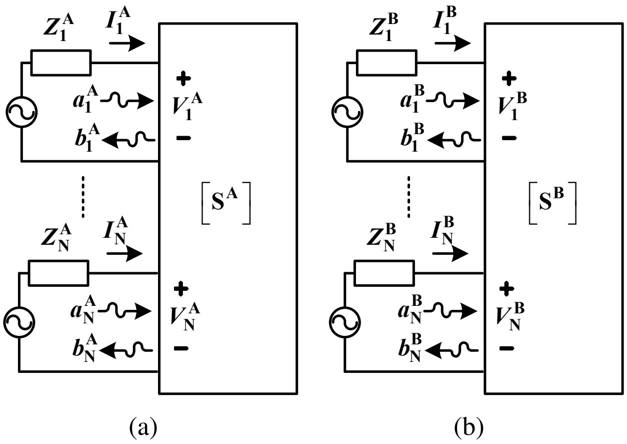


Figure 1: N-port networks with ports using (a) mode set A and (b) mode set B

as linear combinations of any or all of the port voltages and currents of network A. That is,

$$\begin{aligned} V_{\mathrm{n}}^{\mathrm{B}} &= k_{\mathrm{n1}}^{\mathrm{v}} V_1^{\mathrm{A}} + k_{\mathrm{n2}}^{\mathrm{v}} V_2^{\mathrm{A}} + \cdots + k_{\mathrm{nN}}^{\mathrm{v}} V_{\mathrm{N}}^{\mathrm{A}} \\ I_{\mathrm{n}}^{\mathrm{B}} &= k_{\mathrm{n1}}^{\mathrm{i}} I_1^{\mathrm{A}} + k_{\mathrm{n2}}^{\mathrm{i}} I_2^{\mathrm{A}} + \cdots + k_{\mathrm{nN}}^{\mathrm{i}} I_{\mathrm{N}}^{\mathrm{A}} \\ &\text{for} \quad \mathrm{n} = 1, ..., N \end{aligned} \tag{1}$$

or

$$\mathbf{V}^{\mathrm{B}} = \mathbf{K}^{\mathrm{v}} \mathbf{V}^{\mathrm{A}} \tag{2a}$$

$$\mathbf{I}^{\mathrm{B}} = \mathbf{K}^{\mathrm{i}} \mathbf{I}^{\mathrm{A}} \tag{2b}$$

where $\mathbf{V}^{\mathrm{A,B}}$ and $\mathbf{I}^{\mathrm{A,B}}$ denote $[N \times 1]$ matrices containing the port voltages and currents of networks A and B, and $\mathbf{K}^{\mathrm{v,i}}$ are $[N \times N]$ matrices as in (3) and (4).

$$\mathbf{K}^{\mathrm{v}} = \begin{bmatrix} k_{11}^{\mathrm{v}} & k_{12}^{\mathrm{v}} & \cdots & k_{1\mathrm{N}}^{\mathrm{v}} \\ \vdots & \vdots & \ddots & \vdots \\ k_{\mathrm{N1}}^{\mathrm{v}} & k_{\mathrm{N2}}^{\mathrm{v}} & \cdots & k_{\mathrm{NN}}^{\mathrm{v}} \end{bmatrix} \tag{3}$$

$$\mathbf{K}^{\mathrm{i}} = \begin{bmatrix} k_{11}^{\mathrm{i}} & k_{12}^{\mathrm{i}} & \cdots & k_{1\mathrm{N}}^{\mathrm{i}} \\ \vdots & \vdots & \ddots & \vdots \\ k_{\mathrm{N1}}^{\mathrm{i}} & k_{\mathrm{N2}}^{\mathrm{i}} & \cdots & k_{\mathrm{NN}}^{\mathrm{i}} \end{bmatrix} \tag{4}$$

From [1], $\mathbf{K}^{\mathrm{v}}$ and $\mathbf{K}^{\mathrm{i}}$ must be related by (5), with $\mathcal{I}_{\mathrm{N}}$ denoting an $[N \times N]$ identity matrix, for conservation of power under the transformation to be guaranteed.

$$\mathbf{K}^{\mathrm{v}\dagger} \mathbf{K}^{\mathrm{i}} = \mathcal{I}_{\mathrm{N}} \tag{5}$$

The incident and reflected waves at each port of both networks A and B can be related to the port voltage and current at that port using the classical power-wave definitions for real port impedances as

$$\mathbf{V}^{\mathrm{A,B}} = \left(\mathbf{Z}^{\mathrm{A,B}}\right)^{\frac{1}{2}} \left(\mathbf{a}^{\mathrm{A,B}} + \mathbf{b}^{\mathrm{A,B}}\right) \tag{6a}$$

$$\mathbf{I}^{\mathrm{A,B}} = \left(\mathbf{Z}^{\mathrm{A,B}}\right)^{-\frac{1}{2}} \left(\mathbf{a}^{\mathrm{A,B}} - \mathbf{b}^{\mathrm{A,B}}\right) \tag{6b}$$

where $\{\mathbf{a}^{\mathrm{A,B}}, \mathbf{b}^{\mathrm{A,B}}\}$ denote $[N \times 1]$ matrices containing the complex magnitudes of the incident and reflected waves respectively, and $\mathbf{Z}^{\mathrm{A,B}}$ are diagonal matrices containing the characteristic port impedances as shown in (7).

$$\mathbf{Z}^{\mathrm{A,B}} = \begin{bmatrix} Z_1^{\mathrm{A,B}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{\mathrm{N}}^{\mathrm{A,B}} \end{bmatrix} \tag{7}$$

Defining the matrices

$$\mathbf{M}_{\mathrm{S}} = \frac{1}{2}\left(\mathbf{Z}^{\mathrm{B}}\right)^{-\frac{1}{2}} \mathbf{K}^{\mathrm{v}} \left(\mathbf{Z}^{\mathrm{A}}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\mathbf{Z}^{\mathrm{B}}\right)^{\frac{1}{2}} \mathbf{K}^{\mathrm{i}} \left(\mathbf{Z}^{\mathrm{A}}\right)^{-\frac{1}{2}} \tag{8a}$$

$$\mathbf{M}_{\mathrm{C}} = \frac{1}{2}\left(\mathbf{Z}^{\mathrm{B}}\right)^{-\frac{1}{2}} \mathbf{K}^{\mathrm{v}} \left(\mathbf{Z}^{\mathrm{A}}\right)^{\frac{1}{2}} - \frac{1}{2}\left(\mathbf{Z}^{\mathrm{B}}\right)^{\frac{1}{2}} \mathbf{K}^{\mathrm{i}} \left(\mathbf{Z}^{\mathrm{A}}\right)^{-\frac{1}{2}} \tag{8b}$$

the incident and reflected waves of network B can be expressed in terms of those of network A by (9a) and (9b).

$$\mathbf{a}^{\mathrm{B}} = \mathbf{M}_{\mathrm{S}}\mathbf{a}^{\mathrm{A}} + \mathbf{M}_{\mathrm{C}}\mathbf{b}^{\mathrm{A}} \tag{9a}$$

$$\mathbf{b}^{\mathrm{B}} = \mathbf{M}_{\mathrm{C}}\mathbf{a}^{\mathrm{A}} + \mathbf{M}_{\mathrm{S}}\mathbf{b}^{\mathrm{A}} \tag{9b}$$

Finally, with $\mathbf{b}^{\mathrm{A}} = \mathbf{S}^{\mathrm{A}}\mathbf{a}^{\mathrm{A}}$ and $\mathbf{b}^{\mathrm{B}} = \mathbf{S}^{\mathrm{B}}\mathbf{a}^{\mathrm{B}}$, it follows from (9a) and (9b) that $\mathbf{S}^{\mathrm{B}}$ can be expressed in terms of $\mathbf{S}^{\mathrm{A}}$ by (10).

$$\mathbf{S}^{\mathrm{B}} = \left(\mathbf{M}_{\mathrm{C}} + \mathbf{M}_{\mathrm{S}}\mathbf{S}^{\mathrm{A}}\right)\left(\mathbf{M}_{\mathrm{S}} + \mathbf{M}_{\mathrm{C}}\mathbf{S}^{\mathrm{A}}\right)^{-1} \tag{10}$$

A very important special case occurs when the port impedances of network B are related to the port impedances of network A by (11).

$$\mathbf{Z}^{\mathrm{B}} = \mathbf{K}^{\mathrm{v}}\,\mathbf{Z}^{\mathrm{A}}\,\left(\mathbf{K}^{\mathrm{i}}\right)^{-1} \tag{11}$$

In this case, $\mathbf{M}_{\mathrm{C}} = \mathbf{0}$, and the transformation in (10) simply becomes that shown in (12).

$$\mathbf{S}^{\mathrm{B}} = \mathbf{M}_{\mathrm{S}}\mathbf{S}^{\mathrm{A}}\left(\mathbf{M}_{\mathrm{S}}\right)^{-1} \tag{12}$$

where

$$\mathbf{M}_{\mathrm{S}} = \left(\mathbf{Z}^{\mathrm{B}}\right)^{-\frac{1}{2}} \mathbf{K}^{\mathrm{v}} \left(\mathbf{Z}^{\mathrm{A}}\right)^{\frac{1}{2}} \tag{13}$$

Note that this special case is the one most widely used for the transformation of single-ended S-parameters to mixed-mode S-parameters.

3 SIX-CONDUCTOR TRANSMISSION LINE

The application of the transformation to reduce model complexity can be illustrated by the use of multi-line transmission lines, such as the six-line transmission line shown in Fig. 2. The lines are arranged as three pairs, positioned at a fixed radius from the centre. A dielectric half-cylinder is inserted in the centre of the line to provide a discontinuity. For this example, the line is 50mm in length, with an outer radius of 5mm. Each conductor has a radius of 0.5mm, and is positioned at a radius of 3mm from the centre, with each pair at a 120 degree angle with respect to the other pairs. The dielectric disc has a permittivity of 20, and is 1mm in thickness.

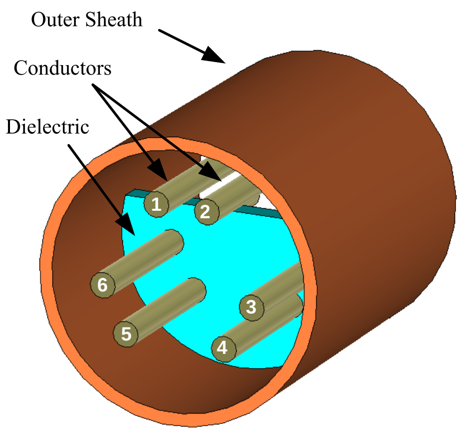


Figure 2: Six-conductor transmission line

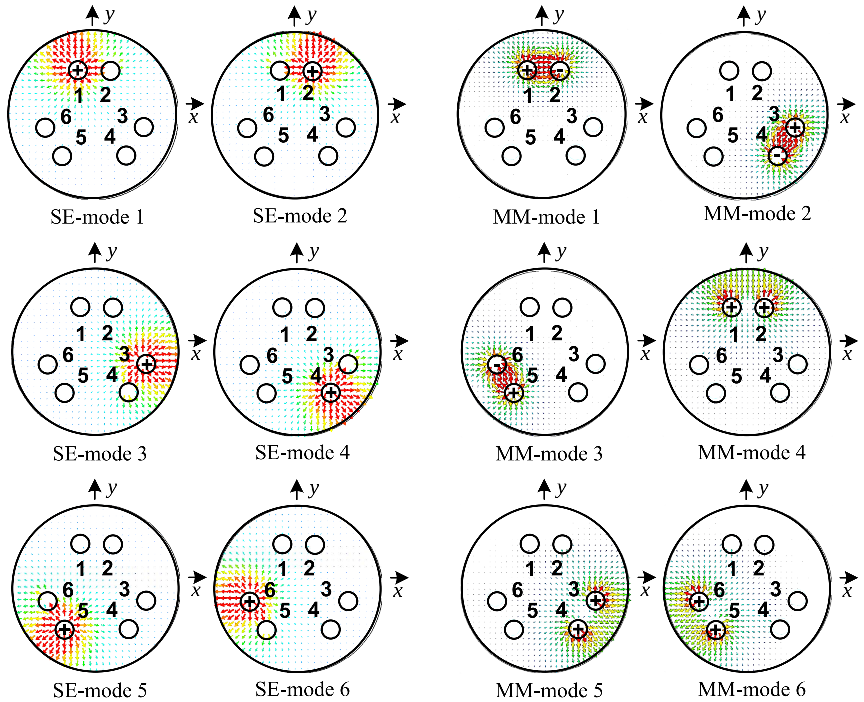
An infinite number of sets of S-parameters can be calculated for this structure. For the purposes of this paper, only two will be shown. Firstly, each conductor can be excited at each of its ports, with all the others terminated in 50 ohm loads. Such an analysis can readily be performed using a single-ended multipin waveguide feed in CST Microwave Studio, with a fast two-dimensional analysis to solve the port impedance for each mode, and a full three-dimensional analysis for the S-matrix. The six port modes established in this way at each port will be denoted as *SE-modes*. The electric field patterns for this set are shown in Fig. 3(a). For six lines at each of two ports, this results in a 12x12 complex S-matrix. For simplicity, only a dependence on frequency will be used here. To model this problem using surrogate models, 144 one-variable models are therefore required. While this can be reduced substantially using reciprocity and symmetry in the longitudinal direction, the dielectric disc reduces the number of symmetries in the transverse plane.

Using the generalized multi-mode S-parameter transformation, various other sets of S-parameters can be computed from the single-ended set, without any additional full-wave analyses. A typical set for this structure is the one obtained by exciting each set of two conductors with both common-mode and differential-mode signals at each port. These excitations as well as their respective electric field distributions are shown in Fig. 3(b) and denoted as *MM-modes*. To calculate the MM S-matrix, any set of port impedances can in principle be used. Here, the port impedance calculated for each MM-mode by a fast two-dimensional analysis in CST is used. For this analysis, each mode of the set is excited using a multi-pin feed with the indicated polarity.

From the two-dimensional CST analysis, and with the single-ended case represented by network A (denoted *SE*) and the multi-mode case by network B (denoted *MM*), the impedance matrices necessary for the transformation in (10) are shown in (14).

$$\begin{aligned} \mathbf{Z}^{\mathrm{SE}} &= diag\,(50) \\ \mathbf{Z}^{\mathrm{MM}} &= diag\,(138, 138, 138, 138, 138, 138, 70, 70, 70, 70, 70, 70) \end{aligned} \tag{14}$$

From standard definitions for differential and common-mode voltages and currents, the matrices



(a) SE port modes (b) MM port modes

Figure 3: Electric field distributions for port modes

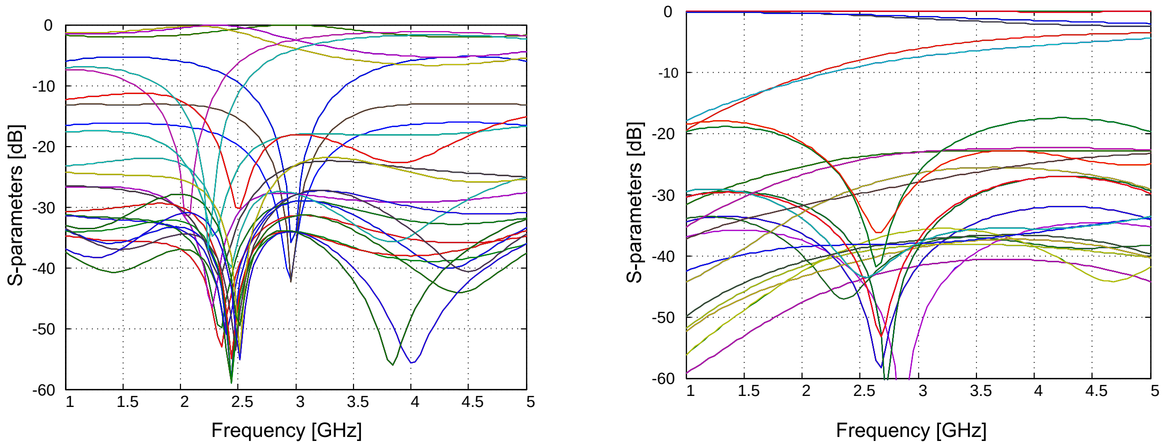
$\mathbf{K}^{\text{v,i}}$ can be constructed as in (15)

$$\mathbf{K}^{\text{v}} = \begin{bmatrix} \mathbf{K}^{\text{p}} \\ \frac{1}{2}|\mathbf{K}^{\text{p}}| \end{bmatrix} \qquad \mathbf{K}^{\text{p}} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{15}$$

with $\mathbf{K}^{\text{i}} = \left(\mathbf{K}^{\text{v}\dagger}\right)^{-1}$ from (5).

The magnitudes of the two sets of 144 S-parameters are shown in Fig. 4. It is clear that in the multi-mode set, a number of S-parameters are small enough to be neglected (¡-60dB across the range), resulting in fewer parameters which need to be modelled. In addition, the non-zero S-parameters frequently occur in identical sets, and in general show less variation as functions of frequency than the single-ended set. This requires lower order surrogate functions for a similar modelling accuracy. Taken in combination, the change in the choice of excitation functions will result in a significant simplification of the required multiport surrogate model.

The given multi-mode set is of course only one example, and in general, each problem can be analyzed in order to find the optimum transformation matrix which will result in the smallest S-parameter set which can be modelled with the lowest order functions. The transformation technique has very few limitations mathematically, and can be included into most surrogate algorithms very easily as a data pre-conditioning step.



(a) Single-ended (b) Multi-mode

Figure 4: Single-ended and multi-mode S-parameters

4 CONCLUSION

This paper shows how the generalized multi-mode scattering parameter transformation can be used to reduce the required number and order of modelling functions required to create an accurate surrogate model. A twelve-port, six-line transmission line problem is used here as an example, but the technique is not limited to specific numbers or lines, or any specific sets of excitations.

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