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A SIMPLIFIED MODEL OF A STEEL COLUMN SUBJECTED TO IMPACT

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Abstract. *The impact response of structures becomes of great interest for recent research activities due to the concern for public safety regarding the accidents (e.g. a vehicle collision onto the building structure). In the event of impact, the structure can be expected to undergo large plastic deformation which is required to absorb kinetic energy generated from the moving vehicle. In this paper, the nonlinear response of a steel column subjected to transversal vehicle collision is studied. The deformation of the column is assumed to occur at critical locations where the generalized plastic hinges are formed. A simplified model which consists of rigid bars connected by elasto-plastic hinges is presented. To include the effect of the surrounding structure, an elastic spring and a mass are attached to the top end of the column where a constant compressive load is also applied. The elliptical yield surfaces in a stress resultant space that account for the interaction between bending moment and axial force are used. Regarding the impact load, the unilateral contact is adopted while friction is ignored. Because of the reason that the velocities and the contact force cannot be defined at impact point of time, the differential measures are written to combine the smooth and non-smooth parts. The set-value force laws including Newton's impact law are adopted to describe the contact condition. Concerning with the time integration, the classical midpoint rule scheme is employed to integrate the equations of motion and the flow rule of the hinges. The paper also presents numerical applications of the model to show its ability to describe the nonlinear dynamic behavior of a steel column under transversal vehicle collision.*

1 INTRODUCTION

Typical frame buildings, when subjected to extreme loading such as explosion or impact, may have high risks to experience progressive collapse. It is suggested that such progressive collapse is initiated by local damage of the supporting elements, i.e. columns. There are at least two directions to follow in order to prevent the progressive collapse. The first direction is to assume that one of the columns has failed due to the mentioned exceptional loading and thus removed. The response of the remaining structures are then investigated. This type of analysis can be found in [1, 2, 3, 4]. The other method is to focus on the response of the suffered column. Several publications as found in [5, 6, 7, 8] and the references cited therein have worked on the behaviour of reinforced concrete structures against extreme loading. However, due to the fact that concrete has low ductility compared to steel, the design concept that bases on stiff member behaviour brings about massive concrete columns. Yet, the design of steel building against impact loading is not clearly explained in current practical norms [9, 10, 11, 12]. With high ductility, steel members are allowed to have large deformation, which is beneficial to the response of the structure under impact. The structure with large displacement has the ability to absorb the kinetic energy generating from the moving vehicle. Some authors such as in [13, 14, 15] used an equivalent single degree of freedom system which is proposed by Biggs in [16] in order to study the dynamical response of the column subjected to extreme loading. Despite of the practical convenience, the method is not able to include membrane effect and the geometric nonlinearity. The influence of axial force on the plastic bending moment is also not included. This paper describes a simplified nonlinear model of a steel column subjected to a compression load and a transverse vehicle collision at any point along the column length for any support conditions. The inelastic behaviour of the column system is taken into account by employing the generalized plastic hinge method. With this model, the deformation of the column is assumed to concentrate at the generalized plastic hinges that form at critical locations. The column elements between the hinges are considered as rigid. The non-smooth problem of impact contains the discontinuity of the velocity. A set of measure - differential equations are used to describe the impact and impact - free motion. The numerical integration of the equations are solved by adopting a classical midpoint rule.

2 DESCRIPTION OF THE SIMPLIFIED MODEL

Described by Fig. 2, the model of the steel column consists of two rigid bars and four generalized elastic-plastic hinges. The top of the column is attached to a spring and applied by a compressive load. Illustrated in Fig. 1.a, the generalized plastic hinges which are modelled by a combined axial-rotational spring as shown in Fig. 1.b situate at both ends of the column and at the location where the force is applied. The material is assumed to be elastic perfectly plastic and the strain rate is ignored. Only axial and flexural deformations are considered whereas shear deformations are neglected. The elongation and shortening of the column are assumed to occur along the column.

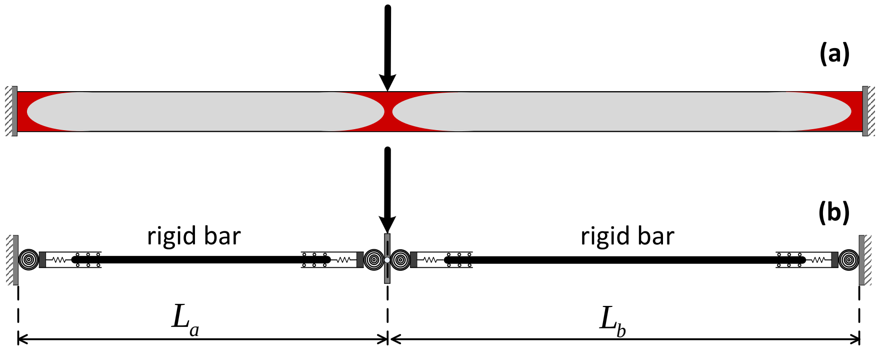

Figure 1: (a): plastic hinge formation. (b): simplified model.

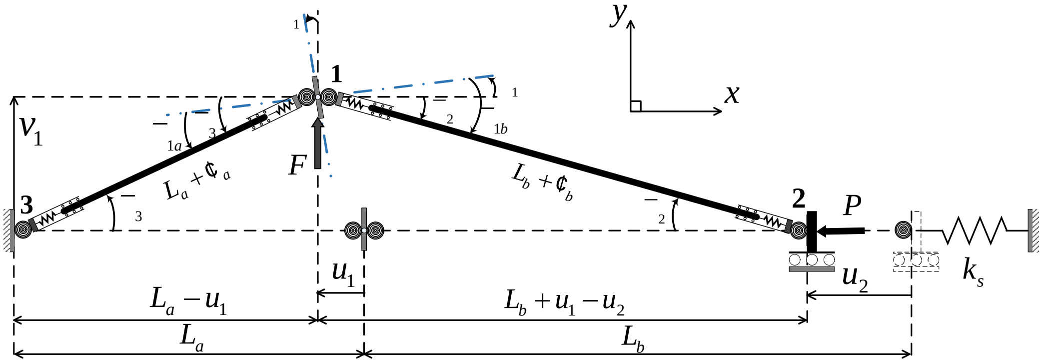

Figure 2: Presentation of simplified model

3 KINEMATIC OF THE MODEL

The kinematic of the model is shown in Fig.2. The model has four degrees of freedom, and the displacement vector is defined by:

$$\boldsymbol{U} = [\, v_1,\, u_1,\, \beta_1,\, u_2 \,]^{\mathrm{T}} \tag{1}$$

The deformation vector $\boldsymbol{E}$ is obtained:

$$\boldsymbol{E} = [\, \delta_a,\, \theta_{1a},\, \delta_b,\, \theta_{1b},\, u_2,\, \theta_2,\, \theta_3 \,]^{\mathrm{T}} \tag{2}$$

The components of the deformation vector $\boldsymbol{E}$ are related to the components of the displacement vector through:

$$\theta_2 = \arctan\left(\frac{v_1}{L_b + u_1 - u_2}\right) \tag{3}$$

$$\theta_3 = \arctan\left(\frac{v_1}{L_a - u_1}\right) \tag{4}$$

$$\theta_{1a} = \theta_3 - \beta_1 \tag{5}$$

$$\theta_{1b} = \theta_2 + \beta_1 \tag{6}$$

$$\delta_a = \sqrt{v_1^2 + (L_a - u_1)^2} - L_a \tag{7}$$

$$\delta_b = \sqrt{v_1^2 + (L_b + u_1 - u_2)^2} - L_b \tag{8}$$

The incremental relation between the displacements and deformations is obtained by differentiating the above equations:

$$\delta \boldsymbol{E} = \frac{\partial \boldsymbol{E}(\boldsymbol{U})}{\partial \boldsymbol{U}} \delta \boldsymbol{U} = \boldsymbol{B}\, \delta \boldsymbol{U} \tag{9}$$

in which

$$\boldsymbol{B} = \begin{bmatrix} \sin\theta_3 & -\cos\theta_3 & 0 & 0 \\ \dfrac{\cos\theta_3}{\tilde{L}_a} & \dfrac{\sin\theta_3}{\tilde{L}_a} & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & -\cos\theta_2 \\ \dfrac{\cos\theta_2}{\tilde{L}_b} & -\dfrac{\sin\theta_2}{\tilde{L}_b} & 1 & \dfrac{\sin\theta_2}{\tilde{L}_b} \\ 0 & 0 & 0 & 1 \\ \dfrac{\cos\theta_2}{\tilde{L}_b} & -\dfrac{\sin\theta_2}{\tilde{L}_b} & 0 & \dfrac{\sin\theta_2}{\tilde{L}_b} \\ \dfrac{\cos\theta_3}{\tilde{L}_a} & \dfrac{\sin\theta_3}{\tilde{L}_a} & 0 & 0 \end{bmatrix} \tag{10}$$

where $\tilde{L}_a = L_a + \delta_a$ and $\tilde{L}_b = L_b + \delta_b$. By inserting Eqs. (3-8) into Eq. (10), the transformation matrix $\boldsymbol{B}$ is completely in function of $\boldsymbol{U}$.

4 STATIC EQUILIBRIUM OF THE SYSTEM

The equation of the virtual work can be expressed as:

$$\delta \boldsymbol{E}^{\mathrm{T}} \boldsymbol{S} = \delta \boldsymbol{U}^{\mathrm{T}} \boldsymbol{f}^{ext} \tag{11}$$

where $\boldsymbol{S}$ is the internal force vector (see Fig. 3) conjugate to the deformation vector $\boldsymbol{E}$. It is given by:

$$\boldsymbol{S} = [\, N_a, M_{1a},\, N_b,\, M_{1b},\, F_2,\, M_2,\, M_3]^{\mathrm{T}} \tag{12}$$

in which N_a and N_b are the normal forces in elements a and b, respectively. F_2 is the force in the spring attached to node 2.

$$F_2 = k_s u_2 \tag{13}$$

The external force vector(See Fig.2) is given by

$$\boldsymbol{f}^{ext} = [\, F\,,\, 0\,,\, 0\,,\, P\,]^{\mathrm{T}} \tag{14}$$

where F and P denote external transversal load and external static compressive load, respectively. Inserting Eq. (9) into Eq. (11) gives the relation between the internal force vector and the external force vector, that is the equilibrium equations:

$$\boldsymbol{f}^{ext} = \boldsymbol{B}^{\mathrm{T}} \boldsymbol{S} \tag{15}$$

Denoting

$$\boldsymbol{f}_g^{int} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{S} \tag{16}$$

the global internal force vector, the equilibrium equation becomes:

$$\boldsymbol{f}^{ext} = \boldsymbol{f}_g^{int} \tag{17}$$

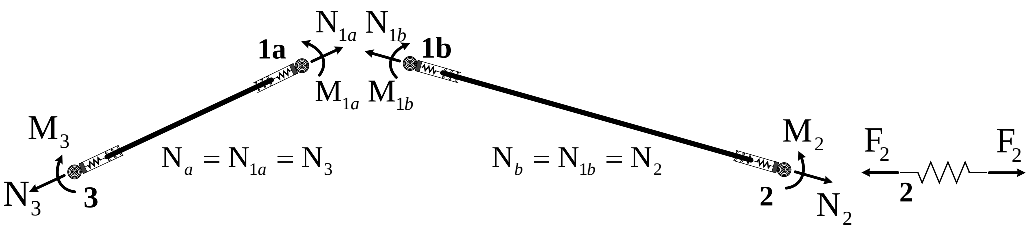

Figure 3: Stress resultant diagram

The elongations associated to the normal forces at each hinge are calculated from:

$$\delta_{1a} = \alpha_a \delta_a \tag{18}$$

$$\delta_{1b} = \alpha_b \delta_b \tag{19}$$

$$\delta_2 = (1 - \alpha_b)\delta_b \tag{20}$$

$$\delta_3 = (1 - \alpha_a)\delta_a \tag{21}$$

The parameters α_a and α_b describe the repartition of the total elongations of bars a and b at their ends.

5 COMPLEX ELASTO-PLASTIC MODELS FOR GENERALIZED PLASTIC HINGE

The elastic behavior of the generalized hinge is uncoupled whereas axial-moment interaction is considered in the plastic range. We adopt the total generalized strain rate decomposition into elastic and plastic parts

$$\dot{\boldsymbol{d}} = \dot{\boldsymbol{d}}^e + \dot{\boldsymbol{d}}^p \tag{22}$$

where $\dot{\boldsymbol{d}} = \left[\,\dot{\theta}, \dot{\delta}\,\right]^{\mathrm{T}}$. For an associated flow rule, the direction of the generalized plastic strain rate vector is given by the gradient to the yield function, with its magnitude given by the plastic multiplier rate $\dot{\lambda}$:

$$\dot{\boldsymbol{d}}^p = \dot{\lambda}\,\frac{\partial \Phi}{\partial \boldsymbol{s}} \tag{23}$$

where $\boldsymbol{s} = [\,M, N\,]^{\mathrm{T}}$ is the generalized stress vector containing the bending and axial forces in the hinge. The plastic multiplier $\dot{\lambda}$ is determined by the classical complementary conditions:

$$\dot{\lambda} \geq 0, \qquad \Phi(M,N) \leq 0, \qquad \dot{\lambda}\Phi(M,N) = 0 \tag{24}$$

The above relations characterize the loading/unloading conditions. For $\dot{\lambda} > 0$, persistency of the plastic state during plastic flow requires:

$$\dot{\lambda}\,\dot{\Phi}(M,N) = 0 \tag{25}$$

In this paper, we consider a family of symmetric and convex yield surface of superelliptic shape:

$$\Phi(M,N) = \|\boldsymbol{s}\|_{\beta_p} - 1 = 0 \tag{26}$$

where $\| \bullet \|_{\beta_p}$ denotes the superelliptic norm:

$$\|\boldsymbol{s}\|_{\beta_p} = \left(\left| \frac{M}{M^p} \right|^{\beta} + \left| \frac{N}{N^p} \right|^{\beta} \right)^{\frac{1}{\beta}} \tag{27}$$

with $1 \leq \beta < \infty$. The curve intersects x-axis at N^p and $-N^p$; it intersect the y-axis at M^p and $-M^p$. The coefficient β, called roundness factor, controls the shape of the yield criterion. This shape evolves from a parallelogram ($\beta = 1$) to a rectangular($\beta \to \infty$). The limit curve becomes a pinched diamond for $\beta < 1$ and is no longer convex. When β goes over 2, the curve become a rounded rectangle. The above family of yield surfaces is further widened by considering generalized superelliptic shapes described by

$$\Phi(M,N) = \left(\left| \frac{M}{M^p} \right|^{\alpha} + \left| \frac{N}{N^p} \right|^{\beta} \right)^{\frac{1}{\gamma}} - 1 \tag{28}$$

For $\alpha = 1$ and $\gamma = 1$ we recover the yield function to the one proposed by Duan and Chen in [17] for a doubly symmetrical steel cross section which is mirrored to give a complete envelop by:

$$\Phi(M,N) = \left| \frac{M}{M^p} \right| + \left| \frac{N}{N^p} \right|^{\beta} - 1 \tag{29}$$

An associated flow rule is adopted where plastic deformation (axial elongation and rotation) are constrained to occur in a direction normal to the yield surface

$$\dot{\theta}^p = \dot{\lambda} \frac{\partial \Phi}{\partial M} = \dot{\lambda} \left(\left| \frac{M}{M^p} \right|^{\alpha} + \left| \frac{N}{N^p} \right|^{\beta} \right)^{\frac{1-\gamma}{\gamma}} \frac{\alpha}{\gamma \left(M^p\right)^{\alpha}} \left|M\right|^{\alpha-1} sgn(M) \tag{30}$$

$$\dot{d}^p = \dot{\lambda} \frac{\partial \Phi}{\partial N} = \dot{\lambda} \left(\left| \frac{M}{M^p} \right|^{\alpha} + \left| \frac{N}{N^p} \right|^{\beta} \right)^{\frac{1-\gamma}{\gamma}} \frac{\beta}{\gamma \left(N^p\right)^{\beta}} \left|N\right|^{\beta-1} sgn(N) \tag{31}$$

where the *signum* function $sgn(\bullet)$ is defined by

$$sgn(x) = \begin{cases} 1 & for \quad x > 0 \\ 0 & for \quad x = 0 \\ -1 & for \quad x < 0 \end{cases} \tag{32}$$

6 NONSMOOTH DYNAMICS: IMPACT LOADING

6.1 Contact model

The model, see Fig. 4, considers the collision between a vehicle, modelled by a rigid point mass m_c, and a column represented by the model proposed in Section 2. The mass of the column is assumed to be concentrated at the impact point. Since the column is modelled by two

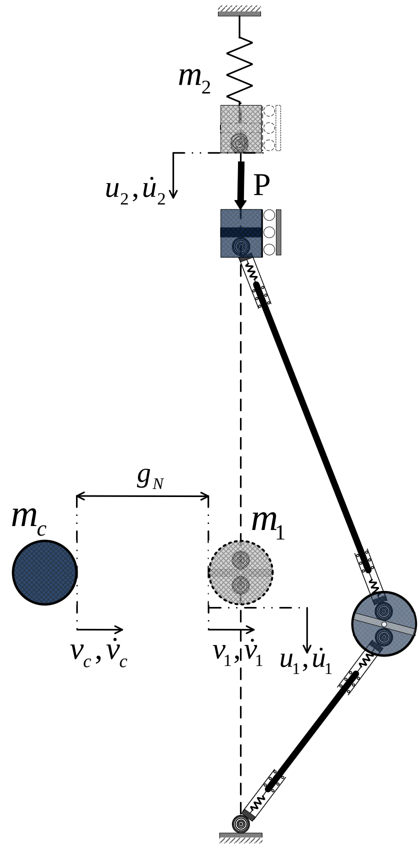

Figure 4: Contact model

rigid bars, the concentrated mass m_1 can be taken as a third of the total mass of the column. Unilateral contact is assumed with a gap function given by:

$$g_N = v_c - v_1 \tag{33}$$

where v_c is the position of the mass m_c and v_1 is the position of the mass m_1. The relative velocity is given by:

$$\gamma_N = \dot{v}_c - \dot{v}_1 \tag{34}$$

6.2 Constraint law

Before the equations of motion of the impacted bodies can be written, the constraint laws must be imposed so that one can detect when contact and impact occur. The constraint law for a unilateral contact is described by a Signorini's law at a position level below:

$$g_N \geq 0\,;\ \lambda_N \geq 0\,;\ g_N \lambda_N = 0 \tag{35}$$

To accommodate energy dissipation during collision and proposed the following impact law:

$$\gamma_N^+ = -\varepsilon \gamma_N^- \tag{36}$$

where γ_N^+ denotes relative velocity after impact and γ_N^- denotes relative velocity before impact. $\varepsilon \in [0, 1]$ is the restitution coefficient; $\varepsilon = 1$ for a purely elastic contact and $\varepsilon = 0$ for a completely inelastic contact. By inserting the Newton's impact law, the constraint law at the velocity level can be written as:

$$\xi_N \geq 0, \qquad -\Lambda_N \leq 0, \qquad \xi_N \Lambda_N = 0 \tag{37}$$

where $\xi_N = \gamma_N^+ + \varepsilon \gamma_N^-$ and Λ_N denotes the percussion force. The above relations (37) can be combined together to give the following variational inequality:

$$(-\Lambda_N) \in \mathbb{R}_0^-, \qquad \xi_N \left(\Lambda_N^* - \Lambda_N\right) \geq 0, \qquad \forall \left(-\Lambda_N^*\right) \in \mathbb{R}_0^- \tag{38}$$

6.3 Equations of Motion

Depending on the value of the gap g_N, see Eq. (35), the equations of motion must be considered into two cases, open contact ($g_N > 0$) and closed contact ($g_N = 0$).

6.3.1 Open contact

The model is illustrated in Fig. 4. In addition to the masses m_1 and m_c, the mass m_2 of the surrounding structure at the top of the column is also considered. The dynamic effect associated with the rotation β_1 (see Fig. 2) of mass m_1 is neglected. Hence, the equations of the open contact motion are:

$$\begin{aligned}
&m_c \, \ddot{v}_c = 0 \\
&m_1 \, \ddot{v}_1 + \boldsymbol{f}_g^{\text{int}}(1) = 0 \\
&m_1 \, \ddot{u}_1 + \boldsymbol{f}_g^{\text{int}}(2) = 0 \\
&m_2 \, \ddot{u}_2 + \boldsymbol{f}_g^{\text{int}}(4) - P = 0 \\
&\boldsymbol{f}_g^{\text{int}}(3) = 0
\end{aligned} \tag{39}$$

where $\boldsymbol{f}_g^{\text{int}} = \boldsymbol{B}^{\mathrm{T}} \boldsymbol{S}$ is the global internal force vector defined in Eq. (16). In this work, midpoint scheme is used to solve the dynamic equations. By introducing midpoint scheme in the equations of motion (Eqs. (39)) at the time $n + \frac{1}{2}$, it is obtained:

$$\begin{aligned}
&\frac{2m_c}{\Delta t}\left(\frac{{v_c}^{n+1} - {v_c}^n}{\Delta t} - {\dot{v}_c}^n\right) = 0 \\
&\frac{2m_1}{\Delta t}\left(\frac{{v_1}^{n+1} - {v_1}^n}{\Delta t} - {\dot{v}_1}^n\right) + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(1) = 0 \\
&\frac{2m_1}{\Delta t}\left(\frac{{u_1}^{n+1} - {u_1}^n}{\Delta t} - {\dot{u}_1}^n\right) + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(2) = 0 \\
&\frac{2m_2}{\Delta t}\left(\frac{{u_2}^{n+1} - {u_2}^n}{\Delta t} - {\dot{u}_2}^n\right) + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(4) - P = 0 \\
&\boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(3) = 0
\end{aligned} \tag{40}$$

Since $\boldsymbol{f}_g^{\text{int}}$ is nonlinear, Newton Raphson iterations are required in order to solve Eq. (40).

6.3.2 Closed contact

During closed contact, the velocity may jump at the time instances when the impacts occur. At those times, the velocity is not differentiable and the contact force is impulsive. A set of differential measures are adopted to write a combined equation of motion so that one does not have to write separately the equation for smooth part and impact part of the close contact. The equations of motion of the colliding masses, m_c and m_1 for a closed contact motion using a set of measure equation is now written as:

$$\begin{aligned} & m_c\, d\dot{v}_c = d\Lambda_N \\ & m_1\, d\dot{v}_1 + \boldsymbol{f}_g^{\text{int}}(1)dt = -d\Lambda_N \\ & g_N = 0 \;,\; \xi_N \geq 0 \end{aligned} \tag{41}$$

where the sets of measure for the velocity and contact force are given by:

$$d\dot{v} = \dot{v}\, dt + (\dot{v}^+ - \dot{v}^-) \tag{42}$$

$$d\Lambda_N = \lambda_N dt + \Lambda_N d\eta \tag{43}$$

By using

$$\int_n^{n+1} \boldsymbol{f}_g^{\text{int}} dt = \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}} \Delta t \;;\; \int_n^{n+1} d\Lambda_N = P_N \;;\; \int_n^{n+1} d\dot{v} = \dot{v}^{n+1} - \dot{v}^n \tag{44}$$

Eqs. (41) can be integrated between t_n and t_{n+1}. By considering also the equations associated with other degrees of freedom, it is obtained:

$$\begin{aligned} & m_c({\dot{v}_c}^{n+1} - {\dot{v}_c}^{n}) = P_N \\ & m_1({\dot{v}_1}^{n+1} - {\dot{v}_1}^{n}) + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(1)\Delta t = -P_N \\ & m_1\, \ddot{u}_1 + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(2) = 0 \\ & m_2\, \ddot{u}_2 + \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(4) - P = 0 \\ & \boldsymbol{f}_g^{\text{int},n+\frac{1}{2}}(3) = 0 \end{aligned} \tag{45}$$

Before solving Eqs. (45), the constraint law in Eq. (37) is considered in order to determine if the contact (or impulsive) force P_N is present or not. The following methodology is adopted. First, the contact(or impulsive) force is assumed to be zero and Eqs. (45) are solved by using the midpoint scheme. Then, ξ_N, defined in Eq. (37) is calculated as:

$$\xi_N = \gamma_N^{n+1} + \varepsilon \gamma_N^n \tag{46}$$

If ξ_N is positive, then the prediction of no contact force is true. If ξ_N is negative, then the contact or impulsive force exists and has a positive value. In such case, the equation $\xi_N = 0$, is added to Eqs. (45) in order to calculate both the kinematic quantities and the contact (or impulsive) force.

7 NUMERICAL EXAMPLES

7.1 Parameters

The purpose of the examples presented in this section is to show the ability of the proposed model in capturing the main features of the steel column's behavior under the vehicle impact. A numerical example of a build - in column of type HEB 220 impacted by a rigid mass is given. The parameters of the problem are given in Table 1. The other necessary parameters are described as follows. The stiffness of the elastic spring, the compressive load and the mass attached to top end of the column are respectively provided: $k_s = 10\,\mathrm{MN/m}$, $P = 0.5\,\mathrm{MN}$ and $m_2 = 0\,\mathrm{kg}$.

Table 1: Parameters of the model

Material properties		
Young modulus	E	$210\,000\,\mathrm{MPa}$
Yield strength	f_y	$355\,\mathrm{MPa}$
Column Type: HEB 220		
Section area	A	$91\,\mathrm{Cm}^2$
Moment of inertia	I	$8\,091\,\mathrm{Cm}^4$
Nominal weight	m_l	$70.1\,\mathrm{kg/m}$
Length	L_a	$1\,\mathrm{m}$
	L_b	$3\,\mathrm{m}$
Bending resistance	M^p	$0.307\,\mathrm{MNm}$
Axial resistance	N^p	$3.23\,\mathrm{MN}$
Impactor		
Mass	m_c	$1\,500\,\mathrm{kg}$
Initial velocity	v_c^0	$10\,\mathrm{m/s}^2$
initial gap	g_N	$0\,\mathrm{m}$
Restitution coefficient	ε	0

The elastic stiffness of the axial and rotational springs for a build - in column are provided as:

$$k_{1a}^{\theta} = k_3^{\theta} = \frac{6EI}{L_a} \tag{47}$$

$$k_{1b}^{\theta} = k_2^{\theta} = \frac{6EI}{L_b} \tag{48}$$

$$k_{1a}^{\delta} = k_3^{\delta} = \frac{2EA}{L_a} \tag{49}$$

$$k_{1b}^{\delta} = k_2^{\delta} = \frac{2EA}{L_b} \tag{50}$$

Fig. 5 illustrates the evolution of the horizontal displacement of both masses with time. Immediately from the beginning, both masses come into contact and remain in such motion until the energy absorbing capacity overcomes the motion and both masses are pushed back and then lose contact. Fig. 6 show the nonlinear changes of vertical displacements with time. The normal forces in each hinges start with an initial compression value as shown in Fig. 7. As long as the impact loading is applied, the normal forces begin to decrease their values toward zero. From the evolution of the bending moments in Fig. 8, one can infer that each hinge plasticizes at a different time.

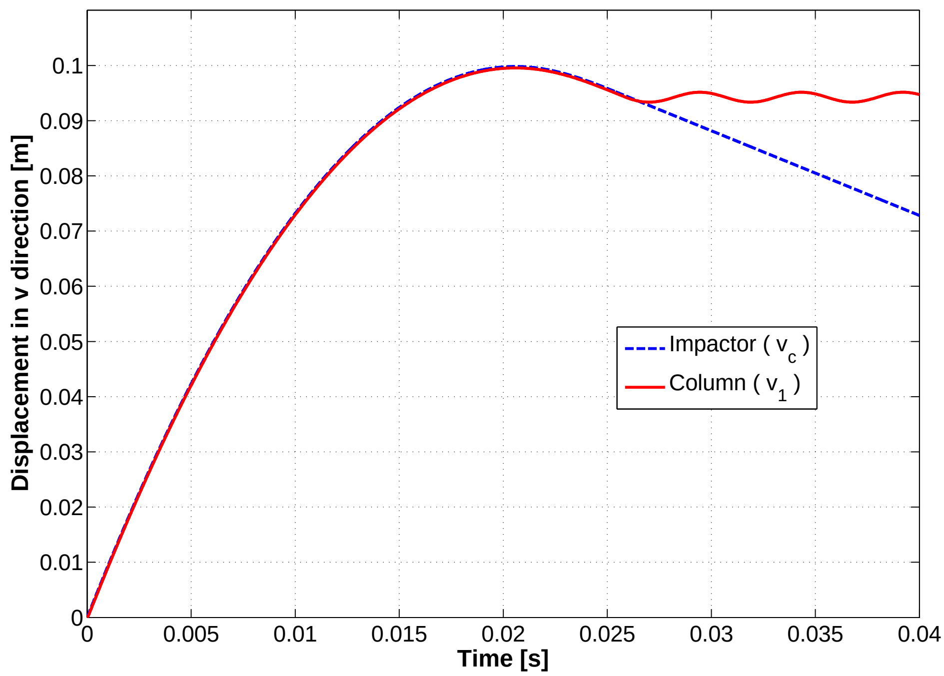

Figure 5: Evolution of horizontal displacement v_1

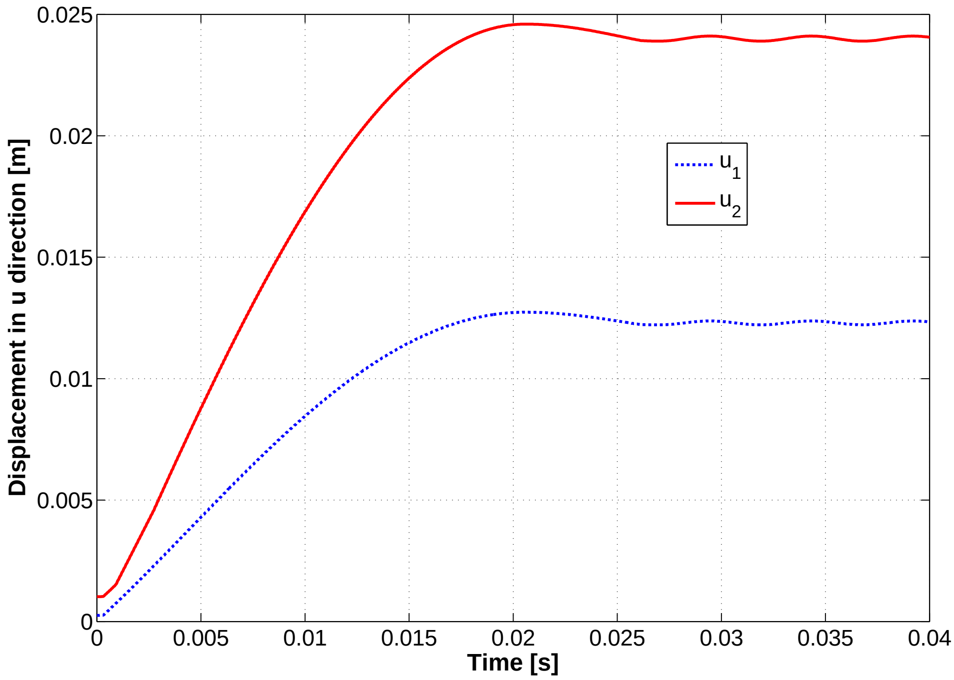

Figure 6: Evolution of vertical displacement u_1 and u_2

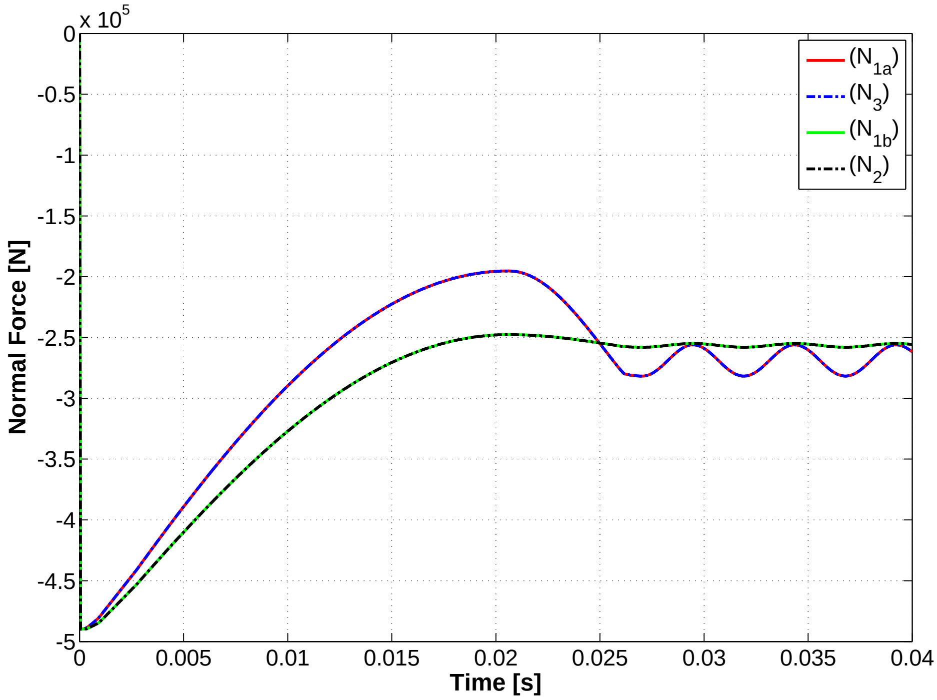

Figure 7: Evolution of normal forces

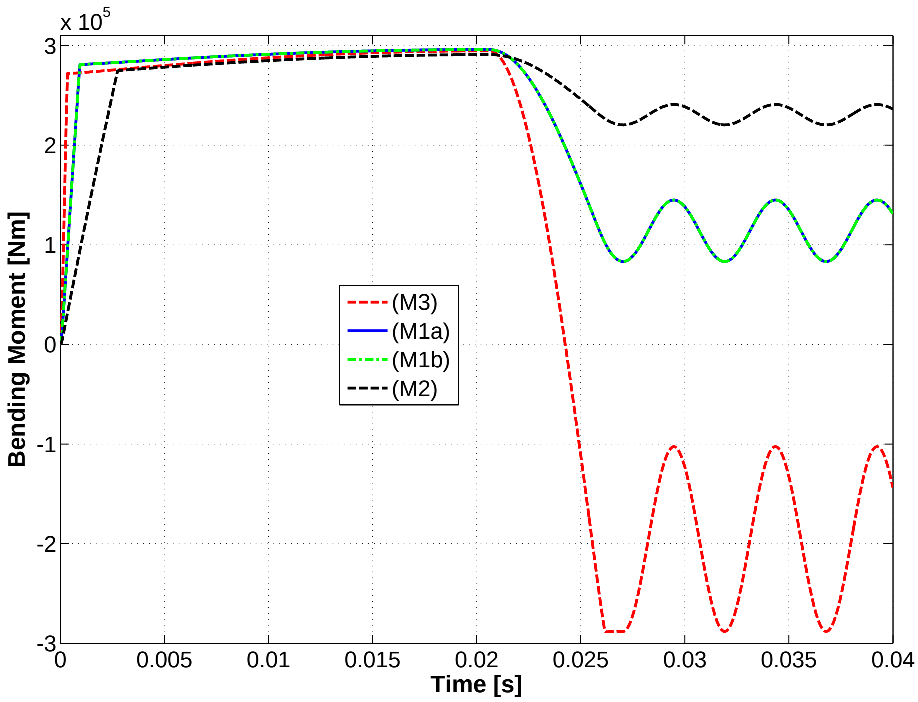

Figure 8: Evolution of bending moments

8 CONCLUSION

In the paper, the new nonlinear model of a steel column subjected to a transversal vehicle mass was presented. The nonlinear geometry and material are included in the model. The generalized plastic hinge method is adopted in order to model a column by using two rigid elements connected with elasto - plastic hinges. The yield criterion od the hinges are governed by a variety of super - elliptic shape functions. For the non -smooth problem, the constraint laws such as Signorini's force law and Newton impact law are imposed to write the equations of motion. A set of differential measure equations are adopted to write overall equations of close - contact motion in order to describe both the impact and impact - free motion. The integration of the equations of motion are executed using the classical mid - point rule scheme.

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