ANALYSIS OF LAMINATED COMPOSITE PLATES USING ISOGEOOMETRIC COLLOCATION METHOD

G. S. Pavan 1, and K. S. Nanjunda Rao 2

1 Research Scholar
Department of Civil Engineering, Indian Institute of Science, Bangalore, India
e-mail: pavangs@civil.iisc.ernet.in

2 Principal Research Scientist
Department of Civil Engineering, Indian Institute of Science, Bangalore, India
e-mail: ksn@civil.iisc.ernet.in

Keywords: Isogeometric collocation Method, NURBS, Laminated composite plate, Sandwich plate, Reissner Mindlin plate theory

Abstract. Isogeometric analysis (IGA) is a relatively new class of computational methods primarily aimed at integrating finite element analysis with Computer Aided Design (CAD). IGA adopts Non-Uniform Rational B-Spline (NURBS) functions as the basis functions to represent geometry and approximate the unknown field variable. The standard Galerkin based isogeometric approach has been extensively explored by researchers during the past decade and is being continued. Apart from the standard Galerkin based isogeometric approach, several other numerical schemes exist within the framework of isogeometric methods. Isogeometric collocation method is one such relatively new numerical procedure. Isogeometric collocation methods discretize the governing PDEs in strong form. In this study, the potential of isogeometric collocation methods for the static analysis of laminated composite plates using Reissner-Mindlin plate theory is explored. Results of benchmark problems on bending of rectangular laminated composite plates subjected to sinusoidal and uniform loading are presented. The results obtained from the proposed isogeometric collocation method for the analysis of laminated composite plates have been assessed by comparing them with the solutions available in literature.
1 INTRODUCTION

Isogeometric analysis (IGA) is a relatively new class of numerical methods proposed by Hughes et al. [1] in 2005 mainly aimed at integrating Computer Aided Design (CAD) and finite element analysis. IGA simplifies the complex procedure of mesh generation and mesh refinement thereby lowering the huge computational cost incurred during the analysis of large systems. IGA achieves this smooth flow of information by adopting the same NURBS functions employed to represent geometry by CAD models to approximate the unknown field variable in an isoparametric fashion. NURBS functions possess high continuity properties which lends IGA a significant edge over traditional finite element methods during discretization of higher order partial differential equations.

Over the last decade, IGA has been successfully applied on wide ranging topics such as solid and structural mechanics [2–10], fluid mechanics [11–17], phase field modeling [18], fluid structure interaction [19–21] etc. One of the important issue related to the efficient implementation of IGA was the development of an efficient integration rule for the Galerkin based IGA. The typical Gauss quadrature rules adopted for element wise integration in Galerkin based finite element methods is sub-optimal for Galerkin-IGA since it does not take into account the higher inter element continuity of NURBS functions.

One of the recent developments in the field of IGA is the emergence of Isogeometric collocation methods (IGA collocation) proposed by Auricchio et al. [22]. Isogeometric collocation method discretizes the governing differential equations in strong form while retaining the inherent IGA feature of adopting NURBS functions in an isoparametric sense. A comparison study between IGA collocation, Galerkin-IGA and finite element method by Schillinger et al. [23] revealed accuracy of the method achieved at a low cost of computation. The advantages offered by IGA collocation method are more evident when higher order approximations are employed.

Studies on IGA collocation have been performed on solid mechanics [24], phase field modeling [25], contact [26]. IGA collocation method has been successfully applied to Euler - Bernoulli beams and Kirchhoff plates [27]. It is worthy to mention that the high continuity properties possessed by NURBS functions enabled the application of IGA collocation method on Euler-Bernoulli beam and Kirchhoff plate theories which are governed by fourth order differential equations. Studies on performance of IGA collocation on shear deformable beams, rods and plates [28–30] has also been conducted. Kiendl et al. [30] showed that mixed formulation within the framework of IGA collocation is successful in alleviating shear locking observed during lower order approximations for plate bending.

In this paper, we propose IGA collocation method for the analysis of laminated composite plates described by Reissner - Mindlin plate theory. Standard formulation with lateral displacement and two rotations are the three unknown field variables within the framework of IGA collocation. Performance of IGA collocation is assessed by solv-
ing benchmark problems and comparing the results obtained with the ones existing in literature.

2 REISSNER MINDLIN PLATE THEORY FOR LAMINATED COMPOSITE PLATES

A laminated composite plate of total thickness $t$ composed of $N$ orthotropic layers is considered. The undeformed midplane of the plate is taken as the reference plane $\Omega_0$. The kinematic assumptions underlying the Reissner - Mindlin theory are made, namely, cross sections remain straight during deformation but may not remain normal to the reference plane. The displacement field according to Reissner Mindlin plate theory is

\[
\begin{align*}
    u(x, y, z) &= u_0(x, y) + z\phi_x(x, y) \\
    v(x, y, z) &= v_0(x, y) + z\phi_y(x, y) \\
    w(x, y, z) &= w_0(x, y)
\end{align*}
\]

where $u_0, v_0, w_0$ are the displacement components for the midplane in $x$, $y$ and $z$ directions respectively. The rotations in $y$-$z$ plane and $x$-$z$ plane are denoted as $\phi_x$ and $\phi_y$ respectively. The strains associated with the displacement field mentioned in equation (1) are

\[
\begin{align*}
    \epsilon_{xx} &= \frac{\partial u_0}{\partial x} + z\frac{\partial \phi_x}{\partial x} \\
    \epsilon_{yy} &= \frac{\partial v_0}{\partial y} + z\frac{\partial \phi_y}{\partial y} \\
    \gamma_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \\
    \gamma_{xz} &= \frac{\partial w_0}{\partial x} + \phi_x \\
    \epsilon_{zz} &= 0
\end{align*}
\]

The stresses at the $k^{th}$ layer in local coordinates are given by the following constitutive relationship.

\[
\begin{bmatrix}
    \sigma_{xx} \\
    \sigma_{yy} \\
    \tau_{xy} \\
    \tau_{xz} \\
    \tau_{yz}
\end{bmatrix}^k =
\begin{bmatrix}
    Q_{11} & Q_{12} & 0 & 0 & 0 \\
    Q_{21} & Q_{22} & 0 & 0 & 0 \\
    0 & 0 & Q_{66} & 0 & 0 \\
    0 & 0 & 0 & Q_{55} & 0 \\
    0 & 0 & 0 & 0 & Q_{44}
\end{bmatrix}^k
\begin{bmatrix}
    \epsilon_{xx} \\
    \epsilon_{yy} \\
    \gamma_{xy} \\
    \gamma_{xz} \\
    \gamma_{yz}
\end{bmatrix}
\]

where

\[
\begin{align*}
    Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}}, & Q_{12} &= \frac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}}, & Q_{22} &= \frac{E_2}{1-\nu_{12}\nu_{21}}, \\
    Q_{66} &= G_{12}, & Q_{55} &= G_{13}, & Q_{44} &= G_{23}
\end{align*}
\]
where $E_1$ and $E_2$ are the elastic modulus in directions 1 and 2 respectively. $G_{12}, G_{13}$ and $G_{23}$ are the shear modulus in x-y, x-z and y-z planes respectively. $\nu_{12}$ and $\nu_{21}$ are the two Poisson ratios for orthotropic materials. The material constants in equation (2) $v$ in local coordinates can be expressed in the global coordinates by the following set of relations

\[
\begin{align*}
Q'_{11} &= Q_{11} \cos^4 \theta + 2 (Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\
Q'_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\
Q'_{22} &= Q_{11} \sin^4 \theta + 2 (Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\
Q'_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\
Q_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\
Q_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \\
Q'_{44} &= Q_{44} \cos^2 \theta + Q_{55} \sin^2 \theta \\
Q'_{45} &= (Q_{55} - Q_{44}) \cos \theta \sin \theta \\
Q'_{55} &= Q_{44} \sin^2 \theta + Q_{55} \cos^2 \theta
\end{align*}
\] (4)

Here, $\theta$ is the inclination between the local co-ordinate system at the $k^{th}$ layer and the global co-ordinate axes.

### 2.1 Equations of equilibrium

The equations of equilibrium expressed in terms of the bending moments $M_{xx}, M_{yy}$, the twisting moments $M_{xy}$, shear forces $Q_x, Q_y$ and the in-plane forces $N_{xx}, N_{yy}$ and $N_{xy}$ are given below

\[
\begin{align*}
N_{xx,x} + N_{xy,y} &= 0 \tag{5a} \\
N_{xy,x} + N_{yy,y} &= 0 \tag{5b} \\
Q_{x,x} + Q_{y,y} &= -f \tag{5c} \\
M_{xx,x} + M_{xy,y} - Q_x &= 0 \tag{5d} \\
M_{yy,y} + M_{yx,x} - Q_y &= 0 \tag{5e}
\end{align*}
\]

The stress resultants in equations (5a) are related to displacement variables by the following relations
\[
\begin{align*}
\{ N_{xx} \} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial x} - \frac{\partial v_0}{\partial y} \right\} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right\} \\
\{ N_{yy} \} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right\} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right\} \\
\{ N_{xy} \} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right\} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right\} \\
\{ Q_y \} &= K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \left\{ \frac{\partial w_0}{\partial y} + \phi_y \right\} \left\{ \frac{\partial w_0}{\partial x} + \phi_x \right\} \end{align*}
\]

(6a)

\[
\begin{align*}
\{ M_{xx} \} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial x} - \frac{\partial v_0}{\partial y} \right\} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right\} \\
\{ M_{yy} \} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right\} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right\} \\
\{ M_{xy} \} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \left\{ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right\} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \left\{ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right\} \\
\{ Q_y \} &= K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \left\{ \frac{\partial w_0}{\partial y} + \phi_y \right\} \left\{ \frac{\partial w_0}{\partial x} + \phi_x \right\} \end{align*}
\]

(6b)

\[
\begin{align*}
\{ Q_y \} &= K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \left\{ \frac{\partial w_0}{\partial y} + \phi_y \right\} \left\{ \frac{\partial w_0}{\partial x} + \phi_x \right\} \\
\{ Q_x \} &= \int_{-t/2}^{t/2} Q_{ij}'(1, z, z^2) \, dz \end{align*}
\]

(6c)

The \( A_{ij}, B_{ij} \) and the \( D_{ij} \) present in the above set of equations (6) are termed as extensional stiffness, bending stiffness and bending - extensional stiffness respectively and are defined as following

\[
(A_{ij}, B_{ij}, D_{ij}) = \int_{-t/2}^{t/2} Q_{ij}'(1, z, z^2) \, dz
\]

(7)

Substituting the stress resultant - displacement variable relationships mentioned in equation (6) into (5a), we obtain the equilibrium equations in terms of displacement variables \( u_0, v_0, w_0, \phi_x \) and \( \phi_y \) as shown below
\[ A_{11} \frac{\partial^2 u_0}{\partial x^2} + 2A_{16} \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + A_{12} \frac{\partial^2 v_0}{\partial x \partial y} + A_{16} \frac{\partial^2 v_0}{\partial x^2} + A_{26} \frac{\partial^2 v_0}{\partial y^2} + A_{66} \frac{\partial^2 v_0}{\partial x \partial y} = 0 \] 

\( 8a \)

\[ \frac{\partial^2 \phi_s}{\partial x^2} + 2B_{16} \frac{\partial^2 \phi_s}{\partial x \partial y} + B_{66} \frac{\partial^2 \phi_s}{\partial y^2} + B_{12} \frac{\partial^2 \phi_h}{\partial x \partial y} + B_{16} \frac{\partial^2 \phi_h}{\partial x^2} + B_{26} \frac{\partial^2 \phi_h}{\partial y^2} + B_{66} \frac{\partial^2 \phi_h}{\partial x \partial y} = 0 \]

\( 8b \)

\[ K A_{55} \frac{\partial \phi_s}{\partial x} + K A_{45} \frac{\partial \phi_s}{\partial y} + K A_{44} \frac{\partial \phi_h}{\partial x} + K A_{55} \frac{\partial^2 w_0}{\partial x \partial y} + 2KA_{45} \frac{\partial^2 w_0}{\partial x \partial y} + K A_{44} \frac{\partial^2 w_0}{\partial y^2} = 0 \]

\( 8c \)

The above equilibrium equations eq. (8) are valid in the interior of the domain. The equilibrium equations at the Neumann boundary are given by:

\[ \begin{bmatrix} n_x^2 & n_y^2 & 2n_x n_y \\ -n_x n_y & n_x n_y & n_x^2 - n_y^2 \end{bmatrix} \begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} \tilde{N}_{nn} \\ \tilde{N}_{ns} \end{bmatrix} \]

\( 9a \)

\[ \begin{bmatrix} n_x^2 & n_y^2 & 2n_x n_y \\ -n_x n_y & n_x n_y & n_x^2 - n_y^2 \end{bmatrix} \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \begin{bmatrix} \tilde{M}_{nn} \\ \tilde{M}_{ns} \end{bmatrix} \]

\( 9b \)

\[ \begin{bmatrix} n_x \\ n_y \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} \tilde{Q}_n \end{bmatrix} \]

\( 9c \)

where \( \tilde{N}_{nn} \) & \( \tilde{N}_{ns} \) are the in - plane axial forces specified at the Neumann boundary. Similary, \( \tilde{M}_{nn} \) & \( \tilde{M}_{ns} \) are the bending moments and, \( \tilde{Q}_n \) are the shear forces specified at Neumann boundary.

Using the stress resultant - displacement relationship in eq. (6), we obtain eqs. (9) in terms of displacement variables as given below:
\[ S_{nn} = (n_x^2 A_{11} + n_y^2 A_{12} + 2 n_x n_y A_{16}) \frac{\partial u_0}{\partial x} + (n_x^2 A_{16} + n_y^2 A_{26} + 2 n_x n_y A_{66}) \frac{\partial u_0}{\partial y} + \]
\[ (n_x^2 A_{16} + n_y^2 A_{26} + 2 n_x n_y A_{66}) \frac{\partial v_0}{\partial x} + (n_x^2 A_{12} + n_y^2 A_{22} + 2 n_x n_y A_{26}) \frac{\partial v_0}{\partial y} + \]
\[ (n_x^2 B_{11} + n_y^2 B_{12} + 2 n_x n_y B_{16}) \frac{\partial \phi_x}{\partial x} + (n_x^2 B_{16} + n_y^2 B_{26} + 2 n_x n_y B_{66}) \frac{\partial \phi_x}{\partial y} + \]
\[ (n_x^2 B_{16} + n_y^2 B_{26} + 2 n_x n_y B_{66}) \frac{\partial \phi_y}{\partial x} + (n_x^2 B_{12} + n_y^2 B_{22} + 2 n_x n_y B_{26}) \frac{\partial \phi_y}{\partial y} \] (10a)

\[ S_{nx} = (-n_x n_y A_{11} + n_x n_y A_{12} + (n_x^2 - n_y^2) A_{16}) \frac{\partial u_0}{\partial x} + (-n_x n_y A_{16} + n_x n_y A_{26} + (n_x^2 - n_y^2) A_{66}) \frac{\partial u_0}{\partial y} + \]
\[ + (-n_x n_y A_{16} + n_x n_y A_{26} + (n_x^2 - n_y^2) A_{66}) \frac{\partial v_0}{\partial x} + (-n_x n_y A_{12} + n_x n_y A_{22} + (n_x^2 - n_y^2) A_{26}) \frac{\partial v_0}{\partial y} + \]
\[ + (-n_x n_y B_{11} + n_x n_y B_{12} + (n_x^2 - n_y^2) B_{16}) \frac{\partial \phi_x}{\partial x} + (-n_x n_y B_{16} + n_x n_y B_{26} + (n_x^2 - n_y^2) B_{66}) \frac{\partial \phi_x}{\partial y} + \]
\[ + (-n_x n_y B_{16} + n_x n_y B_{26} + (n_x^2 - n_y^2) B_{66}) \frac{\partial \phi_y}{\partial x} + (-n_x n_y B_{12} + n_x n_y B_{22} + (n_x^2 - n_y^2) B_{26}) \frac{\partial \phi_y}{\partial y} \] (10b)

\[ \tilde{Q}_n = K \left[ A_{44} \frac{\partial \phi_0}{\partial y} + A_{55} \frac{\partial \phi_0}{\partial x} + A_{44} \frac{\partial \phi_0}{\partial y} + A_{45} \frac{\partial \phi_0}{\partial y} n_x + A_{45} \frac{\partial \phi_0}{\partial y} n_y + A_{45} \frac{\partial \phi_0}{\partial y} n_y \right] + K [A_{55} \phi_0 n_x + A_{55} \phi_0 n_y] \]
\[ + K [A_{44} \phi_0 n_x + A_{44} \phi_0 n_y] \] (10c)

\[ \tilde{M}_{nn} = (n_x^2 B_{11} + n_y^2 B_{12} + 2 n_x n_y B_{16}) \frac{\partial u_0}{\partial x} + (n_x^2 B_{16} + n_y^2 B_{26} + 2 n_x n_y B_{66}) \frac{\partial u_0}{\partial y} + \]
\[ (n_x^2 B_{16} + n_y^2 B_{26} + 2 n_x n_y B_{66}) \frac{\partial v_0}{\partial x} + (n_x^2 B_{12} + n_y^2 B_{22} + 2 n_x n_y B_{26}) \frac{\partial v_0}{\partial y} + \]
\[ (n_x^2 D_{11} + n_y^2 D_{12} + 2 n_x n_y D_{16}) \frac{\partial \phi_x}{\partial x} + (n_x^2 D_{16} + n_y^2 D_{26} + 2 n_x n_y D_{66}) \frac{\partial \phi_x}{\partial y} + \]
\[ (n_x^2 D_{16} + n_y^2 D_{26} + 2 n_x n_y D_{66}) \frac{\partial \phi_y}{\partial x} + (n_x^2 D_{12} + n_y^2 D_{22} + 2 n_x n_y D_{26}) \frac{\partial \phi_y}{\partial y} \] (10d)

\[ \tilde{M}_{nx} = (-n_x n_y B_{11} + n_x n_y B_{12} + (n_x^2 - n_y^2) B_{16}) \frac{\partial u_0}{\partial x} + (-n_x n_y B_{16} + n_x n_y B_{26} + (n_x^2 - n_y^2) B_{66}) \frac{\partial u_0}{\partial y} + \]
\[ + (-n_x n_y B_{16} + n_x n_y B_{26} + (n_x^2 - n_y^2) B_{66}) \frac{\partial v_0}{\partial x} + (-n_x n_y B_{12} + n_x n_y B_{22} + (n_x^2 - n_y^2) B_{26}) \frac{\partial v_0}{\partial y} + \]
\[ + (-n_x n_y D_{11} + n_x n_y D_{12} + (n_x^2 - n_y^2) D_{16}) \frac{\partial \phi_x}{\partial x} + (-n_x n_y D_{16} + n_x n_y D_{22} + (n_x^2 - n_y^2) D_{26}) \frac{\partial \phi_x}{\partial y} + \]
\[ + (-n_x n_y D_{16} + n_x n_y D_{26} + (n_x^2 - n_y^2) D_{26}) \frac{\partial \phi_y}{\partial x} + (-n_x n_y D_{12} + n_x n_y D_{22} + (n_x^2 - n_y^2) D_{26}) \frac{\partial \phi_y}{\partial y} \] (10c)

3 Numerical formulation

In order to solve the equations presented in the previous section using Isogeometric collocation method, the unknown field variables are expressed as a linear combination of Non - Uniform Rational B - Splines (NURBS).

3.1 B - Splines and NURBS

B-spline functions are polynomials, say of degree p, are defined piecewise on the parametric space. The parametric space is given by a knot vector. A knot vector \( \Xi \), is a set of non - decreasing coordinates \( \xi_i \) in the parametric space, \( \Xi = \{\xi_1, \xi_2, \xi_3, \ldots, \xi_{n+p+1}\} \)
where \( n \) is the number of basis functions. The space between two consecutive knots in the parametric space is defined as a knot span. Knot values can be repeated at a particular location in the parametric space and are called as multiple knots. The B-splines are \( C^{p-1} \) continuous at a particular knot and \( C^{p-k} \) continuous at multiple knot locations with multiplicity \( k \).

The \( i^{th} \) B-spline basis function of degree \( p \) for a given knot vector \( \Xi \) is defined recursively as shown below:

\[
N_{i,0}(\xi) = \begin{cases} 
1, & \xi_i \leq \xi < \xi_{i+1} \\
0, & \text{otherwise}
\end{cases} \tag{11a}
\]

\[
N_{i,p} = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{11b}
\]

B-Splines in two dimensions are obtained by taking the tensor product of their univariate counterparts as shown below:

\[
B_{ij,pq}(\xi,\eta) = N_{i,p}(\xi) M_{j,q}(\eta) \quad i = 1,\ldots,n \quad j = 1,\ldots,m. \tag{12}
\]

where \( \xi \) and \( \eta \) are the parametric coordinates in two directions.

A bivariate NURBS function \( R_{ij} \) is defined as the weighted bivariate B-Spline \( B_{ij} \) function,

\[
R_{ij}(\xi,\eta) = \frac{B_{ij,pq}(\xi,\eta) \omega_{ij}}{\sum_{k=1}^{n} \sum_{l=1}^{m} B_{ij,pq}(\xi,\eta) \omega_{ij}} \tag{13}
\]

\( \omega_{ij} \) are weights adopted.

The isogeometric class of numerical methods adopt NURBS functions to discretize the unknown field variables and geometry. Likewise, isogeometric collocation methods adopt NURBS to approximate the unknown field variables, namely, \( u, v, w, \phi_x \& \phi_y \) as shown below:
3.2 Collocation using NURBS functions

The discretized strong form of the equilibrium equations are collocated on the images of greville points in physical space. The greville points are defined in the parametric space. The greville points ̂ξi defined for a spline space of degree p and knot vector {ξi, ......, ξi+p+1} are given by

$$\hat{\xi}_i = \xi_i + 1 + \xi_i + 2 + ...... + \xi_i + p$$  (15)

In two dimensional problems like plate problems, a greville point is defined by ( ̂ξi, ̂ηi), which are the greville coordinates in two parametric directions. From eq. (15), it can be observed that some of the greville points are located within the domain and others are located at the boundary. Since the equations of equilibrium are different for the domain interior and boundary, the equations to be collocated need to be chosen suitably depending on the location of the greville point.

Using eqs.(14), we discretize the equilibrium equations eqs. (8) defined in the interior of the domain. The residual of these equations $\left( R_{\Omega}^u, R_{\Omega}^v, R_{\Omega}^w, R_{\Omega}^{\phi_x}, R_{\Omega}^{\phi_y} \right)$ are presented as follows:
\[ \begin{align*}
R^u_\Omega &= A_{11} \frac{\partial^2 u_0^h}{\partial x^2} + 2A_{16} \frac{\partial^2 u_0^h}{\partial x \partial y} + A_{66} \frac{\partial^2 u_0^h}{\partial y^2} + A_{12} \frac{\partial^2 v_0^h}{\partial x \partial y} + A_{16} \frac{\partial^2 v_0^h}{\partial x^2} + A_{26} \frac{\partial^2 v_0^h}{\partial y^2} + A_{66} \frac{\partial^2 v_0^h}{\partial x \partial y} + \\
&+ B_{11} \frac{\partial^2 \phi_0^h}{\partial x^2} + 2B_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{66} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + B_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ A_{26} \frac{\partial^2 v_0^h}{\partial x \partial y} + B_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + A_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + A_{26} \frac{\partial^2 \phi_0^h}{\partial x^2} + A_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ \frac{\partial \phi_0^h}{\partial x} + B_{26} \frac{\partial \phi_0^h}{\partial x^2} + B_{16} \frac{\partial \phi_0^h}{\partial x \partial y} + B_{66} \frac{\partial \phi_0^h}{\partial x \partial y} + A_{12} \frac{\partial \phi_0^h}{\partial x \partial y} + A_{26} \frac{\partial \phi_0^h}{\partial x^2} + A_{66} \frac{\partial \phi_0^h}{\partial x \partial y} + (16a) \\
R^v_\Omega &= A_{16} \frac{\partial^2 u_0^h}{\partial x \partial y} + A_{12} \frac{\partial^2 v_0^h}{\partial x \partial y} + A_{26} \frac{\partial^2 v_0^h}{\partial x^2} + A_{66} \frac{\partial^2 v_0^h}{\partial y^2} + A_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + A_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + A_{26} \frac{\partial^2 \phi_0^h}{\partial x^2} + A_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ A_{26} \frac{\partial^2 v_0^h}{\partial x \partial y} + B_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + A_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + A_{26} \frac{\partial^2 \phi_0^h}{\partial x^2} + A_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ \frac{\partial \phi_0^h}{\partial y} + B_{26} \frac{\partial \phi_0^h}{\partial x^2} + B_{16} \frac{\partial \phi_0^h}{\partial x \partial y} + B_{66} \frac{\partial \phi_0^h}{\partial x \partial y} + A_{12} \frac{\partial \phi_0^h}{\partial x \partial y} + A_{26} \frac{\partial \phi_0^h}{\partial x^2} + A_{66} \frac{\partial \phi_0^h}{\partial x \partial y} + (16b) \\
R^w_\Omega &= KA_{55} \frac{\partial \phi_0^h}{\partial x} + KA_{45} \frac{\partial \phi_0^h}{\partial y} + KA_{44} \frac{\partial \phi_0^h}{\partial x} + KA_{55} \frac{\partial^2 \phi_0^h}{\partial x^2} + 2KA_{45} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ KA_{44} \frac{\partial^2 \phi_0^h}{\partial y^2} + f (16c) \\
R^\phi_{\psi x} &= B_{11} \frac{\partial^2 \phi_0^h}{\partial x^2} + 2B_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{66} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + B_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ D_{11} \frac{\partial^2 \phi_0^h}{\partial x^2} + 2D_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{66} \frac{\partial^2 \phi_0^h}{\partial y^2} + D_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{16} \frac{\partial^2 \phi_0^h}{\partial x^2} + D_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + D_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} - \\
&- KA_{55} \left( \frac{\partial \phi_0^h}{\partial x} + \phi_0^h \right) - KA_{45} \left( \frac{\partial \phi_0^h}{\partial y} + \phi_0^h \right) (16d) \\
R^\phi_{\psi y} &= B_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{12} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + B_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + B_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ D_{10} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + D_{10} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{16} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + D_{26} \frac{\partial^2 \phi_0^h}{\partial y^2} + D_{66} \frac{\partial^2 \phi_0^h}{\partial x \partial y} + \\
&+ D_{22} \frac{\partial^2 \phi_0^h}{\partial x^2} + D_{26} \frac{\partial^2 \phi_0^h}{\partial x \partial y} - KA_{45} \left( \frac{\partial \phi_0^h}{\partial x} + \phi_0^h \right) - KA_{44} \left( \frac{\partial \phi_0^h}{\partial y} + \phi_0^h \right) (16e)
\end{align*} \]

Similarly, residuals of the equilibrium equations valid at Neumann boundaries \( R^\Gamma_u, \ R^\Gamma_w, \ R^\Gamma_{\psi x}, \) \& \( R^\Gamma_{\psi y} \) are obtained by substituting eq. (14) into eqs. (10). The order of polynomial and knot vector are the same for the NURBS functions specified in eqs. (14). The collocation scheme adopted is

\[
R^\Omega_\xi = 0 \quad R^\Omega_\eta = 0 \quad R^\Omega_\psi = 0 \quad R^\Omega_\psi = 0 \quad on \ S \left\{ \left\{ \xi \right\}^\Omega \right\}
\]

\[
R^\Gamma_u = 0 \quad R^\Gamma_v = 0 \quad R^\Gamma_w = 0 \quad R^\Gamma_{\psi x} = 0 \quad R^\Gamma_{\psi y} = 0 \quad on \ S \left\{ \left\{ \xi \right\}^\Gamma \right\}
\]

where \( S \left\{ \left\{ \xi \right\}^\Gamma \right\} \) is the mapping of Greville points from the parametric space to physical space.
4 NUMERICAL RESULTS

4.1 Bending of all round simply supported square cross ply laminated plates

Consider an all round simply supported rectangular laminated plate. The laminate consists of four plies (0/90/90/0) of equal thickness. Let $h$ be the total thickness of the plate. The material properties are

$$E_1 = 25E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.2E_2, \quad \nu_{12} = 0.25, \quad K = 5/6$$  \hspace{1cm} (18)

where the origin of the co-ordinate system is taken at a corner of the plate, $0 \leq x \leq a$ & $0 \leq y \leq b$. The lateral displacement and the stresses evaluated are non-dimensionalized using the following relationship,

$$w^* = w_0 (\frac{a}{2}, \frac{b}{2}, \frac{E_2 h^3}{b^4 q_0}), \quad \sigma_{xx}^* = \sigma_{xx} (\frac{a}{2}, \frac{b}{2}, \frac{h^2}{b^2 q_0}), \quad \sigma_{yy}^* = \sigma_{yy} (\frac{a}{2}, \frac{b}{2}, \frac{h^2}{b^2 q_0}), \quad \sigma_{xy}^* = \sigma_{xy} (\frac{a}{2}, \frac{b}{2}, \frac{h^2}{b^2 q_0}), \quad \sigma_{xz}^* = \sigma_{xz} (\frac{a}{2}, \frac{b}{2}, k = 1, 3, \frac{h}{b q_0})$$  \hspace{1cm} (19)

The results obtained are listed in Table 1 and compared with methods existing in literature. Figure 1 shows the plot of normalized central displacement $w^*$ versus $\sqrt{N_f}$ (square root of the number of degrees of freedom) for two plate width to thickness ratios of 10 & 100. Convergence of normalized central displacement $w^*$ can be observed.

4.2 Bending of all round clamped square sandwich plates

Consider an all round clamped square sandwich plate. The sandwich plate consists of a central core, embedded with face sheets on top and bottom. The face sheets are
Table 1: Simply supported \([0/90/90/0]\) square laminated plate under sinusoidal load.

<table>
<thead>
<tr>
<th>a/h</th>
<th>Method</th>
<th>(w^\ast)</th>
<th>(\sigma_{xx}^\ast)</th>
<th>(\sigma_{yy}^\ast)</th>
<th>(\sigma_{xy}^\ast)</th>
<th>(\tau_{xz}^\ast)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>HSDT [31]</td>
<td>0.4343</td>
<td>0.5507</td>
<td>0.2769</td>
<td>0.0217</td>
<td>0.2948</td>
</tr>
<tr>
<td></td>
<td>HSDT [32]</td>
<td>0.4343</td>
<td>0.5387</td>
<td>0.2708</td>
<td>0.0213</td>
<td>0.2897</td>
</tr>
<tr>
<td></td>
<td>FSDT [33]</td>
<td>0.4337</td>
<td>0.5382</td>
<td>0.2705</td>
<td>0.0213</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>Elasticity ' [34]</td>
<td>0.4347</td>
<td>0.539</td>
<td>0.271</td>
<td>0.0214</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>HSDT [35]</td>
<td>0.4365</td>
<td>0.5413</td>
<td>0.3359</td>
<td>0.0215</td>
<td>0.4106</td>
</tr>
<tr>
<td></td>
<td>Layer-wise [36]</td>
<td>0.4374</td>
<td>0.542</td>
<td>0.2697</td>
<td>0.0216</td>
<td>0.3232</td>
</tr>
<tr>
<td></td>
<td>RBF-PS [37]</td>
<td>0.432</td>
<td>0.5387</td>
<td>0.2697</td>
<td>0.0213</td>
<td>0.3154</td>
</tr>
<tr>
<td></td>
<td>Wavelets [38]</td>
<td>0.4335</td>
<td>0.5381</td>
<td>0.2704</td>
<td>0.0213</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>IGA- galerkin [39]</td>
<td>0.4354</td>
<td>0.5376</td>
<td>0.2702</td>
<td>0.0213</td>
<td>0.1907</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>0.3339</td>
<td>0.4138</td>
<td>0.2079</td>
<td>0.182</td>
<td>1.5795</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>0.3622</td>
<td>0.4493</td>
<td>0.2255</td>
<td>0.0206</td>
<td>1.0969</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>0.4335</td>
<td>0.5379</td>
<td>0.27</td>
<td>0.0235</td>
<td>0.9867</td>
</tr>
<tr>
<td></td>
<td>Quintic</td>
<td>0.4346</td>
<td>0.5392</td>
<td>0.2707</td>
<td>0.0223</td>
<td>0.8195</td>
</tr>
<tr>
<td>10</td>
<td>HSDT [31]</td>
<td>0.7149</td>
<td>0.5589</td>
<td>0.3974</td>
<td>0.0273</td>
<td>0.2697</td>
</tr>
<tr>
<td></td>
<td>HSDT [32]</td>
<td>0.7147</td>
<td>0.5456</td>
<td>0.3888</td>
<td>0.0268</td>
<td>0.264</td>
</tr>
<tr>
<td></td>
<td>FSDT [33]</td>
<td>0.6628</td>
<td>0.4989</td>
<td>0.3615</td>
<td>0.0241</td>
<td>0.1667</td>
</tr>
<tr>
<td></td>
<td>Elasticity [34]</td>
<td>0.743</td>
<td>0.559</td>
<td>0.403</td>
<td>0.0276</td>
<td>0.301</td>
</tr>
<tr>
<td></td>
<td>HSDT [35]</td>
<td>0.7153</td>
<td>0.5466</td>
<td>0.4383</td>
<td>0.0267</td>
<td>0.3347</td>
</tr>
<tr>
<td></td>
<td>Layer-wise [36]</td>
<td>0.7309</td>
<td>0.5496</td>
<td>0.3956</td>
<td>0.0273</td>
<td>0.2888</td>
</tr>
<tr>
<td></td>
<td>RBF-PS [37]</td>
<td>0.7204</td>
<td>0.5609</td>
<td>0.3911</td>
<td>0.0273</td>
<td>0.2843</td>
</tr>
<tr>
<td></td>
<td>Wavelets [38]</td>
<td>0.6627</td>
<td>0.4989</td>
<td>0.3614</td>
<td>0.0241</td>
<td>0.3181</td>
</tr>
<tr>
<td></td>
<td>IGA- galerkin [39]</td>
<td>0.6654</td>
<td>0.4983</td>
<td>0.361</td>
<td>0.0242</td>
<td>0.1669</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>0.6670</td>
<td>0.5022</td>
<td>0.3623</td>
<td>0.0202</td>
<td>0.1742</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>0.6688</td>
<td>0.5038</td>
<td>0.3636</td>
<td>0.0204</td>
<td>0.1741</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>0.6693</td>
<td>0.5043</td>
<td>0.3639</td>
<td>0.0203</td>
<td>0.1744</td>
</tr>
<tr>
<td></td>
<td>Quintic</td>
<td>0.6709</td>
<td>0.5055</td>
<td>0.3648</td>
<td>0.0203</td>
<td>0.1744</td>
</tr>
<tr>
<td>20</td>
<td>HSDT [31]</td>
<td>0.5061</td>
<td>0.5523</td>
<td>0.311</td>
<td>0.0233</td>
<td>0.2883</td>
</tr>
<tr>
<td></td>
<td>HSDT [32]</td>
<td>0.5060</td>
<td>0.5393</td>
<td>0.3043</td>
<td>0.0228</td>
<td>0.2825</td>
</tr>
<tr>
<td></td>
<td>FSDT [33]</td>
<td>0.4912</td>
<td>0.5273</td>
<td>0.2957</td>
<td>0.0221</td>
<td>0.1749</td>
</tr>
<tr>
<td></td>
<td>Elasticity [34]</td>
<td>0.517</td>
<td>0.543</td>
<td>0.309</td>
<td>0.023</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>HSDT [35]</td>
<td>0.507</td>
<td>0.5405</td>
<td>0.3648</td>
<td>0.0228</td>
<td>0.3818</td>
</tr>
<tr>
<td></td>
<td>Layer-wise [36]</td>
<td>0.5121</td>
<td>0.5417</td>
<td>0.3056</td>
<td>0.023</td>
<td>0.3248</td>
</tr>
<tr>
<td></td>
<td>RBF-PS [37]</td>
<td>0.5078</td>
<td>0.5436</td>
<td>0.3052</td>
<td>0.023</td>
<td>0.3066</td>
</tr>
<tr>
<td></td>
<td>Wavelets [38]</td>
<td>0.4912</td>
<td>0.5273</td>
<td>0.2956</td>
<td>0.0221</td>
<td>0.3332</td>
</tr>
<tr>
<td></td>
<td>IGA- galerkin [39]</td>
<td>0.4931</td>
<td>0.5268</td>
<td>0.2953</td>
<td>0.0221</td>
<td>0.1755</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>0.4882</td>
<td>0.5232</td>
<td>0.2923</td>
<td>0.0203</td>
<td>0.1818</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>0.4907</td>
<td>0.5264</td>
<td>0.2942</td>
<td>0.0204</td>
<td>0.1813</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>0.4940</td>
<td>0.5299</td>
<td>0.2962</td>
<td>0.0205</td>
<td>0.1827</td>
</tr>
</tbody>
</table>
Table 2: All round clamped square sandwich plate under uniform load.

<table>
<thead>
<tr>
<th>a/h</th>
<th>Method</th>
<th>( w^* )</th>
<th>( \sigma_{xx}^* )</th>
<th>( \sigma_{yy}^* )</th>
<th>( \sigma_{xy}^* )</th>
<th>( \tau_{xz}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Reddy J N [40]</td>
<td>0.2785</td>
<td>0.5347</td>
<td>0.0094</td>
<td>0.003</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>0.2350</td>
<td>0.4489</td>
<td>0.009</td>
<td>0.0114</td>
<td>0.3225</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>0.2482</td>
<td>0.4749</td>
<td>0.0092</td>
<td>0.0123</td>
<td>0.3098</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>0.2749</td>
<td>0.5264</td>
<td>0.0114</td>
<td>0.0151</td>
<td>0.3368</td>
</tr>
<tr>
<td>50</td>
<td>Reddy J N [40]</td>
<td>0.3111</td>
<td>0.5356</td>
<td>0.0108</td>
<td>0.0039</td>
<td>0.2406</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>0.3044</td>
<td>0.5044</td>
<td>0.0109</td>
<td>0.0139</td>
<td>0.3342</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>0.3085</td>
<td>0.5113</td>
<td>0.0112</td>
<td>0.0143</td>
<td>0.3299</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>0.3162</td>
<td>0.5241</td>
<td>0.0120</td>
<td>0.0152</td>
<td>0.3365</td>
</tr>
<tr>
<td>10</td>
<td>Reddy J N [40]</td>
<td>1.2654</td>
<td>0.5018</td>
<td>0.0550</td>
<td>0.0120</td>
<td>0.2318</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>1.3569</td>
<td>0.4153</td>
<td>0.0361</td>
<td>0.0281</td>
<td>0.2893</td>
</tr>
<tr>
<td></td>
<td>Cubic</td>
<td>1.3591</td>
<td>0.4154</td>
<td>0.0362</td>
<td>0.0284</td>
<td>0.2894</td>
</tr>
<tr>
<td></td>
<td>Quartic</td>
<td>1.3619</td>
<td>0.4162</td>
<td>0.0362</td>
<td>0.0285</td>
<td>0.2899</td>
</tr>
</tbody>
</table>

assumed to be orthotropic and the core is assumed to be transversely isotropic. Let \( h \) be the total thickness of the plate. Thickness of the core is 0.8 times the total thickness while the face sheets are one - tenth the total thickness of the sandwich plate. Material properties of the core are given below

\[
E_1 = 25E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.2E_2, \quad \nu_{12} = 0.25, \quad K = 5/6 \quad (20)
\]

Material properties of the face sheets are provided below

\[
E_1 = E_2 = 10^6, \quad G_{13} = G_{23} = 0.06 \times 10^6\text{psi}, \quad G_{12} = 0.016 \times 10^6\text{psi}, \quad \nu_{12} = 0.2 \quad (21)
\]

The transverse load acting on the rectangular plate is given by \( q(x, y) = q_0 \), where the origin of the co-ordinate system is taken at a corner of the plate, \( 0 \leq x \leq a \) \& \( 0 \leq y \leq b \). The lateral displacement and the stresses evaluated are non- dimensionalized using the following relations.

\[
w^* = w_0(\frac{a}{2} \cdot \frac{b}{2})\frac{E_2h^3}{b^4q_0}, \quad \sigma_{xx}^* = \sigma_{xx}(\frac{a}{2} \cdot \frac{b}{2} \cdot \frac{h}{b^2q_0})^2
\]
\[
\sigma_{yy}^* = \sigma_{yy}(\frac{a}{2} \cdot \frac{b}{2} \cdot \frac{h}{b^2q_0}), \quad \sigma_{xy}^* = \sigma_{xy}(a, b, \frac{h}{b^2q_0})
\]
\[
\sigma_{xz}^* = \sigma_{xz}(\frac{a}{2}, b, 0)\frac{h}{bq_0}
\]

The results obtained are listed in Table 2 and compared with solution existing in literature.
5 CONCLUSIONS

In this paper, analysis of laminated composite plates has been performed using Iso-

geometric collocation method. The method has been assessed for both laminated plate
and sandwich plates under all round simply supported and all round clamped conditions
respectively. The accuracy obtained by this method is comparable with the methods
existing in literature. Shear locking was observed under the thin plate limit when lower
order approximations were adopted. Mixed formulation may be considered in order to
alleviate shear locking. Further research direction would be to study free vibration anal-
ysis, buckling analysis and extension of present formulation under large deformations.

References

elements, nurbs, exact geometry and mesh refinement,” Computer methods in ap-

the assumed natural strain method to alleviate locking in solid-shell nurbs-based

geometric structural analysis,” Computer methods in applied mechanics and engi-

structural vibrations,” Computer methods in applied mechanics and engi-

1034, 2011.

[6] T. Elguedj, Y. Bazilevs, V. M. Calo, and T. J. Hughes, “B and f projection meth-
ods for nearly incompressible linear and non-linear elasticity and plasticity using
higher-order nurbs elements,” Computer methods in applied mechanics and engi-

of eigenvalue, boundary-value, and initial-value problems,” Computer Methods in

[8] S. Morganti, F. Auricchio, D. Benson, F. Gambarin, S. Hartmann, T. Hughes, and
A. Reali, “Patient-specific isogeometric structural analysis of aortic valve closure,”


