ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

MODELLING THE PROCESS OF SEQUENTIAL EXCAVATION WITH THE BOUNDARY ELEMENT METHOD

C. Duenser[1] **and G. Beer**[1,2]

[1]Institute for Structural Analysis, Graz University of Technology
Lessingstrasse 25, 8010 Graz, Austria
e-mail: duenser@tugraz.at

[2] Centre for Geotechnical and Materials Modeling University of Newcastle
Callaghan, NSW 2308, Australia
e-mail: beer@tugraz.at

Keywords: Sequential Excavation, BEM, BETI, DDM.

Abstract. *The Boundary Element Method (BEM) is ideally suited for the simulation of underground constructions like tunnels or caverns. Such structures are modelled with the BEM inside an infinite or semi-infinite domain. As the radiation condition is fulfilled by the BEM no truncation of the domain is necessary. Only the surface of the structure (e.g. tunnel) has to be discretised by boundary elements (BE). An accurate simulation of the tunnelling process has to consider the sequential excavation where parts of the rock mass are excavated at different time and location. This special constructional condition has a direct influence onto the simulation model. In this work different methods are presented which consider the sequential excavation.*
The first method is the discretisation of the problem by multiple BE regions (MRBEM). Each region, which will be excavated during the excavation process, is discretised by a separate finite BE region. These regions are embedded inside an infinite region which represent the infinite extend of the domain. Thus, a system of BE regions arise which have to be coupled at their common interfaces. Two coupling strategies, the Boundary Element Tearing and Interconnecting Method (BETI) and the method of Interface Coupling (IC) will be presented to solve the sequential tunnel excavation.
The second method uses only a single BE region (SRBEM) for every step of excavation. For each load step the geometry/mesh has to be updated. Thus, the mesh of the previous load step will be extended by the surface of the new excavation volume of the current load step. Beside the geometry update an essential part of this method is an accurate evaluation of the excavation loading. The excavation loads for each excavation step are tractions applied at the part of the boundary surface just generated by the geometry update. These tractions depend on all previous load steps and will be evaluated by a calculation of internal results in the interior of the single region. The internal results can be either stresses or displacements.
In this work the modelling strategies of the MRBEM and SRBEM approach will be presented. On a realistic tunnel example the accuracy of the results for the mentioned methods will be shown as well as the performance of the calculations.

1 INTRODUCTION

Among other characteristics the construction of tunnels within the New Austrian Tunnelling Method is characterised by the sequential excavation of the tunnel. A typical sequential excavation is shown in the Fig. 1 where top heading and bench excavation is performed. Thus the cross section of the tunnel is divided into two parts, top heading and bench. Within the sequential excavation those volume parts are excavated at different time and location. As shown in the figure, in the longitudinal direction of the tunnel the top heading excavation is more advanced as the bench excavation at the same time.

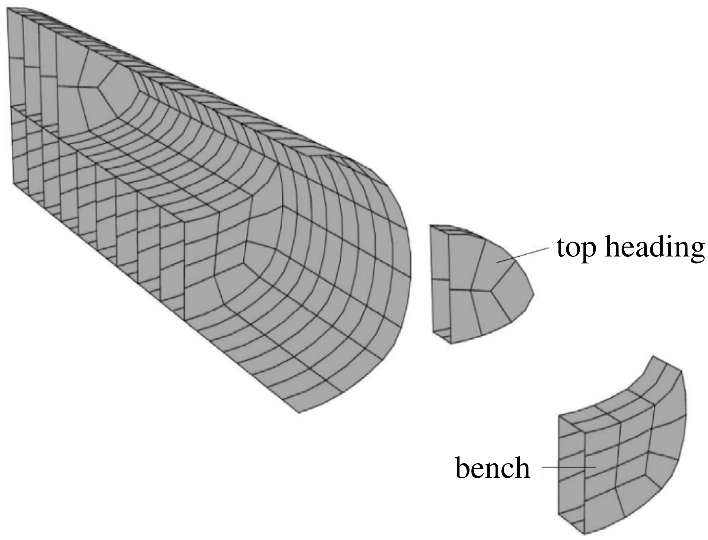

Figure 1: Sequential tunnel excavation

The modelling of the tunnel excavation is done either in an infinite or semi-infinite domain, depending on the depth of the tunnel from the ground surface. Using the Boundary Element Method (BEM) for the simulation of such problems the radiation condition is fulfilled implicitly within the solution, thus no truncation of the mesh and special boundary conditions are necessary. Using the BEM only the surface of the problem has to be discretised. Thus the mesh generation is drastically reduced for such simulation problems. This will be shown by a 3D example in section 4.

2 BOUNDARY INTEGRAL EQUATION

The basis of the current work is the displacement boundary integral equation (see [1]) which is:

$$\mathbf{C}(\mathbf{y})\mathbf{u}(\mathbf{y}) + \oint_{\Gamma} \mathbf{T}(\mathbf{y},\mathbf{x})\mathbf{u}(\mathbf{x})\,\mathrm{d}\Gamma = \int_{\Gamma} \mathbf{U}(\mathbf{y},\mathbf{x})\mathbf{t}(\mathbf{x})\,\mathrm{d}\Gamma \tag{1}$$

where $\mathbf{U}(\mathbf{y},\mathbf{x})$ and $\mathbf{T}(\mathbf{y},\mathbf{x})$ are the fundamental solutions for displacements and tractions and $\mathbf{u}(\mathbf{x})$ and $\mathbf{t}(\mathbf{x})$ are the boundary displacements and tractions, respectively. $\mathbf{C}(\mathbf{y})$ is an integral free term depending on the geometrical conditions at the boundary point $\mathbf{y}$. The boundary integral equation (1) is valid for a single region whose boundary is discretised with boundary elements. Due to the discretisation the boundary Γ is divided into elements E and nodes N.

Thus, the integral equation (1) is transformed to:

$$\mathbf{C}\mathbf{u}_i + \sum_{e=1}^{E}\sum_{n=1}^{N} \mathbf{\Delta T}_{ni}^{e}\mathbf{u}_n^e = \sum_{e=1}^{E}\sum_{n=1}^{N} \mathbf{\Delta U}_{ni}^{e}\mathbf{t}_n^e \tag{2}$$

$\mathbf{\Delta T}_{ni}^{e}$ and $\mathbf{\Delta U}_{ni}^{e}$ are integrated kernel matrices with respect to the collocation node i and element n. Eq. 2 is evaluated for all collocation points i and the coefficient matrices $\mathbf{\Delta T}_{ni}^{e}$ and $\mathbf{\Delta U}_{ni}^{e}$ are assembled into matrices $[\Delta T]$ and $[\Delta U]$, whereas the following equation arise:

$$[\Delta T]\{u\} = [\Delta U]\{t\} \tag{3}$$

Eq. 3 allows the solution of a single boundary element region. At the nodes of the boundary either displacements or tractions are known. The values for the unknown boundary conditions (BC's) are solved by rearranging Eq. 3. The corresponding columns of matrices $[\Delta T]$ and $[\Delta U]$ related to the unknown BC' are shifted to the left side and the columns of the matrices $[\Delta T]$ and $[\Delta U]$ related to the known BC's are multiplied with the values of the known BC's and form the right hand side vector $\{f\}$ of the following equation:

$$[A]\{x\} = \{f\} \tag{4}$$

In case of a mixed boundary value problem the content of the solution vector $\{x\}$ are either displacements or tractions, matrix $[A]$ is filled up either with columns of matrix $[\Delta T]$ or $[\Delta U]$.

3 MODELLING THE SEQUENTIAL EXCAVATION PROBLEM

In the particular case of a sequential excavation rock volumes are excavated gradually. Thus the simulation is performed step wise. In each step in general one or more rock volumes may be excavated. The simulation model has to meet these requirements to evaluate the deformation and stress state [2] of the tunnel progress. With the BEM this process can be simulated by applying a Multiple Region BEM (MRBEM) or a Single Region BEM (SRBEM). Using a MRBEM approach each region removed during the excavation process is discretised by a BEM region. The regions are coupled at their common interfaces and form the MREBM system. Using the SRBEM only a single region is necessary to solve the tunnelling problem for a certain step of excavation.

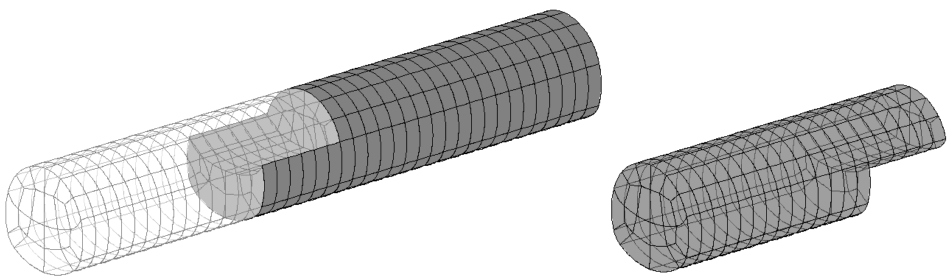

Figure 2: MRBEM model (left) and SRBEM model (right)

In Fig. 2 the meshes are shown which are necessary to solve the tunnelling problem for a certain excavation state either with the MRBEM (left mesh) or with the SRBEM method (right mesh) [3]. The mesh of the MRBEM model consists of several finite regions which are embedded in an infinite region. The finite regions are deactivated from the calculation model during the simulation process according to a predetermined sequence.

With the SRBEM model a single BEM region is discretised which represents the actual excavation state of the tunnel, which is the surface of the volume already excavated. The mesh size at the beginning of the analysis is very small and is increasing together with the progress of the tunnel excavation.

3.1 Multiple region BEM

Currently two coupling methods are used to simulate the sequential excavation.These two methods are explained next.

3.1.1 Interface coupling method (IC)

For the interface coupling method [1] stiffness matrices are evaluated which are related only to the interface of a region.

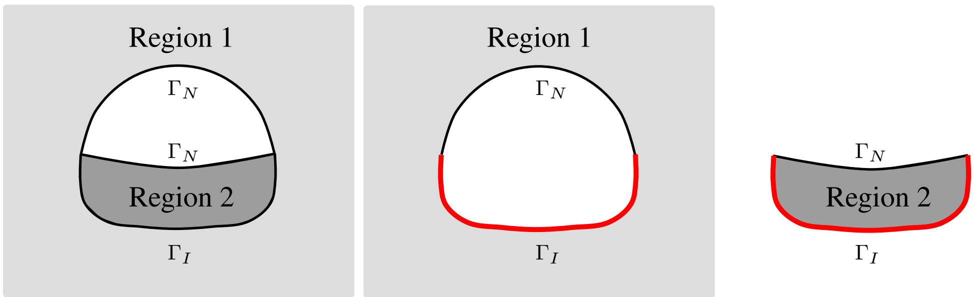

Figure 3: Interface coupling (IC)

Starting with the assembled equation system (3), which is

$$[\Delta T]\{u\} = [\Delta U]\{t\} \tag{5}$$

and assuming that no Dirichlet boundary conditions are present (which in tunnelling is usually the case) the boundary can be divided into a Neumann and Interface part. This is shown in Fig. 3 where 2 regions are coupled at the common interface Γ_I. Due to the different boundary condition types Eq. 5 can be separated to:

$$\begin{bmatrix} [U_I] & [U_N] \end{bmatrix} \left\{ \begin{matrix} \{t_I\} \\ \{t_N\} \end{matrix} \right\} = \begin{bmatrix} [T_I] & [T_N] \end{bmatrix} \left\{ \begin{matrix} \{u_I\} \\ \{u_N\} \end{matrix} \right\} \tag{6}$$

The traction t_N are the known boundary condition values and will be shifted to the right side together with the kernel matrix $[U_N]$. u_N is the unknown vector of displacements of the Neumann boundary and this will be shifted to the left together with the kernel matrix $[T_N]$.

$$\begin{bmatrix} [U_I] & -[T_N] \end{bmatrix} \left\{ \begin{matrix} \{t_I\} \\ \{u_N\} \end{matrix} \right\} = \begin{bmatrix} [T_I] & -[U_N] \end{bmatrix} \left\{ \begin{matrix} \{u_I\} \\ \{t_N\} \end{matrix} \right\} \tag{7}$$

The matrix on the left side of Eq. 7 is renamed by $[A]$ and on the right hand side the vector of interface displacements $\{u_I\}$ are separated from the traction vector $\{t_N\}$ belonging to the

Neumann boundary:

$$\begin{bmatrix} & A & \end{bmatrix} \left\{ \begin{array}{c} \{t_I\} \\ \{u_N\} \end{array} \right\} = [T_I]\,\{u_I\} + \begin{bmatrix} -[U_N] \end{bmatrix} \left\{ \ \{t_N\} \ \right\} \tag{8}$$

Solving this equation for $\{t_I\}$ and $\{u_N\}$ will result to the following:

$$\left\{ \begin{array}{c} \{t_I\} \\ \{u_N\} \end{array} \right\} = [A]^{-1}[T_I]\,\{u_I\} + [A]^{-1}\big[-[U_N]\big] \left\{ \ \{t_N\} \ \right\} \tag{9}$$

The matrix product $[A]^{-1}[T_I]$ will result to the stiffness matrix $[K]^*$ and a flexibility matrix $[D]$, and the second part of the right hand side of Eq. 9, as vector $\{t_N\}$ is known, will give a known vector shown next:

$$\left\{ \begin{array}{c} \{t_I\} \\ \{u_N\} \end{array} \right\} = \left[\begin{array}{c} [K]^* \\ [D] \end{array} \right] \{u_I\} + \left\{ \begin{array}{c} \{t_{IN}\} \\ \{u_{NN}\} \end{array} \right\} \tag{10}$$

The first equation of 10 represents the interface tractions $\{t_I\}$. Multiplying it with the mass matrix $[M]_r$ of region r, the tractions are replaced by equivalent nodal point forces. A coupling to a Finite Element region would be possible know. This equation is shown next:

$$\{f_I\}_r = [K]_r\{u_I\}_r + \{f_{IN}\}_r \tag{11}$$

The final forces $\{f_I\}_r$ at the interface of region r are the forces due to the interface displacements $\{u_I\}_r$ plus the forces at the interface due to the loading (given tractions $\{t_N\}_r$. The interface forces $\{f_I\}_r$ as well as the interface displacements $\{u_I\}_r$ are unknown at the present state. These unknowns are evaluated together with all other regions. Thus, $[K]_r$ and $\{f_{IN}\}_r$ are evaluated for every region r of the multiple region system and the final system of equation can be assembled under the following conditions:

– Equilibrium of forces at the interface:

$$\{f_I\}_1 + \{f_I\}_2 + ... + \{f_I\}_r = 0 \tag{12}$$

Eq.12 states that the forces at the interface of all adjacent regions are in equilibrium.

– Compatibility of displacements at the interface:

$$\{u_I\}_m = \{u_I\}_n \tag{13}$$

Equation (13) states that the displacements at the interface of adjoining regions n and m are equal.

Considering these conditions, Eq.11 of every region is assembled into a global equation system which is shown as following:

$$\{f_I\}_r = [K]^{sys}\{u_I\} + \{f_N\} = 0 \tag{14}$$

where $[K]^{sys}$ is the assembled stiffness matrix related to all coupling interfaces of the system. $\{f_{IN}\}$ is the right hand side vector related to the loading of the system and $\{u_I\}$ is the vector of interface displacements. From this equation the interface displacements are solved. Once $\{u_I\}$ is known all remaining unknowns (tractions at the interface $\{t_I\}$ and displacements at the Neumann boundary $\{u_N\}$) can be evaluated using Eq.10.

3.1.2 Boundary element tearing and interconnecting method (BETI)

The Boundary Element Tearing and Interconnecting Method (BETI) is a domain decomposition method based on the Symmetric Galerkin BEM [4] similar to the Finite Element Tearing and Interconnecting Method (FETI) for the FEM introduced by [5]. In this work the collocation BEM is used to set up the equation system similar to the original BETI method. Applying this method for each region a stiffness matrix based on the DOFs of the whole region surface is calculated which is in contrast to the method IC applied in section 3.1.1, where the stiffness matrix is based on the coupled DOFs only. From Eq.3 the boundary tractions of a region are calculated as following:

$$[\Delta U]^{-1}[\Delta T]\{u\} = \{t\} \tag{15}$$

As in the IC method tractions are converted to work equivalent nodal point forces by multiplying Eq.15 with the mass matrix $[M]$:

$$[M][\Delta U]^{-1}[\Delta T]\{u\} = [M]\{t\} = \{f\} \tag{16}$$

where the stiffness matrix $[K]$ is:

$$[K] = [M][\Delta U]^{-1}[\Delta T] \tag{17}$$

Inserting this into Equation (16) will result in the well known relation between displacements and forces:

$$[K]\{u\} = \{f\} \tag{18}$$

As in the IC method two conditions have to be satisfied to formulate a coupled system consisting off r boundary element regions:

- Equilibrium
- Compatibility

The equilibrium state of a region can be described by using Equation (18):

$$[K]\{u\} = \{f_N\} + [B]^T\{\lambda\} \tag{19}$$

whereas the force vector on the right hand side of Equation (18) is split into:

$$\{f\} = \{f_N\} + [B]^T\{\lambda\} \tag{20}$$

and inserted into Equation (19). $[K]\{u\}$ are the forces at the boundary of the region due to deformation, $\{f_N\}$ is the force vector of the given loading (Neumann boundary conditions) and $[B]^T\{\lambda\}$ are the coupling forces (Lagrange multipliers) to the neighbouring regions.

The compatibility of a system of r regions can be written in following form:

$$[B]_1\{u\}_1 + [B]_2\{u\}_2 + \cdots + [B]_r\{u\}_r = \{b\} \tag{21}$$

Equation (21) either guaranties that the displacements at the interface of adjacent regions are equal or that the displacements at the Dirichlet boundary are equal to the applied Dirichlet boundary conditions which are entries of vector $\{b\}$.

The final system of equation of a coupled system of r boundary element regions is shown as following:

$$\begin{bmatrix} [K]_1 & & 0 & & -[B]_1^T \\ & [K]_2 & & & -[B]_2^T \\ 0 & & \ddots & & \vdots \\ & & & [K]_r & -[B]_r^T \\ [B]_1 & [B]_2 & \cdots & [B]_r & 0 \end{bmatrix} \cdot \begin{Bmatrix} \{u\}_1 \\ \{u\}_2 \\ \vdots \\ \{u\}_r \\ \{\lambda\} \end{Bmatrix} = \begin{Bmatrix} \{f_N\}_1 \\ \{f_N\}_2 \\ \vdots \\ \{f_N\}_r \\ \{b\} \end{Bmatrix} \tag{22}$$

Equations 1 to r of Eq.22 are representing the equilibrium of each region and the last equation of Eq.22 guaranties compatibility of displacements at every node at the interface of adjacent regions and at the nodes corresponding to the Dirichlet boundary.

In the implementation of the BETI method Eq.22 is not assembled to an equation system. The equation system Eq.22 is condensed to the solution of the coupling forces λ (Lagrange multipliers). This is done by inserting equations 1 to r into the last equation of Eq.22. From this equation λ is solved either directly or iteratively with a BiCGSTAB iterative solver. As the stiffness matrix $[K]_r$ of a finite region (floating region) is singular [6] special treatment of rigid body motions have to be considered. The whole solution formulation is shown in detail by [4].

3.1.3 Comparison of coupling methods

The main advantage of the BETI method is that the stiffness matrix of each region has to be calculated only once and in the case of a sequential tunnel excavation these matrices can be used for each load step of excavation. Using the BETI method for this application type the stiffness matrices are independent on the changing boundary conditions. Changing boundary conditions due to sequential excavation are considered by the coupling matrix $[B]_r$ of Equation (22). The coupling matrices have to be computed repeatedly for each calculation step. As those matrices are sparsely populated they are implemented as sparse matrices. The effort to set up those matrices is small and it is insignificant compared to the overall computing time. The way how the equation system is formulated makes the treatment of operations related to a region independently from the other regions. Thus, the BETI method is ideally suited for parallelisation techniques.

The advantage of the IC method is that the size of the stiffness matrix is related to the number of DOFs at the interface of the coupled system of regions. In the case of a sequential excavation the coupling surfaces are reduced from one excavation step to the other. In each load step one or more regions are deactivated from the simulation model. Due to the deactivation the boundary condition of surfaces adjacent to the deactivated regions change from Interface condition to Neumann condition. Thus, the size of the assembled system stiffness matrix reduces from one load step to the other and the solution of the equation system gets faster. For regions for which a change of boundary conditions happens the stiffness matrix has to be calculated newly. Compared to the BETI method stiffness matrices do not remain constant throughout the entire analysis of such an excavation simulation.

3.2 Single region BEM

In Fig.4 the excavation using single regions [7] are shown on a simple example in 2D for two excavation steps. The simulation considers a top heading excavation in the first excavation step and a bench excavation in the second step by using a single BEM region for each step. The region for the first step is the boundary of the top heading and the region for the second step is the boundary of the final tunnel surface (the outer boundary of top heading and bench together).

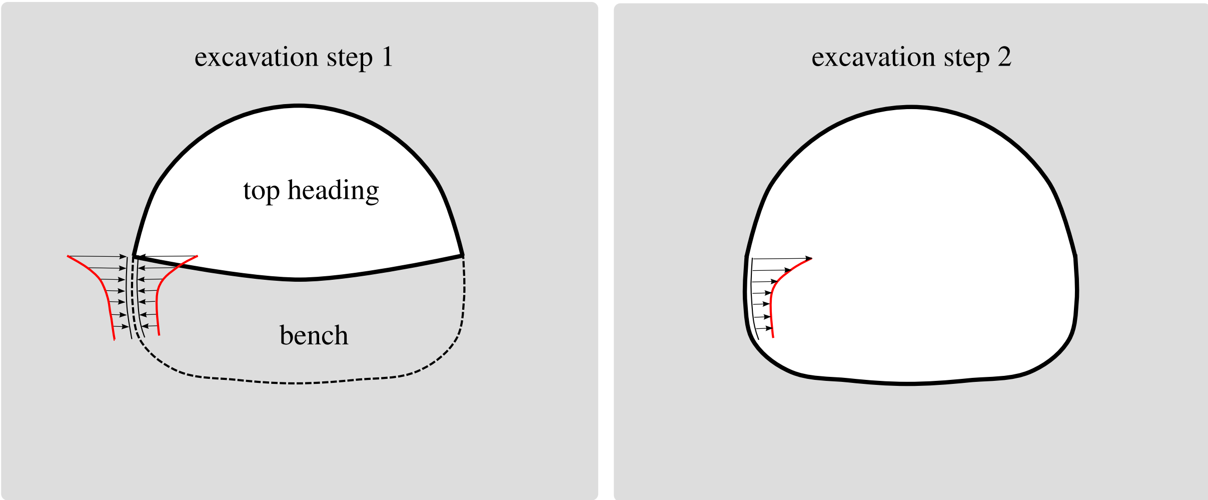

Figure 4: Single region BEM (SRBEM) excavation

The crucial task is the evaluation of the excavation loading (boundary tractions) for the subsequent excavation step. For the first step the excavation loading (boundary traction) is given from the known primary stress field at the whole surface of the top heading region. Applying Eq.3 with known tractions the displacements are calculated for the top heading boundary. In a post-processing algorithm the tractions along the boundary of the bench (dashed line in Fig.4) inside of region 1 (top heading region) are calculated. These tractions are applied at the region of the subsequent load step (shown in Fig.4 on the right) and again Eq.3 is used to solve this load step.
For the evaluation of the tractions inside a region two methods have been investigated. The simpler method is the direct evaluation of the stress at points inside the region using the stress boundary integral equation. As the stress at the corners of the bench (shown in Fig.4) are singular, an evaluation of the stress is difficult as well as the distribution of the stress towards that corner point. The results due to this method are not satisfactory, thus a second method was investigated where displacements are calculated at the inner points. With this method an intermediate calculation for the region which is excavated (only the bench region!) is necessary. The mentioned displacements will be the known Dirichlet boundary conditions and the tractions will be calculated by this intermediate single region calculation. In excavation step 2 this traction will be applied as loading. A detailed explanation about the key issue of evaluating excavation loads can be found in [8].

In comparison to the the coupling methods with this method the mesh of the model has to be adapted at each step of excavation due to the change of the tunnel surface. The simulation is starting with a very small mesh and in every subsequent excavation step the existing mesh has to be extended by the surface of the removed volume of excavation. At the end of the excavation simulation (all regions are excavated) the mesh size is the same as for the coupling methods. This will be shown in the 3D example shown next.

4 EXAMPLE - CROSSING PASSAGE - NEW YORK UNDERGROUND

In the following an example of a sequential tunnel excavation is shown. More precisely it is the crossing passage of two tunnels of the New York Underground. In the planning phase 3D-CAD planning documents were available. In order to create the BEM model these data was used to mesh the geometry. The initial CAD model of the construction design was too detailed in order to use it for the numerical discretisation and therefore the CAD geometry description had to be simplified. After some adaptation and simplification of the CAD model it was imported to the pre-processor CUBIT 2014 [9] and the mesh was automatically generated. The final mesh is shown in Fig.5 and was modelled with linear (4 nodes) quadrilateral boundary elements (BE). At all four ends of the simulation model infinite BE's [10] are situated to consider correct boundary conditions in the longitudinal direction of the tunnels. The mesh consists of approximately 20 000 nodes and 7 800 elements.

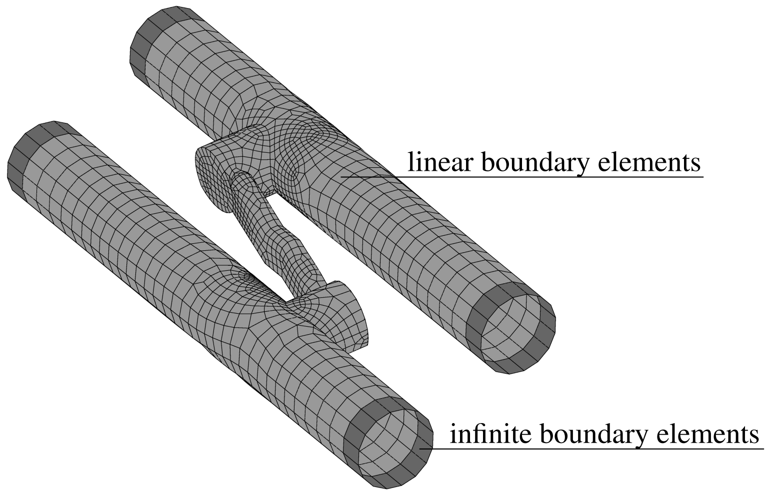

Figure 5: Mesh for the BEM analysis

In Fig.6 the sequence of the sequential excavation is shown. In sum 12 load cases (indicated by LC#) are considered. In the first 6 load cases the main tunnels are excavated, from LC7 to LC12 the crossing passage is excavated. The meshes for each LC shown in Fig.6 correspond to the single regions used in the SRBEM simulation method. The excavation model starts with a small mesh which is extended from LC to LC.

For both coupling methods (IC and BETI) the mesh of LC12 (shown in Fig.6) would be the infinite region, which is filled with finite regions, the region volumes which are excavated from LC to LC. Thus, the simulation for the coupling methods starts with a large model (with all coupled regions) which is reduced by finite regions during the excavation process.

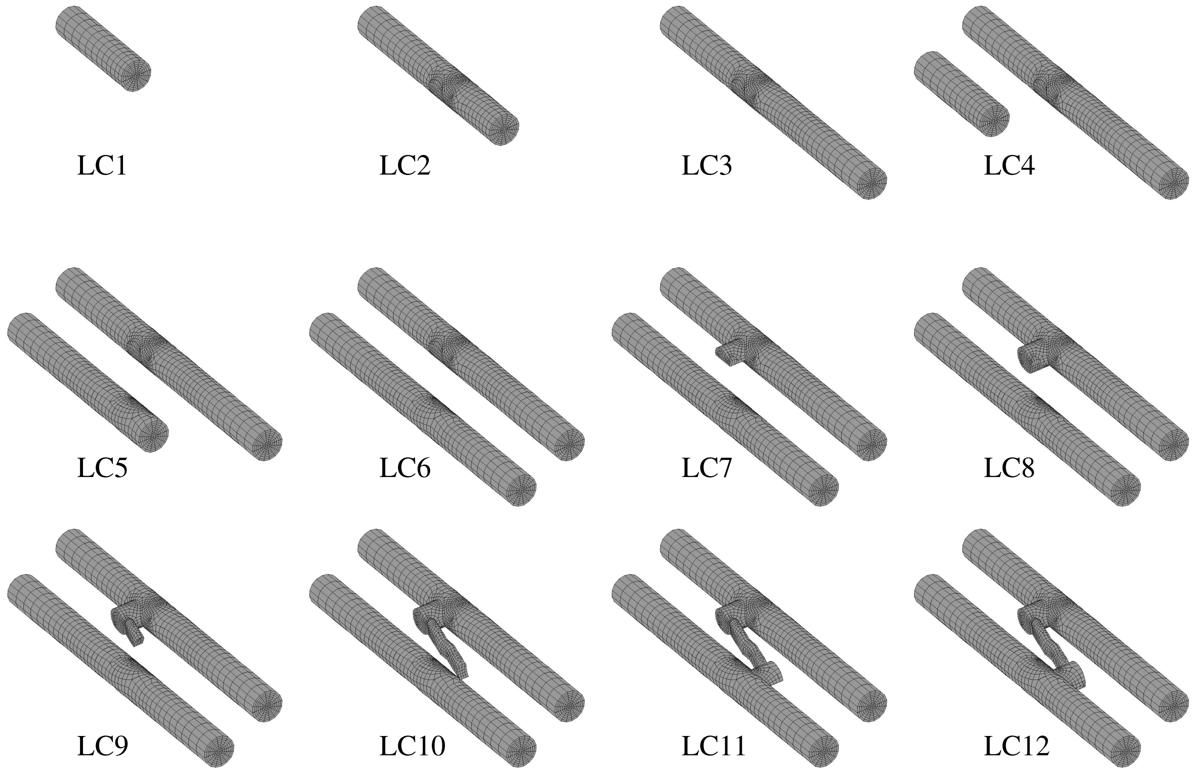

Figure 6: Excavation sequence LC1 to LC12

In Fig.7 contour lines of displacements in z-direction for the fully excavated tunnels and crossing passage are shown. The maximum settlement is 8.1 cm at the joining openings of the crossing passage and the upper main tunnel. The maximum heave is about 8.3 cm at the joining tunnels of the crossing passage and the bottom main tunnel.

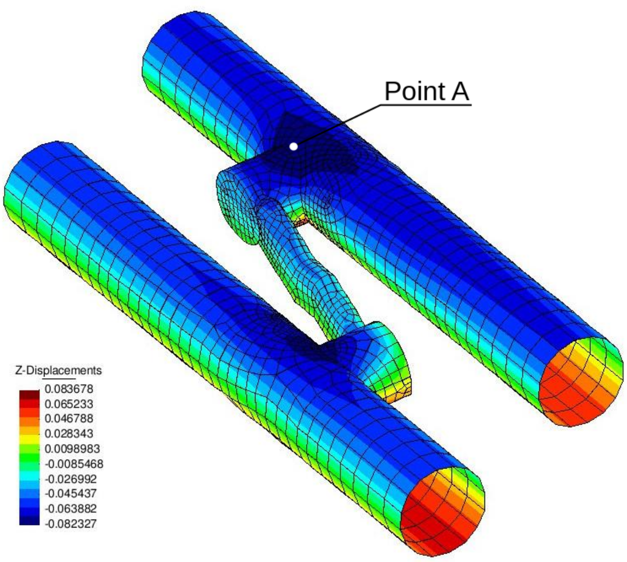

Figure 7: Contour lines of z-displacements for LC12

In the following diagram of Fig.8 vertical displacements for node A (location shown in Fig.7) of all load steps are displayed. The calculations were done for the three methods of excavation explained in section 3:

- Single region BEM method (SRBEM)
- Multiple region BEM method / interface coupling (MRBEM IC)
- Multiple region BEM method / BETI (MRBEM BETI)

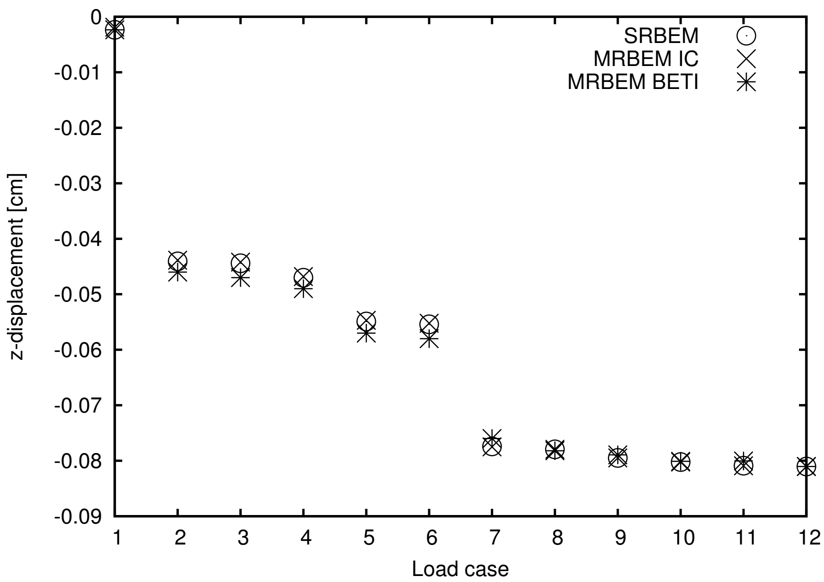

Figure 8: z-displacements of point A for LC1 to LC12

The displacements are displayed for each load case and each simulation method. It is shown that for the SRBEM and for the MRBEM IC very similar results are obtained. The results for the MRBEM BETI are slightly different from the two other methods.

Modelling method		**calculation time [h]**		
	1 CPU	**8 CPU's**	**16 CPU's**	**32 CPU's**
SRBEM	3,51	1,06	0,76	0,66
MRBEM – IC	12,39	2,50	1,67	1,40
MRBEM – BETI	5,41	0,87	0,55	0,58

Table 1: Computation times for different number of CPU's

The calculation times for the three simulation methods are shown in Table1 depending on the number of CPU's used. Without any parallelisation (calculation with 1 CPU) the calculation time for the MRBEM IC is the largest of the three methods. For this method the stiffness matrix of BEM regions with changing boundary conditions has to be calculated again from one load step to the other. This is the main reason for the lack of efficiency of this method (IC). For the present example the calculation time for the SRBEM is similar to the one of the MRBEM

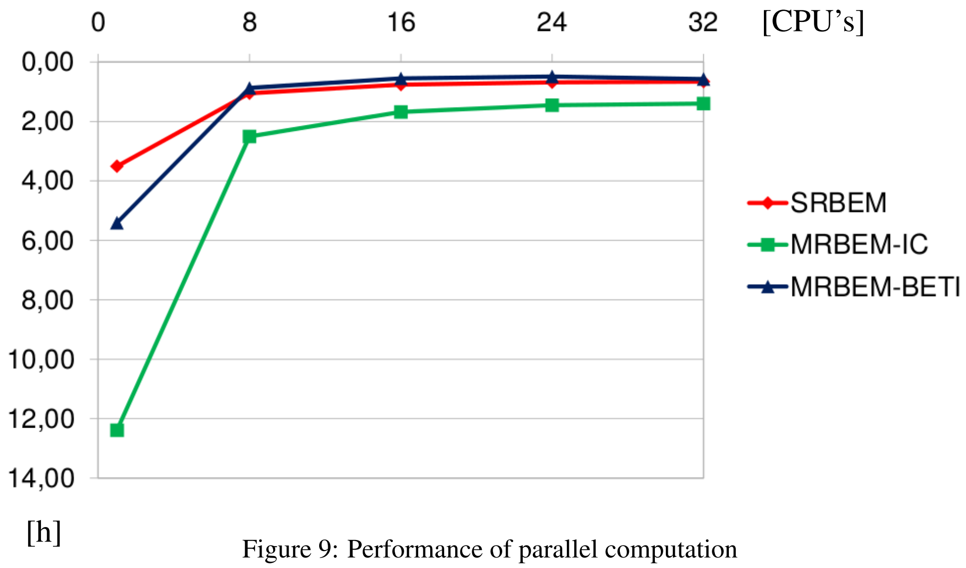

Figure 9: Performance of parallel computation

BETI. At the first excavation step the SRBEM method always starts with a very small mesh (small equations system), the size of the equation system increases from step to step as the surface of the excavation volume grows. For the MRBEM BETI method the stiffness matrices of all regions are calculated only once at the beginning of the analysis. They dont change any more during the whole analysis. This is the main advantage of the MRBEM BETI against the MRBEM IC method.

In Fig.9 the performance of the three simulation models is shown taking advantage of parallelisation. As shown the time difference for the overall computation of this excavation problem is decreasing as the number of CPU's increase.

At the current state of experience an objective statement cant be made whether SRBEM or MRBEM BETI is favourable in relation to computing performance. Maybe for an example of an increased number of load steps it can be expected that the MRBEM BETI method has an advantage over the SRBEM method.

5 CONCLUSIONS

In this work three simulation methods for the calculation of the sequential tunnel excavation using the BEM have been presented. Two coupling methods, the classical multiple region boundary element method with interface coupling (MRBEM IC) and the BETI coupling method (MRBEM BETI) are explained in brevity as well as the single region method (SRBEM).

On a practical example in 3D the results for the three methods are verified and it has been shown that the accuracy of the solutions of all methods is excellent and corresponds well to each other. The performance of parallelisation is demonstrated, whereas the methods SRBEM and MRBEM-BETI have distinct advantages in computing efficiency over the classical interface coupling. Causes and consequences of the three models for the simulation of the sequential tunnel excavation related to their implementation and the final performance have been worked out and are demonstrated by a realistic example in 3D.

REFERENCES

[1] G. Beer, I. Smith and C. Duenser, *The Boundary Element Method with Programming*. SpringerWienNewYork, 2008.

[2] C. Duenser, K. Thoeni, K. Riederer, B. Lindner, G. Beer, New Developments of the Boundary Element Method for Underground Constructions, *International Journal of Geomechanics* **12** (6) 665–675, 2012.

[3] C. Duenser, G. Beer, New Algorithms for the Simulation of the Sequential Tunnel Excavation with the Boundary Element Method, *Computational Methods in Tunnelling - EURO:TUN 2007*, 1–11, 2007.

[4] U. Langer and O. Steinbach, Boundary Element Tearing and Interconnecting Methods. *Computing*, **71**, 205–228, 2003.

[5] C. Farhat, F.-X. Roux, A method of finite element tearing and interconnecting and its parallel solution algorithm, *International Journal for Numerical Methods in Engineering*, **32** (6), 1205–1227, 1991.

[6] G. Of, O. Steinbach, The all-floating boundary element tearing and interconnecting method, *Journal of Numerical Mathematics* **17**, 277–298, 2009.

[7] C. Duenser, B. Lindner, G. Beer, Simulation of sequential Tunnel Excavation/Construction with the Boundary Element Method, in: *Computational Methods in Tunneling - EURO:TUN 2009, Aedificatio Publishers*, 147–154, 2009.

[8] C. Duenser and G. Beer, Simulation of sequential excavation with the Boundary Element Method. *Computers and Geotechnics*, **44**, 157–166, 2012.

[9] CUBIT, Preprocessor - Geometry and Mesh Generation Toolkit, Scandia National Laboratories, https://cubit.scandia.gov/

[10] W. Moser, C. Duenser, G. Beer, Mapped infinite elements for three-dimensional multi-region boundary element analysis, *International Journal for Numerical Methods in Engineering* **61** (3), 317–328, 2004.