

RECENT ADVANCES IN SURROGATE MODELING OF REFLECTOR ANTENNA SYSTEMS

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Abstract. *This paper discusses some of the recent advances in the surrogate based modeling and optimization of reflector antenna systems, and presents several examples. Much of the focus is on the design of reflector systems for radio astronomy applications, where especially tight specifications are placed on some of the important performance metrics. In several cases the response surfaces of these performance metrics are computationally expensive to compute, and, as an added complication, they are often conflicting and need to be optimized in a multi-objective (MO) or Pareto framework. The large numbers of function evaluations required in these optimization loops make direct full wave simulation in the loop intractable, and surrogate based methods may in these cases lead to reliable global MO optimizations. Two main characteristics of reflector antenna responses of interest are exploited, namely their slow (or periodic) variation with design parameters, as well as the availability of a computationally cheap geometric optics approximation.*

1 INTRODUCTION

Reflector antenna systems are widely used in personal, commercial, scientific, and military applications, with ever increasing demands on their performance. Due to their typically large electrical size, the design of such systems traditionally relied heavily on approximated electromagnetic (EM) methods - most commonly Geometric Optics (GO) and Physical Optics (PO). These methods tend to display increasing accuracy with electrical size of the analyzed system, with the computational time normally scaling as N . An obvious drawback of using these methods is their inability to effectively and accurately model some of the (often important) finer details of the structures, including feed/dish interactions, strut and supporting structure influence, and large curvatures in the dish surfaces. With the enormous increase in availability of powerful personal and cluster computing facilities, as well as commercial computational electromagnetic (CEM) codes capable of solving a wide range of EM problems using full wave methods, it has become tractable in recent times to, if not design, at least simulate the performance of reflector systems much more accurately. An obvious drawback of full wave methods is the often prohibitively slow simulation times when it is to be used in an optimization loop.

Given this framework of the availability of well-developed fast approximate methods, as well as accurate full wave solutions, the use of surrogate based optimization (SBO) schemes for the design of reflector antennas is an obvious extension to the classical design methods. In this context SBO refers to a variety of methods used to shift the optimization task onto some surrogate model which is based on the fast approximated model (and is therefore fast to evaluate), but aligned in some way to the slow but accurate model (and is therefore more accurate than the approximate model – even if just in a specific region of interest in the parameter space).

This paper will discuss some of the recent advances in this field and provide examples of how SBO has been used to develop design methodologies for advanced reflector antenna systems. Much of the focus will be on the design of reflector systems for radio astronomy applications, where especially tight specifications are placed on some of the important performance metrics. In several cases the response surfaces of these performance metrics are computationally expensive to compute, and, as an added complication, they are often conflicting and need to be optimized in a multi-objective (MO) or Pareto framework. The large numbers of function evaluations required in these optimization loops make direct full wave simulation in the loop intractable, and SBO methods may in these cases lead to reliable global MO optimizations.

The paper is organized to first discuss the basics of reflector antenna systems analysis and some typical performance metrics. Given this background, the difficulty in producing accurate optimized designs will be highlighted, and discussions presented on surrogate modeling based methods that can be used to accelerate the design process. Several examples of different design applications will be presented, which include the design of the dish surfaces for a given feed radiation pattern, as well as the design of feed antennas for given reflector configurations.

2 REFLECTOR ANTENNA SYSTEMS BACKGROUND

In the most general terms, a reflector antenna system may be seen (in the transmit sense) as a transducer which transforms a spherical wavefront primary pattern, from some feed antenna, to a collimated plane wave which is radiated as the secondary pattern. This description is helpful in understanding the simplest approximation method used to analyze and design such systems, namely GO. The main assumption of GO is that the reflector surfaces are electrically very large and smooth, leading to specular reflection conditions for the ray tube approximations used to describe the radiation patterns. From here it is straight forward to analyze and describe the basic

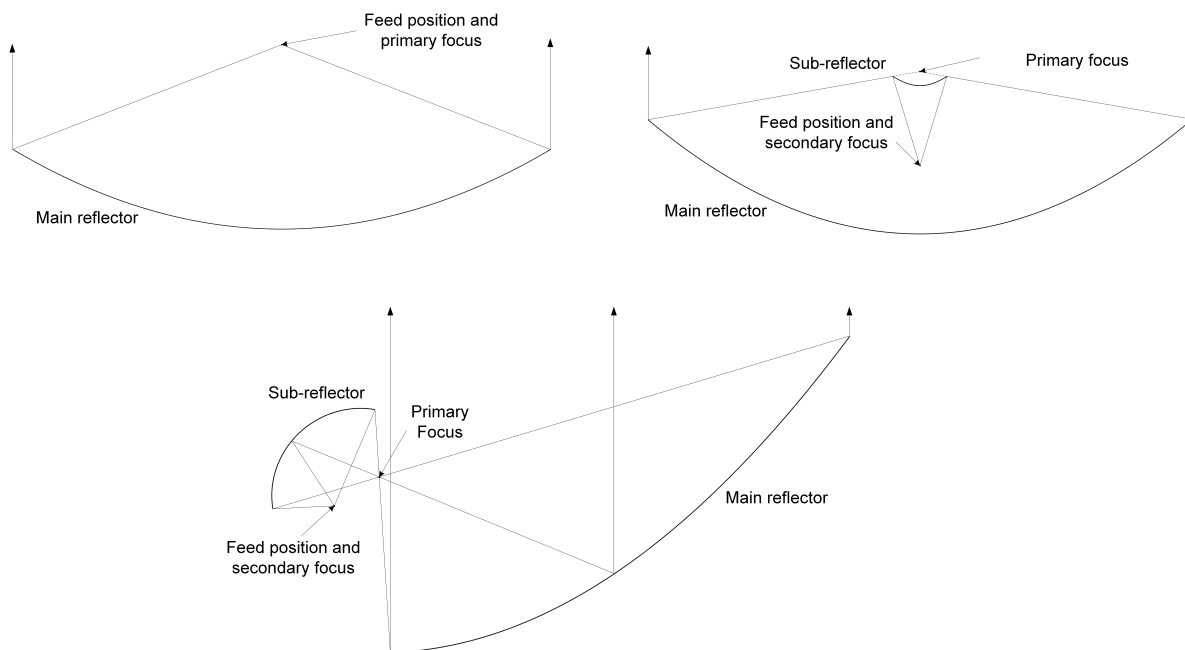


Figure 1: Outline geometry of some popular reflector configurations. Top left: Symmetrical paraboloid; Top right: Symmetrical Cassegrain; Bottom: Offset Gregorian. Ray paths, feed positions and foci for conic section (unshaped) systems are also indicated.

mechanisms of reflector systems – the simplest of which are possibly the family of conic section reflectors. These include the focus fed paraboloid, as well as Cassegrain and Gregorian dual reflector systems [1]. All these systems may be symmetrical or offset, with GO based design details available in the open literature [2, 3, 4, 5, 6], and sketches of some of the configurations shown in Fig. 1.

To achieve optimal performance, which typically involves maximizing the gain or receiving sensitivity (the ratio of effective aperture area and system temperature) while maintaining acceptable sidelobe levels (SLL) and cross polarization isolation (XPI), much effort has gone into developing reflector feed antennas with suitable radiation pattern characteristics [7, 8].

Another method used to improve performance is reflector shaping. Here the reflector antenna surfaces are shaped to provide a desired secondary pattern (normally specified as an aperture distribution) for a given primary radiation pattern. This strategy is most often employed in dual reflector systems, where a constant path length from the focus to the aperture can be enforced, along with specular reflection and power conservation [9, 10, 11, 12].

When designing very high performance reflector systems, and when the reflector surfaces become electrically smaller, the GO approximation no longer provides adequate accuracy. Some examples of such designs include the design of feeds for modern radio telescopes [13], PO synthesis of shaped beam reflectors for satellite applications [14], and design of small microwave link antennas for maximum sensitivity [15]. In all of these designs some other, more accurate, analysis method is employed in the optimization loop – mostly with a significantly design time penalty. The slow analysis time problem becomes especially pronounced when full wave simulations are required in the optimization loop, such as when the feed antenna parameters are varied or the reflector system is tightly coupled with the feed and not electrically large. Another situation that impedes the simulation time is when the receiving sensitivity is a goal function of the design. To calculate this response, the antenna radiometric noise temperature is required, which requires integration of the secondary pattern over the full 4π steradian sphere. Some

methods have been presented to accelerate this calculation, but at a cost in accuracy [16, 17]. The main bottleneck here is the requirement to calculate the secondary pattern over a fine angular grid, which becomes slow even for asymptotic methods such as PO. Some recent advances have accelerated this process as well, but it is most effective for extremely large problems [18].

3 SURROGATE MODELLING METHODS IN REFLECTOR DESIGN

In many reflector antenna applications, especially where pencil beams are required as the secondary pattern, the performance of the system is described by some derived quantities from the secondary pattern. These are normally the gain (described in normalized form as aperture efficiency), the receiving sensitivity, the SLL and XPI. Furthermore, these performance metrics are almost always conflicting, which makes Pareto base multi-objective (MO) optimization in the design process very attractive to quantify the trade-offs. MO optimization, however, typically requires a large number of function evaluations to converge, which may lead to unacceptably long run times if the full reflector analysis is included in the optimization loop. Faster, surrogate models are thus required to make such optimization based designs tractable.

3.1 Interpolation based models

Recently, interpolation based models have been used with some success in analysis and design of high performance reflector antenna systems. Interpolation of the response (hyper)surfaces may be required as functions of frequency, of physical parameters, or both.

A well known effect in many reflector systems is that of chromatic aberration, where a ripple with frequency is observed in the secondary radiation pattern amplitude close to broad-side. The cause of the ripple is interference of radiated fields with different path lengths which arise for a variety of reasons including coupling between the feed and the reflector or sub-reflectors in symmetrical systems [19], or back radiation from the feed or diffraction from the sub-reflector in offset systems [20]. Directly resolving the ripple in, for instance, the aperture efficiency will require a large number of samples in frequency. A Gaussian Process (GP) based interpolation scheme was suggested in [21], which is able to resolve the frequency ripple in offset Gregorian systems using only a fraction of the samples required for Nyquist convergence. An example of the possible performance is shown in Fig. 2, where the specific example is the offset Gregorian reflector system described in [21].

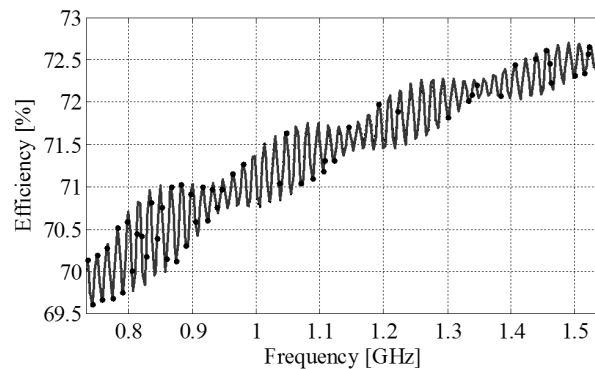


Figure 2: GP interpolation results for the aperture efficiency of an offset Gregorian system as a function of frequency. The training points are indicated as dots, the predicted response as a solid line, and the target response as a dashed line. The target response is mostly not visible as it is behind the interpolated response. Example taken from [21].

Frequency response interpolation is mostly used for detailed analysis of different responses using full wave simulations to model all the interactions between die reflector surfaces and feed. In the optimization based design context, some scalarization is normally performed over the frequency axis. It turns out that, for most design parameters of interest, the scalarized responses are often smooth functions. Some examples from the literature include aperture efficiency as a function of subtended angle in focus fed paraboloids [22], as well as receiving sensitivity, aperture efficiency, SLL and XPI of classical and shaped offset Gregorian systems [23, 24]. An example of some typical response surfaces is given in Section 4. Given these smooth responses as functions of design variables, interpolation based surrogate models can easily be constructed. All optimizations, including MO ones, are then performed on the interpolant instead of the actual simulation models [25].

3.2 Multi-fidelity models

In the above section on interpolation models, the allocation of a base set to ensure a converged interpolant is an important part of the model design, and may still require significant simulation time. This is because all the simulations are performed at the level of a fine model – meaning here the simulation model is as accurate as required for the specific design. The fine model may be GO or PO for the main beam if the reflectors are very large electrically, but in smaller systems, or when receiving sensitivity is required, these models are no longer sufficiently accurate. In this case they may be used as a faster coarse model, to augment the (limited) information from the accurate fine models obtained via, for instance, full wave simulations. Surrogate modeling, in this context, attempts to align the fully sampled coarse model to the sparsely sampled fine model over some region of interest in the design space. This is the multi-fidelity modeling scenario, and it is very well suited to reflector design problems – mainly due to the availability of a physics based (often GO) coarse model which is very fast to evaluate.

Examples of multi-fidelity modeling and design of reflector systems again include modeling of frequency and geometric parameters. In the case of frequency variations, the ripple mentioned in the previous sub-section may also be modeled using a multi-fidelity approach. Since the physical cause of the ripple is well understood, the period of the ripple may be estimated from the physical configuration of the system. This idea was implemented for the aperture efficiency of an offset Gregorian system in [26], where the amplitude of the ripple was estimated using PO simulations which did not include the main reflector. The response constructed in this way therefore used a multi-fidelity model description – the fine model is the full system simulated at a few points in frequency, and the coarse model is constructed using simulations of only the feed and sub-reflector, along with GO, to estimate the expected fields in the direction of the main beam. The models were aligned using a regression scheme, as well as band-limited sampling theory. An example of the possible system performance is shown in Fig. 3, where the specific example is the offset Gregorian reflector system described in [26].

Multi-fidelity models have also been successfully used in MO optimization and entire domain modeling of reflector systems in terms of their geometric parameters. Here the correlation between coarse GO and fine PO or full wave models is exploited by aligning the coarse models to the fine models, in regions of interest in the design space, using two general methods. The alignment methods used are space mapping (SM) [27] as well as regression through the residuals between the coarse and fine models. SM based methods are demonstrated for MO optimization problems in [28], where the shaped surface of an offset Gregorian system is designed for maximum aperture efficiency and minimum SLL, and in [29], where a feed antenna for a

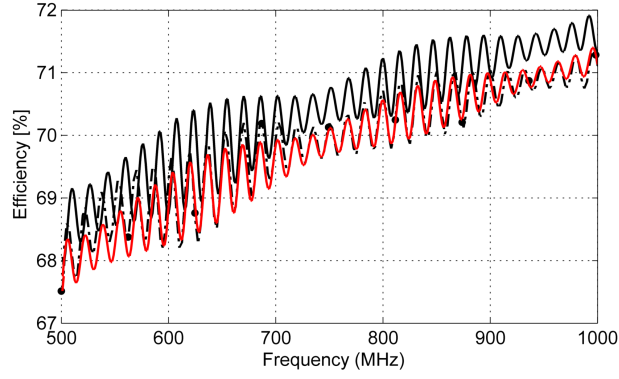


Figure 3: Multi-fidelity modeling results for the aperture efficiency of an offset Gregorian system as a function of frequency. The training points are indicated as dots, the coarse model as a solid black line, the fine model as a dashed line, and the aligned surrogate model as a solid red line. Example taken from [26].

classical Cassegrain system is designed for the same goals. Implicit SM has also been used to accurately model the SLL and aperture efficiency of blocked aperture symmetrical reflector systems over the full design space [30]. The residual regression method was used to model the radiometric noise temperature, used in the receiving sensitivity calculation, of a variety of reflector systems in [31], and for the MO optimization of feed antennas for an offset Gregorian system in [25].

In all these examples interpolation of the coarse model over the parameter space is also employed. An illustrative example is shown in the following section to highlight the main steps in a multi-fidelity MO optimization design procedure.

4 EXAMPLE

An example that illustrates both the interpolation and multi-fidelity modeling schemes, in the MO optimization context, is that discussed in [28]. In this problem the shape of the dishes, described by two parameters σ and b , is to be optimized for maximum aperture efficiency and minimum SLL. A fixed feed pattern is assumed, and details of the geometry is provided in [28]. GO is used as a coarse model, and a PO simulation using the commercial package GRASP [32] is used as the fine model. The optimization algorithm involves the following steps:

1. Allocate a base set X'_B , as a grid over the full parameter space, and evaluate the coarse model $c(X'_B)$.
2. Construct a Kriging interpolant [33] through $c(X'_B)$ to obtain the coarse model over the full parameter space \mathbf{x} as $c(\mathbf{x})$.
3. Allocate an initial fine model base set X_B and evaluate the fine model $f(X_B)$.
4. Construct a SM based surrogate model $s(\mathbf{x})$ using multi-point parameter extraction [27] at X_B .
5. Identify the Pareto front on s using some multi-objective evolutionary algorithm (or a grid search – the model evaluation is fast enough).
6. Sample the Pareto front obtained in step 4 to obtain a new base set X_B ; evaluate the fine model at X_B and compare to the corresponding surrogate model samples.

7. If the termination condition is not satisfied, return to step 4.

Termination of the algorithm is based on the least squares distance between the Pareto set representations obtained in consecutive iterations. The important point here is that the surrogate model is aligned to the fine model only in the region of the estimated Pareto front – it may be quite inaccurate away from the front. Therefore the method is best suited to problems with smooth response surfaces, such as those encountered here. The method is illustrated in Figs. 4 and 5, where the improved alignment of the surrogate model is obvious along the Pareto front for the second iteration. Also note how the Pareto set representation lies on the surrogate model surface after the second iteration, indicating very good alignment with the actual fine model in this region.

5 CONCLUSIONS AND FUTURE WORK

This paper provided a summary of some recent advances in surrogate modeling of reflector antenna systems. Two main characteristics of reflector systems that allow relatively simple, accurate and low cost surrogate models to be developed, are the slow (or periodic) variation of the most important performance metrics as functions of the design parameters, as well as the availability of simple GO models of these responses. These characteristics have been exploited for some simple design cases, and a proof of concept established. Future work include a multitude of extensions to the current design and analysis methods, mostly working towards extending the MO optimization algorithm to more realistic, higher dimensional, design problems. Here more sophisticated sampling schemes should be employed to establish and update the interpolants, while allowing exploration of the search space away from the Pareto front [34].

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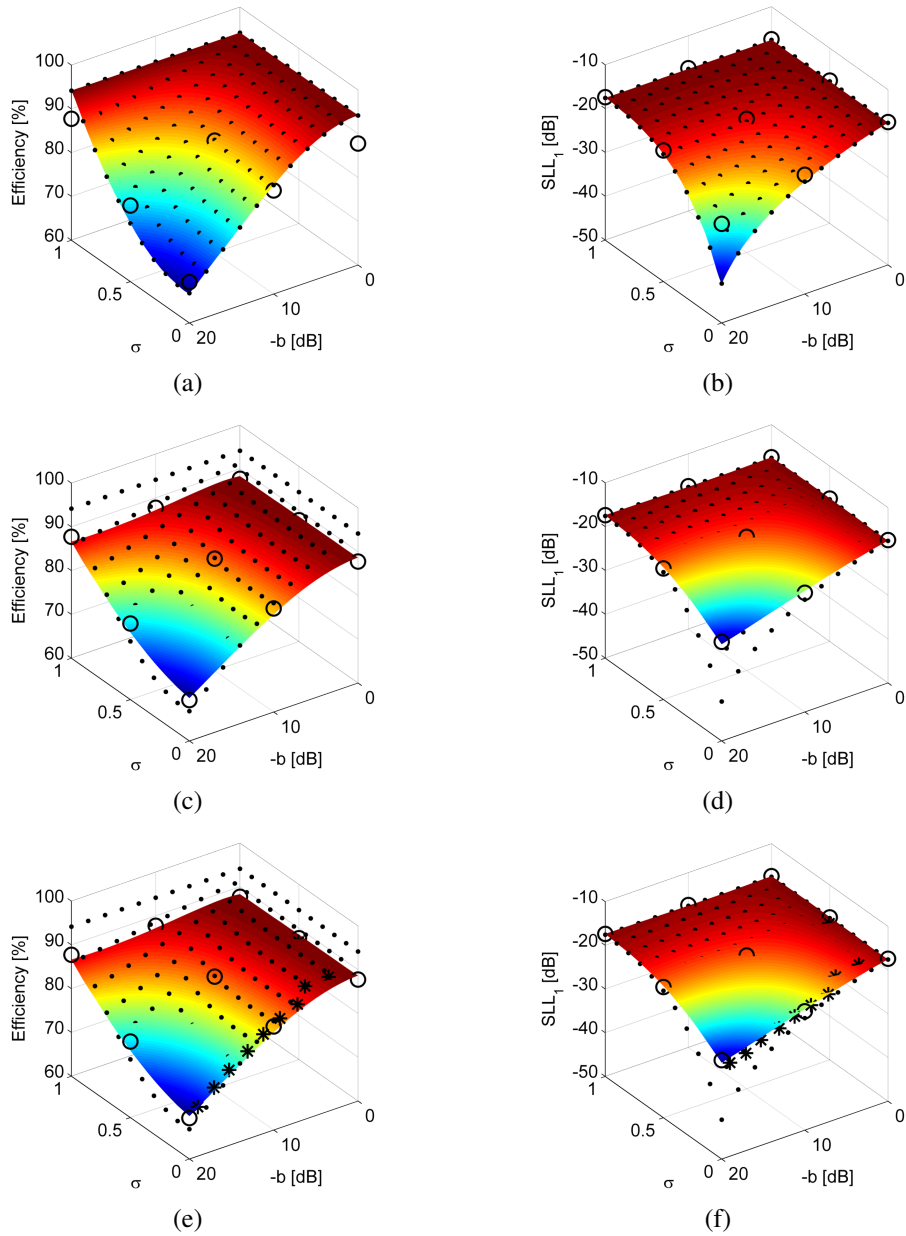


Figure 4: The evolution of the first iteration of the MO optimization algorithm using interpolation and multi-fidelity based models. The left column shows aperture efficiency, and the right the SLL. Dots indicate the coarse model base set X'_B , circles the fine model base set X_B , and stars the sampled Pareto set representation. The surface shows the smooth surrogate model. Steps 1-3 are illustrated in (a,b), step 4 in (c,d) and step 5 in (e,f).

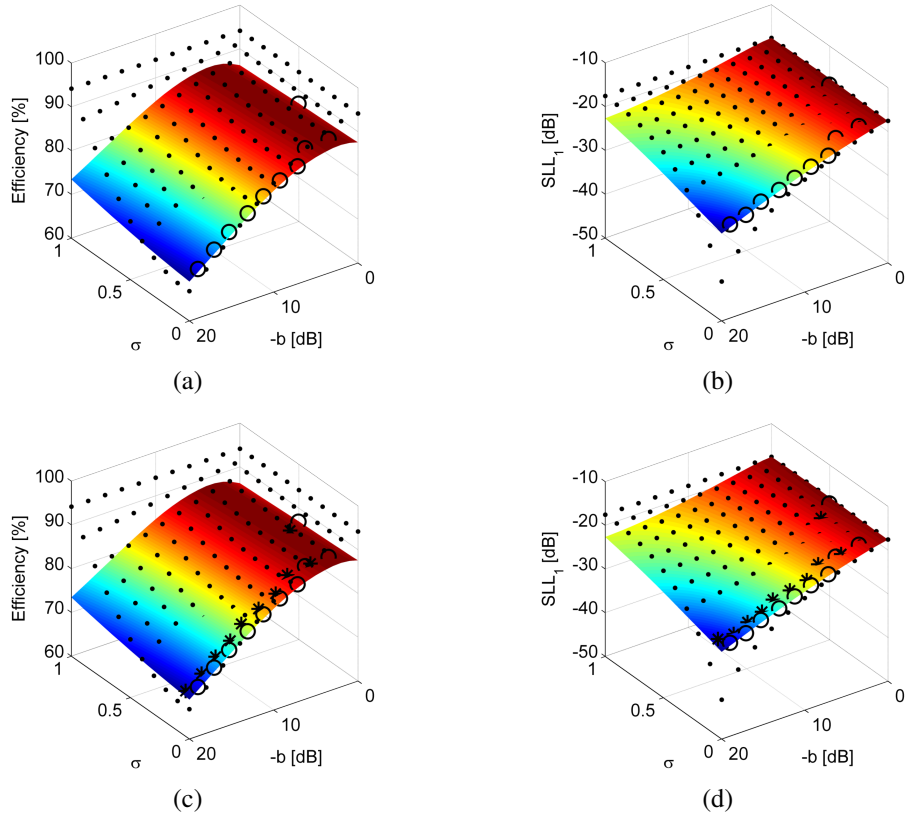


Figure 5: The evolution of the second and final iteration of the MO optimization algorithm using interpolation and multi-fidelity based models. The left column shows aperture efficiency, and the right the SLL. Dots indicate the coarse model base set X'_B , circles the fine model base set X_B , and stars the sampled Pareto set representation. The surface shows the smooth surrogate model. Step 4 is illustrated in (a,b) and step 5 in (c,d).