ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

EVALUATION OF LINE-FITTING METHOD OF MODEL ORDER REDUCTION

V. Makhavikou[1], R. Kasper[1], and D. Vlasenko[2]

[1] Otto von Guericke University
Universitaetsplatz, 2, 39106 Magdeburg, Germany
{valery.makhavikou, roland.kasper}@ovgu.de

[2] Schaeffler Technologies AG & Co. KG
Industriestrasse 1-3, 91074 Herzogenaurach, Germany
dmitry.vlasenko@schaeffler.com

Keywords: Elastic multibody systems, Floating frame of reference, Model order reduction.

Abstract. *Computer aided simulation is an important part of development of modern technical products. Many modern technical systems operate at high speeds and include lightweight components. These conditions usually lead to appearance of deformation effects that must be taken into account in processes of system modeling and simulation.*

In this contribution the dynamics of elastic multibody systems is formulated using the floating frame approach and the finite element method. In many applications a large number of elastic coordinates have to be employed to properly describe body deformations. This results in large simulation costs and memory deficiency problems. The problem can be solved using model order reduction techniques, which approximate a large number of equations by a much smaller number of equations that keep important dynamic properties of the original system.

This work is focused on the evaluation of the line-fitting method that is a linear structure preserving method of model order reduction for elastic multibody simulations. The method is based on the approximation of transfer functions and enables tuning of a reduced order model for certain transfer functions and certain frequency ranges. Line-fitting reduced order models possess such desirable properties as high accuracy, preservation of stability, low order and stiffness.

We apply the line-fitting method to a model of elastic bar, analyze properties of a reduced order model, and compare the results with results of the powerful SEREP reduction approach.

1 INTRODUCTION

1.1 Research field

The subject of this research relates to simulation of elastic multibody systems (EMBS). The term EMBS denotes a system of rigid and elastic bodies interconnected by joints or coupling elements, where the bodies may undergo large rigid body motions and elastic deformations. Under elastic deformations a body returns to its initial state after applied loads are removed. In particular, in this article we focus on linear elastic deformations. It means that we assume small deformations and linear relationship between deformations and applied forces. EMBS appear in applications of many engineering fields: robotics, biomechanics, and vehicle and aircraft dynamics.

Modeling of EMBS is based on methods of multibody system dynamics and the theory of elasticity. The most efficient way to describe the dynamics of EMBS undergoing linear deformations is a floating frame formulation [1]. According to this method the total motion of elastic body is divided into two parts: rigid body motion represented by the motion of body reference frame and deformations with respect to this frame.

Linear elastic deformations are usually modeled by the finite element method which results in a system of ODEs. The problem is that in many cases, the system of equations is extremely high dimensional, and its solution on standard computers is either not feasible or it suffers from vast computational time and memory deficiency problems. The problem can be solved by model order reduction methods. These methods approximate the large set of equations by a small set of equations that keep important dynamic properties of the original system.

1.2 Past studies

Over the last decades a variety of reduction techniques have been developed. The set of classical reduction approaches applied in elastic multibody dynamics consists of the methods based on modal truncation, condensation [2, 3], and component mode synthesis [4, 5, 6]. The traditional reduction methods also include a System Equivalent Reduction Expansion Process (SEREP), which was introduced in [7] and extensively used in structural and multibody mechanics. The classical approaches are implemented in many simulation software and remain state of the art techniques for model order reduction.

More recently, a few alternative reduction methods have come from the field of control theory, namely, techniques based on the singular value decomposition (SVD) using Gramian matrices and moment matching via Krylov subspaces [8, 9, 10, 11]. The methods are aimed at the approximation of input-output behavior of dynamical systems. Nowadays, the application of these techniques in the context of EMBS is on the focus of intensive research.

The comprehensive description of basic model order reduction methods can be found in the books [8, 12]. Relative advantages and disadvantages of widespread reduction approaches are presented in [13, 14].

1.3 Problem statement

In this article we examine a recently developed line-fitting method of model order reduction. The approach was originally proposed in [15]. In addition, the method was tested on academic and industrial application examples and compared with the widely used Craig-Bampton method in [16, 17]. The line-fitting approach is based on an approximation of transfer functions of original finite element model and it enables tuning of a reduced order model for certain transfer functions and certain frequency ranges. In [16, 17] line-fitting reduced order models showed better results in comparison with Craig-Bampton models in the accuracy, order, and stiffness.

The limitation of past studies of the line-fitting method consists in the fact that they focus primarily on the comparison with the Craig-Bampton method. The aim of this contribution is to evaluate the line-fitting method relative to the powerful SEREP approach. Despite the SEREP method is not a typical approach for the approximation of input-output behavior of a system, it can be applied for such applications and provide high quality reduced order models.

The relative comparison of promising reduction approaches is an important task because this contributes to further improvements of methods and to an appropriate choice of methods for certain applications examples.

1.4 Main steps and outline

In order to evaluate reduction approaches, it is necessary to define evaluation parameters, methods, and criteria. Application examples often make the following demands on the reduction approaches:

- high accuracy of reduced model,
- fast simulation of reduced model,
- simple tuning of reduced model for certain transfer functions and certain frequency ranges,
- preservation of model stability,
- preservation of second order structure of equations of motion,
- computational efficiency of reduction method.

We evaluate accuracy of reduced order models using a relative error of natural frequencies, correlation of eigenvectors, and a relative error for system transfer functions. In order to assess simulation speed, we compare an order of reduced models and the largest stiffness. The difficulty of tuning of reduced order model can be assessed using the time needed for the achievement of predefined accuracy demands. Preservation of stability and preservation of second order structure of equations of motion follow from theoretical descriptions of the methods. Finally, we determine computational efficiency of the reduction approaches by analyzing their computational complexity.

Theoretical description of methods allows partial evaluating of their properties. Complete assessment requires application of methods to numerical examples with subsequent analysis of reduced order models.

The structure of this article is as follows. We begin with a peace of theoretical information about equations of motion for EMBS and about the model order reduction. After that we describe SEREP and line-fitting approaches and identify their strength and weaknesses from the theoretical point of view. Next, we apply both methods to a model of elastic bar, validate reduced order models, and compare the results. Finally, we summarize and generalize the obtained results and discuss further work on the subject.

2 FUNDAMENTALS OF ELASTIC MULTIBODY DYNAMICS

2.1 Modeling of multibody dynamics

The equations of motion of a single unconstrained body can be derived using Jordain's principle. This leads to the following representation:

$$\begin{bmatrix} \mathbf{M}_r^i & \mathbf{M}_{re}^i \\ \mathbf{M}_{er}^i & \mathbf{M}_e^i \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_r^i \\ \ddot{\mathbf{q}}_e^i \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{K}_e^i \cdot \mathbf{q}_e^i + \mathbf{D}_e^i \cdot \dot{\mathbf{q}}_e^i \end{Bmatrix} = \begin{Bmatrix} \mathbf{h}_r^i \\ \mathbf{h}_e^i \end{Bmatrix}. \tag{1}$$

Here the superscript i refers to a number of body. The vector of coordinates consists of the rigid body coordinates $\mathbf{q}_r$ and the elastic coordinates $\mathbf{q}_e$. The vector $\mathbf{q}_r$ includes 3 translational and 3 rotational coordinates; the term $\mathbf{q}_e$ has a length N equal to a number of degrees of freedom of the finite element model. The mass matrix includes a rigid body part $\mathbf{M}_r \in \mathbb{R}^{6\times 6}$, an elastic part $\mathbf{M}_e \in \mathbb{R}^{N\times N}$, and coupling parts $\mathbf{M}_{re} = \mathbf{M}_{er}^T$. The matrices $\mathbf{K}_e$ and $\mathbf{D}_e$ represent stiffness and damping matrices of the elastic body. The Coriolis, centrifugal, as well as external forces are summarized in the force vector $\mathbf{h}$.

The equation of motion of the whole EMBS can be written as

$$\mathbf{M}(\mathbf{q}) \cdot \ddot{\mathbf{q}} + \mathbf{D} \cdot \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t), \tag{2}$$

where $\mathbf{q}$ is a vector of generalized coordinates of the whole system, $\mathbf{g}$ includes the vector $\mathbf{h}$ and the vector of constraint forces. More detailed description of EMBS dynamics can be found in [18, 19].

2.2 Modeling of elastic bodies

In this subsection we focus on the concept of model order reduction. Let $\mathbf{u}(t)$ and $\mathbf{y}(t)$ be an input distribution and an output measurement arrays, respectively. The elastic part of the equation of motion (1) is transformed into a linear time invariant second order multi-input and multi-output (MIMO) system

$$\begin{aligned} &\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e(t) + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e(t) + \mathbf{K}_e \cdot \mathbf{q}_e(t) = \mathbf{B}_e \cdot \mathbf{u}(t) \\ &\mathbf{y}(t) = \mathbf{C}_e \cdot \mathbf{q}_e(t) \end{aligned} \tag{3}$$

with the input matrix $\mathbf{B}_e \in \mathbb{R}^{N\times p}$ and the output matrix $\mathbf{C}_e \in \mathbb{R}^{r\times N}$, where p and r are numbers of input and output coordinates, respectively.

Due to the representation of elastic multibody systems using the floating frame approach, the elastic degrees of freedom can be reduced by means of methods of linear model order reduction. The basic idea of model reduction is to approximate the original set of ODEs by a low-dimensional set of equations that preserves significant characteristics of the full model. The large vector of elastic coordinates $\mathbf{q}_e \in \mathbb{R}^N$ is approximated using a vector $\tilde{\mathbf{q}}_e \in \mathbb{R}^n$ of a smaller dimension $n \ll N$ as

$$\mathbf{q}_e(t) \approx \mathbf{V} \cdot \tilde{\mathbf{q}}_e(t), \tag{4}$$

with $\mathbf{V} \in \mathbb{R}^{N\times n}$ being a projection matrix.

The transformation (4) leads to the following equations of motion:

$$\begin{aligned} &\tilde{\mathbf{M}}_e \cdot \ddot{\tilde{\mathbf{q}}}_e(t) + \tilde{\mathbf{D}}_e \cdot \dot{\tilde{\mathbf{q}}}_e(t) + \tilde{\mathbf{K}}_e \cdot \tilde{\mathbf{q}}_e(t) = \tilde{\mathbf{B}}_e \cdot \mathbf{u}(t) \\ &\mathbf{y}(t) = \tilde{\mathbf{C}}_e \cdot \tilde{\mathbf{q}}_e(t) \end{aligned} \tag{5}$$

with the reduced matrices $\tilde{\mathbf{M}}_e = \mathbf{V}^T \cdot \mathbf{M}_e \cdot \mathbf{V}$, $\tilde{\mathbf{D}}_e = \mathbf{V}^T \cdot \mathbf{D}_e \cdot \mathbf{V}$, $\tilde{\mathbf{K}}_e = \mathbf{V}^T \cdot \mathbf{K}_e \cdot \mathbf{V} \in \mathbb{R}^{n\times n}$, and $\tilde{\mathbf{B}}_e = \mathbf{V}^T \cdot \mathbf{B}_e \in \mathbb{R}^{n\times p}$, $\tilde{\mathbf{C}}_e = \mathbf{C}_e \cdot \mathbf{V} \in \mathbb{R}^{r\times n}$.

The difference of model order reduction methods lies in the way they generate the transformation matrix $\mathbf{V}$.

3 MODEL ORDER REDUCTION METHODS

The task of model reduction methods is to find a coordinate transformation matrix **V** in the relation (4). This section shows the constuction of **V** according to the line-fitting method and the SEREP approach. We also point out advantages and disadvantages of both methods relative to the requiremets stated in the introduction section.

3.1 SEREP

The idea of SEREP method is to include to a reduced order model only c modes and r degrees of freedom that are necessary to the model. Here c is defined using a bandwidth of excitation loads and r is defined based on the number of degrees of freedom where external loads are applied or model output coordinates are defined.

According to the SEREP method, all degrees of freedom of finite element model are divided into two sets: retained and truncated. These degrees of freedom are denoted with the indexes r and t, respectively. The elastic part of equations of motion from (1) with no damping can be written as

$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{h}_e \tag{6}$$

For the sake of simplicity we omit the index e for the subsequent equations in this section. Partition of (6) into retained and truncated coordinates yields:

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rt} \\ \mathbf{M}_{rt} & \mathbf{M}_{tt} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_t \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{rr} & \mathbf{K}_{rt} \\ \mathbf{K}_{rt} & \mathbf{K}_{tt} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{q}_r \\ \mathbf{q}_t \end{bmatrix} = \begin{bmatrix} \mathbf{h}_r \\ \mathbf{h}_t \end{bmatrix} \tag{7}$$

The coordinate transformation from physical coordinates to modal coordinates is defined as

$$\mathbf{q} = \mathbf{U} \cdot \mathbf{x} \tag{8}$$

where $\mathbf{U}$ and $\mathbf{x}$ are a modal matrix and modal coordinates. According to the SEREP method, the physical coordinates are approximated by a subset of modal vectors and corresponding modal coordinates as follows

$$\mathbf{q} = \mathbf{U}_c \cdot \mathbf{x}_c \tag{9}$$

Next, the approximation of physical coordinates is partitioned as

$$\begin{bmatrix} \mathbf{q}_r \\ \mathbf{q}_t \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{cr} \\ \mathbf{U}_{ct} \end{bmatrix} \cdot \mathbf{x}_c \tag{10}$$

Solving of $\mathbf{q}_r = \mathbf{U}_{cr} \cdot \mathbf{x}_c$ for $\mathbf{x}_c$ gives

$$\mathbf{x}_c = \mathbf{U}_{cr}^{+} \cdot \mathbf{q}_r \tag{11}$$

where $\mathbf{U}_{cr}^{+} = (\mathbf{U}_{cr}^T \cdot \mathbf{U}_{cr})^{-1} \cdot \mathbf{U}_{cr}^T$ defines a pseudo inverse matrix of $\mathbf{U}_{cr}$. Substituting (11) into (9) yields

$$\mathbf{q} = \mathbf{U}_c \cdot \mathbf{U}_{cr}^{+} \cdot \mathbf{q}_r \tag{12}$$

It follows that the coordinate transformation matrix takes the form

$$\mathbf{V} = \mathbf{U}_c \cdot (\mathbf{U}_{cr}^T \cdot \mathbf{U}_{cr})^{-1} \cdot \mathbf{U}_{cr}^T . \tag{13}$$

The accuracy of eigenfrequencies and eigenvectors retained in SEREP reduced order model is exact [13]. The modes may be arbitrarily selected from the modes in the original model. In addition, the accuracy of the eigensolution of the reduced model does not depend on the location of interface degrees of freedom. The quality of predefined transfer functions in the reduced model is determined by the eigenvectors chosen in the SEREP procedure. The opened questions are what and how many modes to include into the model to meet accuracy demands on the transfer functions of interest.

The order and stiffness of reduced model are defined by the number of chosen interface coordinates and the highest eigenfrequency of retained modes.

The tuning of reduced model for certain transfer functions and certain frequency ranges can be a not trivial task because of the difficult choice of important eingevectors.

The non-proper definition of number of interface coordinates and retained modes can result in the generation of rank-deficient mass and stiffness matrices. In this case, stability of full model is not preserved in the reduced model.

The transformation (13) preserves the second order structure of equations of motion for the reduced system.

Computation of eigensolution takes in general $O(N^3)$ arithmetical operations. In some applications the eigenvalues can be known in advance, e.g., from experimental tests. The computational cost of pseudo inverse using SVD is $O(N^3)$ operations.

3.2 Line-fitting method

The line-fitting method approximates the dynamical behavior of mechanical systems by fitting transfer functions

$$\mathbf{H}(s) = \mathbf{C}_e \cdot (s^2\mathbf{M}_e + s\mathbf{D}_e + \mathbf{K}_e)^{-1} \cdot \mathbf{B}_e \tag{14}$$

that are determined from the system of equations (3) by using the Laplace transformation with a complex frequency $s = i\omega$.

In order to obtain the projection matrix $\mathbf{V}$ without rigid body modes, we exclude the contribution of rigid body motion form (14). The translational and rotational rigid modes can be written as follows:

$$\mathbf{V}_0 = \begin{bmatrix}\mathbf{V}_{0t} & \mathbf{V}_{0r}\end{bmatrix} = \begin{bmatrix}\begin{bmatrix}\vdots \\ [\mathbf{I}]^k \\ \vdots\end{bmatrix} & \begin{bmatrix}\vdots \\ [-\tilde{\mathbf{r}}]^k \\ \vdots\end{bmatrix}\end{bmatrix}, k = 1,.., N. \tag{15}$$

Here the matrix $\mathbf{I}$ is a 3 × 3 identity matrix, the term $[\mathbf{r}]^k$ gives an undeformed position of k-th node of the FE model and consequently $[\tilde{\mathbf{r}}]^k$ annotates an associated skew symmetric matrix. The given calculation of $\mathbf{V}_0$ is valid for finite element nodes that exhibit three translational degrees of freedom.

The part of transfer matrix that corresponds to rigid body motion can be found as

$$\mathbf{H}_0(s) = \mathbf{C}_0 \cdot (s^2\mathbf{M}_0 + s\mathbf{D}_0 + \mathbf{K}_0)^{-1} \cdot \mathbf{B}_0 \tag{16}$$

with $\mathbf{M}_0 = \mathbf{V}_0^T \cdot \mathbf{M}_e \cdot \mathbf{V}_0$, $\mathbf{D}_0 = \mathbf{V}_0^T \cdot \mathbf{D}_e \cdot \mathbf{V}_0$, $\mathbf{K}_0 = \mathbf{V}_0^T \cdot \mathbf{K}_e \cdot \mathbf{V}_0 \in \mathbb{R}^{6\times6}$, and transformed input and output matrices $\mathbf{B}_0 = \mathbf{V}^T \cdot \mathbf{B}_e \in \mathbb{R}^{6\times p}$, $\mathbf{C}_0 = \mathbf{C}_e \cdot \mathbf{V}_0 \in \mathbb{R}^{r\times6}$. The component of transfer matrix (14) corresponding to elastic deformations is defined as

$$\mathbf{H}_1(s) = \mathbf{H}(s) - \mathbf{H}_0(s) \tag{17}$$

Further, we are looking for the transformation matrix **V**. We want to retain the output coordinates in the reduced order model, therefore the upper part of the coordinate transformation matrix **V** is an r × r identity matrix:

$$\mathbf{V} = \begin{bmatrix} \mathbf{I} \\ \mathbf{W} \end{bmatrix} \tag{18}$$

The matrix **W** can be found from the relation of non-output coordinates and output coordinates in terms of their transfer functions as

$$\mathbf{H}_1^{\perp}(s) \approx \mathbf{W} \cdot \mathbf{H}_1(s) \tag{19}$$

We construct the matrix **W** as a trade-off between frequency points of interest $s_k = i \cdot 2\pi f_k, k = 1..z$. The equation (19) is turned into

$$\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \dots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix} = \mathbf{W} \cdot \begin{bmatrix} \mathbf{H}_1(s_1) & \dots & \mathbf{H}_1(s_z) \end{bmatrix} \tag{20}$$

The decomposition of (20) into real and imaginary parts yields:

$$\begin{gathered} \begin{bmatrix} \Re(\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \dots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix}) & \Im(\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \dots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix}) \end{bmatrix} = \\ \mathbf{W} \cdot \begin{bmatrix} \Re(\begin{bmatrix} \mathbf{H}_1(s_1) & \dots & \mathbf{H}_1(s_z) \end{bmatrix}) & \Im(\begin{bmatrix} \mathbf{H}_1(s_1) & \dots & \mathbf{H}_1(s_z) \end{bmatrix}) \end{bmatrix} \end{gathered} \tag{21}$$

that can be rewritten as

$$\begin{gathered} \mathbf{T}_n = \mathbf{W} \cdot \mathbf{T}_o \\ \mathbf{T}_o^T \cdot \mathbf{W} = \mathbf{T}_n^T \end{gathered} \tag{22}$$

The matrices $\mathbf{T}_o^T \in \mathbb{R}^{2pz \times r}$ and $\mathbf{T}_n^T \in \mathbb{R}^{2pz \times (N-r)}$, where p, r, z are the number of inputs, outputs, and reference frequency points, respectively. The order of reduced model n is equal to the number of outputs r. The matrix **W** can be found as a least-squares solution of (22). After the projection matrix **V** is obtained, the reduced order model is constructed using (5).

The main positions of reference frequency points have to be points of resonances and anti-resonances. Besides, additional reference frequency points can be used to enhance quality of reduced model in special frequency ranges or points.

The error of predefined transfer functions in the line-fitting method depends on a residuum in equation (22). The error can be influenced by a number of interface coordinates, position of interface coordinates, and a choice of reference frequency points. As for the majority of other reduction methods, the error in this approach is unknown before the reduction, but it can be calculated after the reduction. Recommendations on the achievement of highly precise results are given in [15].

Simulation speed depends in addition to other factors on the number of coordinates of reduced order model and stiffness of the system. According to the results presented in [16, 17], line-fitting models require relative low number of coordinates to achieve desired accuracy demands. The stiffness is defined by the highest eigenfrequency of the system. The line-fitting method can influence the accuracy of eigenfrequencies contained in the predefined frequency range, but the spectrum of model beyond this range is uncontrolled. For this reason, no statements about the limits of the spectrum can be made. However, numeric examples in [16, 17] show that the stiffness of line-fitting models is significantly lower than the stiffness of Craig-Bampton models.

The tuning of reduced model for certain transfer functions and certain frequency ranges is a straight forward process. The local accuracy of transfer functions can be improved by additional reference frequency points, whereas the accuracy over a whole frequency range of interest is enhanced by increasing the order of the reduced model.

The preservation of stability is an important property of reduction approaches because this ensures that a reduced model does not cause any type of failure to elastic multibody systems. It was proven in [15] that the line-fitting technique meets this essential requirement.

The systems reduced by the line-fitting method preserve the second order structure of equations of motion. This enables integration of reduced order models into a multibody system of second order type.

The main computational burdens of this reduction approach consist in the calculation of transfer function matrices for several frequency points and the solution of large least-squares problem. In applications, where the input-output behavior of a system is under consideration, the most of reduction approaches require calculation of transfer function matrices for all points in a predefined frequency range to evaluate accuracy of reduced order model. Therefore the values of transfer function matrices at several frequency points are considered as given information. In the case where the values of transfer function matrices are unknown, the complexity of their computation has a cubic dependency of N. The computational cost of least squares solution of equation (22) is $O(zn^2N)$. As before, z is the number of reference frequency points, n is the number of interface coordinates, and N is the number of coordinates for the original finite element model. The total computational cost of the line-fitting method is larger in comparison with the Craig-Bampton approach, but it remains acceptable for moderately dimensional finite element models.

4 NUMERICAL EXAMPLE

In this section we reduce a finite element model of unconstrained elastic bar illustrated in Figure 1. The SEREP and line-fitting methods are applied and results are compared.

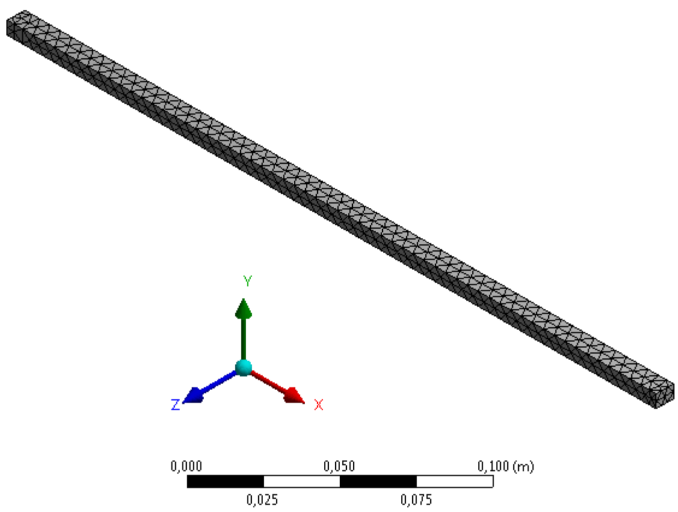

Figure 1: Finite element model of elastic bar.

4.1 Model description

The model of the bar has the following characteristics: size 6×8×300 mm, mass 0,1 kg, Young's modulus $2 \cdot 10^{10}$ Pa, damping factor beta 10^{-5}, number of degrees of freedom 1656. As a frequency range of interest we define an interval [0, 1000] Hz. The interval contains 6 zero eigenfrequencies and 6 deformation eigenfrequencies of the model. The set of input and output degrees of freedom for the bar is demonstrated in Figure 2. Here input and output coordinates coincide with each other.

Figure 2: Input and output coordinates of the model.

4.2 Initialization of reduction methods

For the line-fitting method the frequency range of interest, input and output coordinates are considered as given information. The task of the engineer is to choose an appropriate order of reduced model, to define interface degrees of freedom, and to allocate reference frequency points. The order of reduced model depends on a number of transfer functions that need to be tuned and a number of eigenfrequencies contained in a frequency range of interest. According to [15], the order equal to a double number of eigenfrequencies in the frequency range of interest is usually enough to achieve proper accuracy of reduced model. In the case of the bar model, we define the order of reduced model to be 14. After that the same number of interface degrees of freedom must be chosen. Nine interface degrees of freedom are assigned to nine user defined input and output coordinates, while the remaining coordinates are uniformly distributed throughout the model. The interface coordinates are located as it is shown in Figure 3.

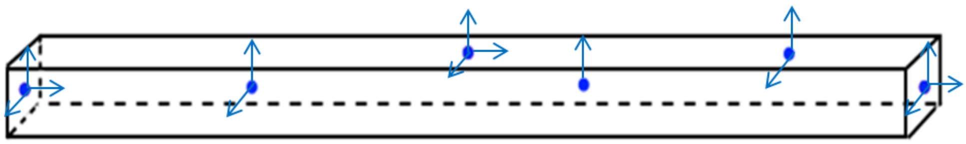

Figure 3: Interface coordinates for the reduced order model.

In order to choose reference frequency points, we compute transfer functions specified by the interface DoFs. The set of reference frequency points is common for all interface transfer functions; therefore it is possible to obtain a complete set of frequency points analyzing only a few of them. The reference points are defined at resonance, antiresonance, and some intermediate frequencies of the interval [0,1000] Hz, see Figure 4. For this model 20 frequency points of interest are assigned.

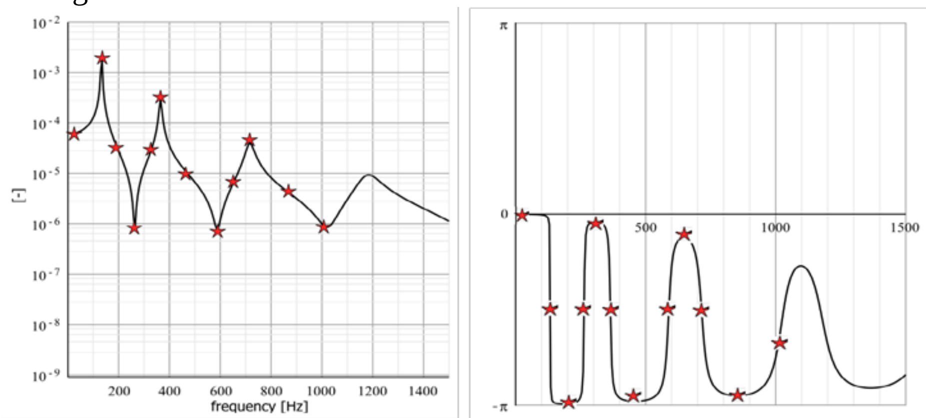

Figure 4: Choice of reference frequency points.

In order to initialize the SEREP method, it is necessary to choose interface coordinates and modes retained in the reduced model. We defined nine interface coordinates shown in Figure 2 as degrees of freedom of reduced order model. In order to avoid rank-deficiency problems in the SEREP method, we assigned the same amount of modes as the number of retained coordinates. Thus, the set of chosen modes consists of nine deformation modes with lowest frequencies. It also follows that the number of retained modes is larger than the amount of modes in the frequency range of interest.

4.3 Evaluation of reduced order models

For the comparison of the reduction methods we employ three criteria: normalized relative eigenfrequencies difference, modal assurance criterion, and a relative error of transfer functions. Figure 5 represents the result of the former test for the eigenfrequencies in the range of interest [0, 1000] Hz.

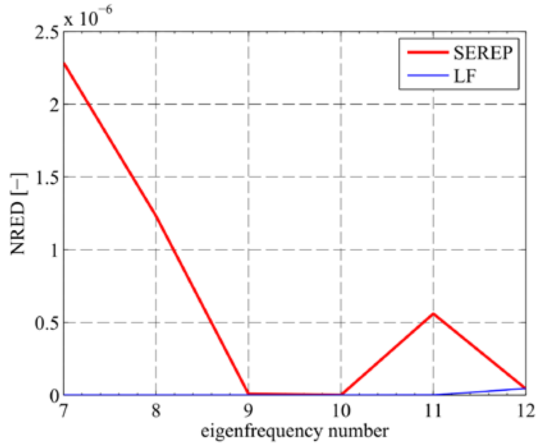

Figure 5: Relative error of non-zero eigenfrequencies for reduced models.

The diagram shows that the eigenfrequencies in the predefined frequency range are accurately approximated by both methods, but the precision is considerably better for the line-fitting model.

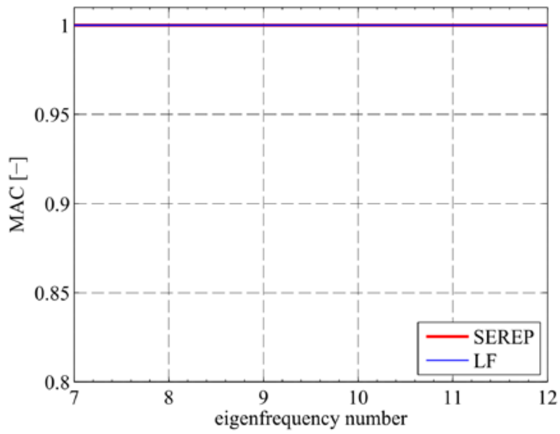

Figure 5: Correlation of deformation modes.

The results of MAC test are illustrated in Figure 6. The diagram shows that MAC values for all modes in the predefined frequency range are higher than 0.8. This implies a high correlation of deformation forms of the reduced and full models and, as a result, a qualitatively successful approximation.

Further we evaluate dynamics properties of reduced order models using transfer functions. For multi-input multi-output systems it is problematically to analyze errors of transfer functions separately, therefore the error of reduction is usually evaluated using the formula:

$$\varepsilon(\omega) = \frac{\left\| \mathbf{H}(i\omega) - \tilde{\mathbf{H}}(i\omega) \right\|_F}{\left\| \mathbf{H}(i\omega) \right\|_F} \tag{23}$$

where $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are transfer matrices of original and reduced systems, and $\|\cdot\|_F$ denotes the Frobenius norm. The function $\varepsilon(\omega)$ shows a total error introduced by a model reduction approach in a certain frequency range. Figure 6 represents a relative error of transfer functions for both the reduced order models and interface degrees of freedom shown in Figure 2.

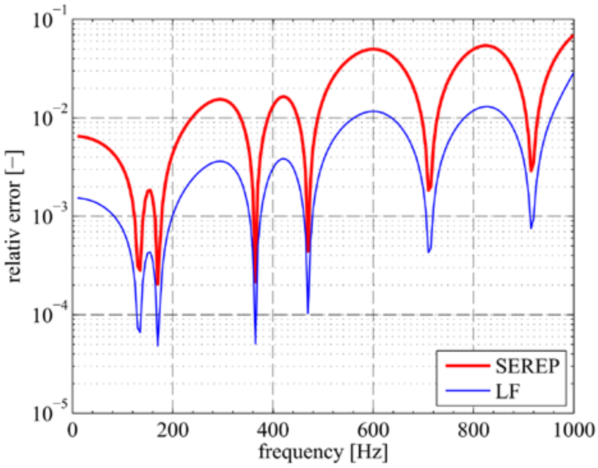

Figure 6: Relative error of frequency response.

The plot shows that the line-fitting method approximated the relevant transfer functions over the whole interval considerably more accurately than the SEREP approach. The error of line-fitting model does not exceed 3%. The SEREP method provides results with errors up to 7%. Increasing of order of SEREP model and the amount of modes to 14 improves the accuracy of transfer functions, but the line-fitting model is still superior, see Figure 7.

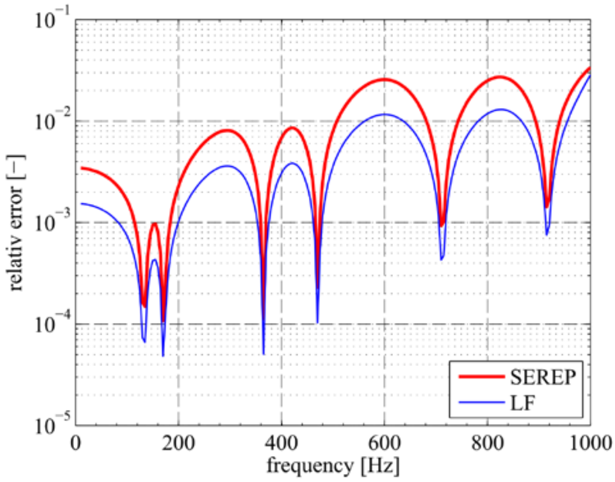

Figure 7: Relative error of frequency response with the enhanced SEREP model.

5 SUMMARY AND FURTHER RESEARCH

In this paper we evaluated the line-fitting method relative to the demands defined by application examples. The demands included a high accuracy of reduced order model, fast simulation, simple tuning of reduced model for certain transfer functions and certain frequency ranges, preservation of model stability, preservation of second order structure of equations of motion, and computational efficiency of reduction method. The evaluation was made based on the theoretical description and results of the numerical example. The analysis showed that the line-fitting method satisfies all the requirements for moderately dimensional finite element models.

In addition, the line-fitting method was compared with the widely used SEREP approach. We found that the line-fitting approach outperforms the SEREP method in accuracy, possibility to tune reduced order models for certain transfer functions and certain frequency ranges, and preservation of stability. Both methods preserve the second order structure of equations of motion. The relative drawback of the line-fitting method is the higher computational cost of a coordinate transformation matrix. Besides, the stiffness of the line-fitting model was higher relative to the stiffness of SEREP model.

Further we intend to improve computational efficiency of the line-fitting method and to compare the approach with reduction techniques from the control theory.

REFERENCES

[1] T. Wasfy, A. Noor, Computational strategies for flexible multibody systems. *Applied Mechanics Reviews*, Bd. 56, Nr. 6, 553-613, 2003.

[2] J. Guyan, Reduction of stiffness and mass matrices. *AIAA Journal*, Bd. 3, 380-380, 1965.

[3] A. Leung, An accurate method of dynamic condensation in structural analysis. *International Journal for Numerical Methods in Engineering*, Bd. 12, Nr. 11, 1705-1715, 1978.

[4] M. C. C. Bampton, R. R. Craig, Coupling of substructures for dynamic analyses. *AIAA*, Bd. 6, 1313-1319, 1968.

[5] R. Craig, Coupling of substructures for dynamic analyses: an overview. *Proceedings of the AIAA Dynamics Specialists Conference*, Atlanta, USA, 2000.

[6] S. Dietz, *Vibration and Fatigue Analysis of Vehicle Systems Using Component Modes*. VDI-Verlag, 1999.

[7] J. O'Callahan, P. Avitabile, R. Riemer, System Equivalent Reduction Expansion Process (SEREP). In *7. International Modal Analysis Conference*, Las Vegas, 1989.

[8] A. C. Antoulas, *Approximation of Large-Scale Dynamical Systems*. Society for Industrial and Applied Mathematics, 2005.

[9] Z. Bai, Y. Su, Dimension reduction of second-order dynamical systems via a second-order Arnoldi method, *SIAM J. Sci. Comp.*, Vol.25, No.5, 1692-1709, 2005.

[10] B. Salimbahrami, B. Lohmann, Order reduction of large scale second-order systems using krylov subspace methods. *Linear Algebra and its Applications*, 415, 385-405, 2006.

[11] P. Benner, J. Saak, Efficient balancing-based MOR for large-scale second-order systems. *Mathematical and Computer Modelling of Dynamical Systems,* Bd. 17, Nr. 2, 123-143, 2011.

[12] Z.-Q. Qu, *Model Order Reduction Techniques: With Applications in Finite Element Analysis*. Springer London, London, 2004.

[13] P. Koutsovasilis, *Model order reduction in structural mechanics: Coupling the rigid and elastic multi body dynamics,* VDI-Verl., Duesseldorf, 2009.

[14] B. Besselink, U. Tabak, A. Lutowska, van de Wouw, N., H. Nijmeijer, D. J. Rixen, M. E. Hochstenbach, W.H.A. Schilders. A comparison of model reduction techniques from structural dynamics, numerical mathematics and systems and control. *Journal of Sound and Vibration*, 332(19), 4403-4422, 2013.

[15] V. Makhavikou, *Line-fitting method of model order reduction for elastic multibody systems*, PhD thesis, Otto von Guericke University, Magdeburg, 2015.

[16] V. Makhavikou, R. Kasper, D. Vlasenko, Method of model reduction for elastic multibody systems, *Proceedings of 11th World Congress on Computational Mechanics, 5th European Conference on Computational Mechanics*, 475-486, Barcelona, July 20-25, 2014.

[17] V. Makhavikou, R. Kasper, D. Vlasenko, Line-fitting method of model order reduction in a context of elastic multibody simulation, *Proceedings of 12th Magdeburger Maschinenbau-tage,* Otto von Guericke University, Magdeburg, September, 2015.

[18] R. Schwertassek, O. Wallrapp, *Dynamik flexibler Mehrkoerpersysteme: Methoden der Mechanik zum rechnergestützten Entwurf und zur Analyse mechatronischer Systeme*, Braunschweig, Vieweg, 1999.

[19] A. A. Shabana, *Dynamics of multibody systems*, Cambridge and U.K. and New York, Cambridge University Press, 1998.