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IN-HOUSE MULTIBODY HUMAN MODEL BASED ON EULER PARAMETERS FOR THE FAST IMPACT SCENARIO CALCULATION

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Abstract. *Purpose of this paper is to build an in-house spatial model of a human body based on multibody approaches, that could be utilized in numerical simulations of various impact scenarios. The model currently contains 17 rigid bodies and 16 spherical joints. The joints are modelled as range-unlimited in order to keep model robust for the further analysis. The limits of the motion will be further modelled within internal stiffness. However the approach of the spatial joints with unlimited motion suffers with so-called singular positions and thus the numerical calculation can loose its stability. To solve this problem, the concept of four Euler parameters instead of widely used three Euler angles is used here. The dimensions and masses of the particular segments are based on the real data of the Czech population measurement in 80s. The model is parametrized with the height and weight of the particular human. All the dimensions are calculated with respect to these values, which makes the model scalable to capture human population diversity. The contact problem of the human body with the external shape or self-contact between segments is solved using the continuous contact force model, where normal and tangential forces are evaluated as a function of the penetration, stiffness and coefficients of restitution. Algorithm for ellipsoid-to-plane contact is applied here. MATLAB software is used to build the in-house software for a human body model. The model will be further validated to capture the real human kinematics and to achieve human-like behaviour under different impact conditions.*

1 INTRODUCTION

Modelling of the human body is an expanding discipline of biomechanical research. With the advantages of the fast computer processing, the researchers have started to use computers for the biomechanical modelling more often. The goals ate to investigate numerical models of the human by different approaches for different situations. The example of the highly expanding area of human modelling is safety/automotive field, where virtual models of the human body or the parts of the body are often used to predict or eliminate fatal injuries or death. For this purpose, the detail and deformable finite element modelling method is usually applied. Such complex models of particular tissues are widely developed. With the proper description of materials and geometry, these models can serve to calculate deformations, velocities, accelerations, stresses or rupture of any human segments. On the other hand, there might be some cases, where such complex description are inefficient or inappropriate. If we are interested only in global behaviour (kinematics and dynamics) of the system (human body) the detailed description within finite elements are inefficient and time and cost consuming. Let us imagine a simple example of a man pedalling his bike. If we consider the man and the bike as one mechanical system and the goal is to describe the kinematics of the bike based on position of the legs, the knowledge of stresses and deformations of the segments or internal organs is not required. For such purpose the classical description of mechanisms can be enough to capture the motion. Moreover, the complex description can be considered with the articulated rigid body approach. If the researchers are interested in muscle forces, the musculoskeletal modelling method is an appropriate tool. Such model consists of constrained rigid bodies (articulated rigid body, multibody system) and the model of muscles. On the other hand, the requirements of the detailed tissue injury and deformations always require application of the finite element modelling.

Purpose of this work is to build an in-house software for fast simulations of the global behaviour of the scalable human body under different loading scenarios. Authors use the multibody (articulated rigid bodies) approach for the description of the human body. Such method allows to calculate the motion under various loading in a real time simulation (or close to real time). Authors replaced the well known Euler angles with the principle of Euler parameters. The Euler angles is a widely used method for description of the spatial rotational motion of the rigid body. However, this approach can suffer from the singular positions [1] , when the angle of nutation is close to $\pm i\pi, i \in < 0, 1, 2... >$. To solve this problem, the principle of Euler parameters [2, 3, 4, 5], that can be used without the singularities, was applied here.

2 METHODS

2.1 Euler parameters

Four Euler parameters are used for the parametrization of the 3 degrees-of-freedom spatial (spherical) motion of the rigid body. Therefore, these parameters are not totally independent and the constraint equation between Euler parameters can be define as

$$\sum_{k=1}^{4} (\beta_k)^2 = 1, \tag{1}$$

or written using vector of Euler parameters $\boldsymbol{\beta}$ as

$$\boldsymbol{\beta}^T \boldsymbol{\beta} - 1 = 0. \tag{2}$$

When using e.g. Lagrange equations for the formulation of system dynamics, the second derivative of the constraint equations is further added to the equation of motion. Thus the Eq. 2 is twice differentiated and one obtain

$$\boldsymbol{\beta}^T\ddot{\boldsymbol{\beta}} = -\dot{\boldsymbol{\beta}}^T\dot{\boldsymbol{\beta}}. \tag{3}$$

2.1.1 Euler parameters in terms of Euler angles

Euler parameters define the 3 degrees-of-freedom rotation usually described using three Euler angles (precession ψ, nutation θ and spin ϕ). Since the Euler parameters can be hard to interpret in the physical meaning, it is useful to define Euler parameters as functions of well known Euler angles as

$$\beta_1 = \cos\left[\frac{1}{2}(\psi+\phi)\right]\cos\frac{\theta}{2}, \tag{4}$$

$$\beta_2 = \cos\left[\frac{1}{2}(\psi-\phi)\right]\sin\frac{\theta}{2}, \tag{5}$$

$$\beta_3 = \sin\left[\frac{1}{2}(\psi-\phi)\right]\sin\frac{\theta}{2}, \tag{6}$$

$$\beta_4 = \sin\left[\frac{1}{2}(\psi+\phi)\right]\cos\frac{\theta}{2}, \tag{7}$$

and similarly the derivatives of Euler angles as a function of Euler velocities

$$\dot{\beta}_1 = -\sin\left[\frac{1}{2}(\psi+\phi)\right]\ \frac{1}{2}(\dot{\psi}+\dot{\phi})\cos\frac{\theta}{2} - \cos\left[\frac{1}{2}(\psi+\phi)\right]\sin\frac{\theta}{2}\,\frac{\dot{\theta}}{2}, \tag{8}$$

$$\dot{\beta}_2 = -\sin\left[\frac{1}{2}(\psi-\phi)\right]\ \frac{1}{2}(\dot{\psi}-\dot{\phi})\sin\frac{\theta}{2} + \cos\left[\frac{1}{2}(\psi-\phi)\right]\cos\frac{\theta}{2}\,\frac{\dot{\theta}}{2}, \tag{9}$$

$$\dot{\beta}_3 = \cos\left[\frac{1}{2}(\psi-\phi)\right]\ \frac{1}{2}(\dot{\psi}-\dot{\phi})\sin\frac{\theta}{2} + \sin\left[\frac{1}{2}(\psi-\phi)\right]\cos\frac{\theta}{2}\,\frac{\dot{\theta}}{2}, \tag{10}$$

$$\dot{\beta}_4 = \cos\left[\frac{1}{2}(\psi+\phi)\right]\ \frac{1}{2}(\dot{\psi}+\dot{\phi})\cos\frac{\theta}{2} - \sin\left[\frac{1}{2}(\psi+\phi)\right]\sin\frac{\theta}{2}\,\frac{\dot{\theta}}{2}. \tag{11}$$

2.2 Equations of motion

2.2.1 Equations of motion of unconstrained bodies

Equations of motion of one unconstrained (free) body were defined based on the notation shown in [4]. Here, the body has 7 coordinates (three translational coordinates and four Euler parameters). The dynamic equations of motion of such unconstrained body can be define as

$$\underbrace{\begin{bmatrix}\mathbf{m}^i_{RR} & \mathbf{0}\\ \mathbf{0} & \mathbf{m}^i_{\beta\beta}\end{bmatrix}}_{\mathbf{M}^i}\underbrace{\begin{bmatrix}\ddot{\mathbf{R}}^i\\ \ddot{\boldsymbol{\beta}}^i\end{bmatrix}}_{\mathbf{q}^i} = \underbrace{\begin{bmatrix}(\mathbf{Q}^i_e)_R\\ (\mathbf{Q}^i_e)_\beta\end{bmatrix}}_{\mathbf{Q}^i_e} + \underbrace{\begin{bmatrix}\mathbf{0}\\ (\mathbf{Q}^i_\nu)_\beta\end{bmatrix}}_{\mathbf{Q}^i_\nu}, \qquad \text{i=1, 2, ...,n}, \tag{12}$$

where $\mathbf{M}^i$ is the mass matrix of i-th body, $\mathbf{q}^i$ is the vector of 7 generalized coordinates of the body and $(\mathbf{Q}^i_e)_R$ and $(\mathbf{Q}^i_e)_\beta$ are the vectors of generalized external forces associated with the generalized translations and orientations, respectively. Vector $\mathbf{Q}^i_\nu$ is so called quadratic velocity vector [4].

Equation (12) is the matrix equation that governs the unconstrained motion of the rigid body. This equation was simplified for the centroidal case, i.e when the origin of the local body-fixed coordinate system is rigidly attached to the centre of mass. When the sub-matrices are expressed in terms of the vector of Euler parameters, the overall equations of motion of the unconstrained body can be written in the following form

$$\underbrace{\begin{bmatrix} m^i\boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}^i_{cog}\boldsymbol{G}^i \\ 0 & \boldsymbol{\beta}^{iT} \end{bmatrix}}_{\mathbf{M}^i} \underbrace{\begin{bmatrix} \ddot{\mathbf{R}}^i \\ \ddot{\boldsymbol{\beta}}^i \end{bmatrix}}_{\mathbf{q}^i} = \underbrace{\begin{bmatrix} \sum\limits_{j=1}^{n_F} \boldsymbol{F}^i_j \\ \boldsymbol{G}^{iT}[\sum\limits_{k=1}^{n_M} \boldsymbol{M}^i_j + \sum\limits_{j=1}^{n_F} \boldsymbol{u}^i_k \times \boldsymbol{F}^i_k] \end{bmatrix}}_{\mathbf{Q}^i_e} + \\ + \underbrace{\begin{bmatrix} \mathbf{0} \\ - \begin{bmatrix} \boldsymbol{I}^i_{cog}\dot{\boldsymbol{G}}^i\dot{\boldsymbol{\beta}}^i - (\boldsymbol{G}^i\dot{\boldsymbol{\beta}}^i \times \boldsymbol{I}^i_{cog}\boldsymbol{G}^i\dot{\boldsymbol{\beta}}^i) \\ \dot{\boldsymbol{\beta}}^{iT}\dot{\boldsymbol{\beta}}^i \end{bmatrix} \end{bmatrix}}_{\mathbf{Q}^i_\nu}, \qquad \text{i=1, 2, ...,n,} \tag{13}$$

where

$$\boldsymbol{I}_{cog} = \boldsymbol{A}\overline{\boldsymbol{I}}_{cog}\boldsymbol{A}^T \tag{14}$$

is the inertia matrix defined in the global coordinate system, in which matrix $\boldsymbol{A}$ is the direction cosine matrix (transformation matrix between local and global coordinate systems) and is defined as

$$\boldsymbol{A} = \begin{bmatrix} 1-2\beta_3^2-2\beta_4^2 & 2(\beta_2\beta_3-\beta_1\beta_4) & 2(\beta_2\beta_4+\beta_1\beta_3) \\ 2(\beta_2\beta_3+\beta_1\beta_4) & 1-2\beta_2^2-2\beta_4^2 & 2(\beta_3\beta_4-\beta_1\beta_2) \\ 2(\beta_2\beta_4-\beta_1\beta_3) & 2(\beta_3\beta_4+\beta_1\beta_2) & 1-2\beta_2^2-2\beta_3^2 \end{bmatrix}. \tag{15}$$

Matrices $\boldsymbol{G}$ and $\overline{\boldsymbol{G}}$ are defined (in the global and the local coordinate system, respectively) as

$$\boldsymbol{G} = 2\begin{bmatrix} -\beta_2 & \beta_1 & -\beta_4 & \beta_3 \\ -\beta_3 & \beta_4 & \beta_1 & -\beta_2 \\ -\beta_4 & -\beta_3 & \beta_2 & \beta_1 \end{bmatrix} \tag{16}$$

and

$$\overline{\boldsymbol{G}} = 2\begin{bmatrix} -\beta_2 & \beta_1 & \beta_4 & -\beta_3 \\ -\beta_3 & -\beta_4 & \beta_1 & \beta_2 \\ -\beta_4 & \beta_3 & -\beta_2 & \beta_1] \end{bmatrix}. \tag{17}$$

Symbol m^i in equation (13) is mass of i-th body, $\boldsymbol{I}$ is 3x3 identity matrix, $\boldsymbol{F}_i$ is the vector of applied external forces, $\boldsymbol{u}_k$ is the position vector of point P, where an external force acts, n_F is the total number of external forces, $\boldsymbol{M}_i$ is the vector of applied external torques and n_M is the total number of the torques.

2.2.2 Equations of motion of constrained bodies

Let us consider a multibody system containing N bodies. The bodies are interconnected using kinematic joints. The kinematic constraints (equations) of the joints can be defined as

$$\boldsymbol{C}(\boldsymbol{q}, t) = \mathbf{0}. \tag{18}$$

The overall equations of motion of the kinematic system containing N bodies can be written as

$$\begin{bmatrix} \mathbf{M}^1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_{\mathbf{q}^1}^T \\ \mathbf{0} & \mathbf{M}^2 & \cdots & \mathbf{0} & \mathbf{C}_{\mathbf{q}^2}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & 0 & \cdots & \mathbf{M}^N & \mathbf{C}_{\mathbf{q}^N}^T \\ \mathbf{C}_{\mathbf{q}^1} & \mathbf{C}_{\mathbf{q}^2} & \cdots & \mathbf{C}_{\mathbf{q}^N} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}^1 \\ \ddot{\mathbf{q}}^2 \\ \vdots \\ \ddot{\mathbf{q}}^N \\ -\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e^1 + \mathbf{Q}_\nu^1 \\ \mathbf{Q}_e^2 + \mathbf{Q}_\nu^2 \\ \vdots \\ \mathbf{Q}_e^N + \mathbf{Q}_\nu^N \\ \mathbf{Q}_d \end{bmatrix} \tag{19}$$

or written in a more condensed form as

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_q^T \\ \mathbf{C}_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ -\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e + \mathbf{Q}_\nu \\ \mathbf{Q}_d \end{bmatrix}. \tag{20}$$

Vector $\boldsymbol{C_q}$ is a Jacobian matrix that is defined using derivatives of constraint equation (18) with respect to generalized coordinates $\boldsymbol{q}$ as

$$\boldsymbol{C_q} = \begin{bmatrix} \frac{\partial C_x}{\partial q_1} & \cdots & \frac{\partial C_x}{\partial q_N} \\ \frac{\partial C_y}{\partial q_1} & \cdots & \frac{\partial C_y}{\partial q_N} \\ \frac{\partial C_z}{\partial q_1} & \cdots & \frac{\partial C_z}{\partial q_N} \end{bmatrix}. \tag{21}$$

Constraint equation (18) and consequently equation (21) depend on the type of the particular kinematic joint. Vector $\boldsymbol{Q}_d$ is the vector that absorbs quadratic terms in the velocities and it is defined as

$$\mathbf{Q}_d = \begin{bmatrix} -\sum\limits_{\alpha=1}^{N}\sum\limits_{j=1}^{N} \frac{\partial^2 \boldsymbol{C}_1}{\partial q_j \partial q_\alpha} \dot{q}_j \dot{q}_\alpha \\ -\sum\limits_{\alpha=1}^{N}\sum\limits_{j=1}^{N} \frac{\partial^2 \boldsymbol{C}_2}{\partial q_j \partial q_\alpha} \dot{q}_j \dot{q}_\alpha \\ \vdots \\ -\sum\limits_{\alpha=1}^{N}\sum\limits_{j=1}^{N} \frac{\partial^2 \boldsymbol{C}_r}{\partial q_j \partial q_\alpha} \dot{q}_j \dot{q}_\alpha \end{bmatrix}, \tag{22}$$

in which r is the number of constraints in the model.

2.3 Contact treatment

The goal of this work is to develop the human body model, or external shape of the body, respectively, that consists of spheres and ellipsoids linked in the open kinematic tree structure. This model is built on purpose simulations of the global behaviour of the human body under external loading including contact with some infrastructure or self-contact between bodies. Despite the fact that there are no limitations for the shape of the infrastructure, we can always approximate such shape with the finite number of planes. Thus, only the contact between an ellipsoid and a plane is solved. In authors' previous work [6] the algorithm for the solution of the ellipsoid-to-plane contact was presented. The main idea of this method lays in the creation of a new plane, parallel to the initial one and tangential to the ellipsoid, see Figure 1. When common point C of the new plane and of the ellipsoid is detected, the distance between this point and the initial plane can be calculated.

In case of collision the generated contact forces are calculated based on the continuous contact force model [7] as

$$\mathbf{F}_{ij}^c = -K_n \delta_n \mathbf{n}_c - C_n(\mathbf{v}_c \mathbf{n}_c)\mathbf{n}_c + \mathbf{F}_{ij}^t \quad \text{(normal force)}, \tag{23}$$

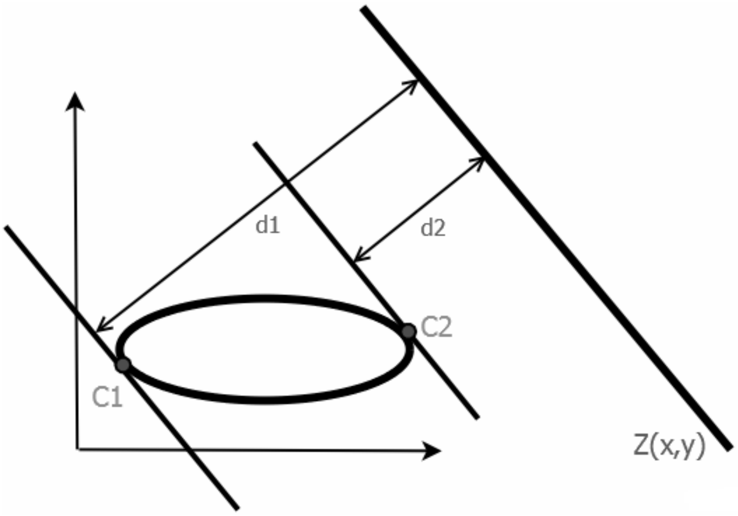

Figure 1: Ellipsoid and parallel planes.

$$\mathbf{F}_{ij}^{t} = -K_t \mathbf{v}_c^t + C_t(\mathbf{v}_c \times \mathbf{n}_c) \times \mathbf{n}_c \quad \text{(tangential force)}, \tag{24}$$

$$\mathbf{M}_{ij}^{c} = \mathbf{R}_{ij}^{C} \times \mathbf{F}_{ij}^{c} \quad \text{(torque of the contact force)}. \tag{25}$$

The contact force expressions (23), (24) and (25) are functions of distance (penetration) δ_n, relative normal velocity $\mathbf{v}_c$ collinear with vector of normal $\mathbf{n}_c$ and tangential velocity vector $\mathbf{v}_c^t$ and the contact parameters; i.e. virtual normal and tangential stiffness K_n and K_t respectively, and virtual coefficients of restitution C_n and C_t. Vector $\mathbf{R}_{ij}^{C}$ is the position vector of the contact point defined in the global coordinate system.

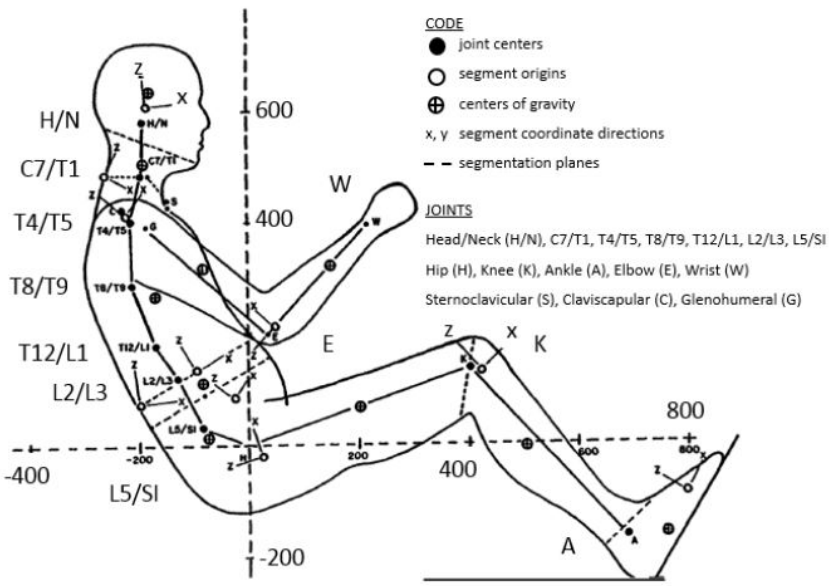

Figure 2: Segmentation of a human body [8].

2.4 Human body model

The basic concept of this work is in the segmentation of the human body defined by Robbins [8]. Human body is divided into the segments, whose motion respects correct kinematics and dynamics of the human body caused be external loading, see Figure 2.

Based on this segmentation, the spatial human body model was created as an open kinematic tree-structure containing 17 rigid bodies. Hynčík [8] suggested the simple 11-bodies structure of the 2D model, as is shown in Figure 3, where all bodies are modelled as ellipses or circles, respectively.

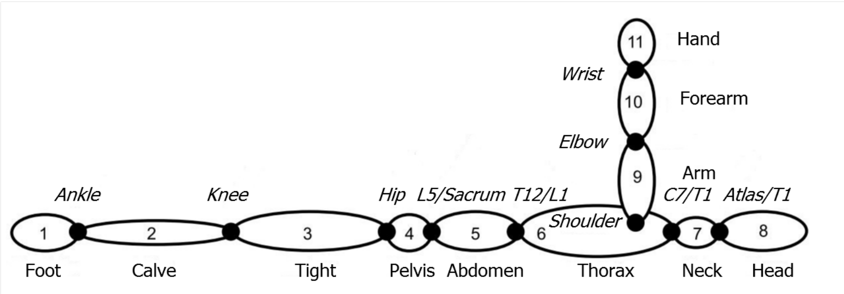

Figure 3: Segmentation of 2D human body model [8].

The spatial segmentation is expanded by the pairs of extremities and thus the model has 17 rigid bodies, based on Figure 2 and Figure 3:

- Pelvis
- Abdomen
- Thorax
- Neck
- Head
- Left and right arms
- Left and right forearms
- Left and right hands
- Left and right tights
- Left and right calves
- Left and right feet

The connections between bodies are realized by means of 16 spherical joints, representing real human's joints:

- Joint between abdomen and pelvis, i.e. vertebrae L5/Sacrum
- Joint between thorax and abdomen, i.e. vertebrae T12/L1
- Joint between neck and thorax, i.e. vertebrae C7/T1
- Joint between head and neck, i.e. vertebrae Atlas/T1
- Left and right shoulder joints
- Left and right elbow joints
- Left and right wrist joints
- Left and right hip joints
- Left and right knee joints
- Left and right ankle joints

2.4.1 Mass distribution, geometry and inertia properties

The proposed model is developed in the way that the total mass and the total height of the human are inputs of the in-house software. The dimensions and masses of particular segments are calculated based on these defined values. The masses of the body segments can be calculated from the total mass and height of the human [9, 10]. The method of Zaciorsky and Salujanov [10] is based on the radioisotope method of experimental measuring on 100 humans and defined coefficients for each body segments. The mass of the segments can be expressed using the simple equation as

$$m_i = \beta_{0i} + \beta_{1i} m + \beta_{2i} v, \tag{26}$$

where m [kg] is the total mass of the body, v [cm] is weight of body and coefficients β are defined in the Table 1.

Name of the segment	$\boldsymbol{\beta_0}$ **[kg]**	$\boldsymbol{\beta_1}$ **[-]**	$\boldsymbol{\beta_2}$ $[\boldsymbol{kg.cm^{-1}}]$
Pelvis	-7.498	0.0976	0.04896
Abdomen	7.181	0.2234	-0.0663
Thorax	8.2144	0.1862	-0.0584
Neck	0.096	0.0031	0.0022
Head	1.2	0.014	0.0123
Arm	0.25	0.03013	-0.0027
Forearm	0.3185	0.01445	-0.00114
Hand	-0.1165	0.0036	0.00175
Tight	-2.649	0.1463	0.0137
Calf	-1.592	0.03616	0.0121
Foot	-0.829	0.0077	0.0073

Table 1: Table of coefficients for calculation of body segment's masses [10].

Geometry of the segments was defined based on the Robbins database [11], where values of human segments' dimensions for the large, medium and small man and woman respectively, were published. Authors use average male of height equals to 180 cm as a reference one (measured in Czech Republic in the year 2011) [12], and all the dimensions from Robbins database (medium male) are related to this value. Thus, the dimensions of each body segment are functions of the total height of the human. Such algorithm makes the proposed in-house model scalable for various humans: height – Robbins database (Figure 2) and weight – Zaciorsky method (26). The inertia properties of the body segments are calculated from mass and geometry of the particular parts.

The in-house software model is parametrized with the height and weight, which makes the model scalable in order to capture the diversity of human population [13, 14, 15].

3 RESULTS

The mathematical methods for the development of the scalable multibody model of the human were described in the previous chapter. The purpose of such model is the description of the global behaviour of the human under different loading scenarios. The examples of the various human body models are shown in the Figure 4. Figure 5 shows the motion of the reference human body model (180 cm, 80 kg) for the prescribed initial velocities at the trunk and the extremities.

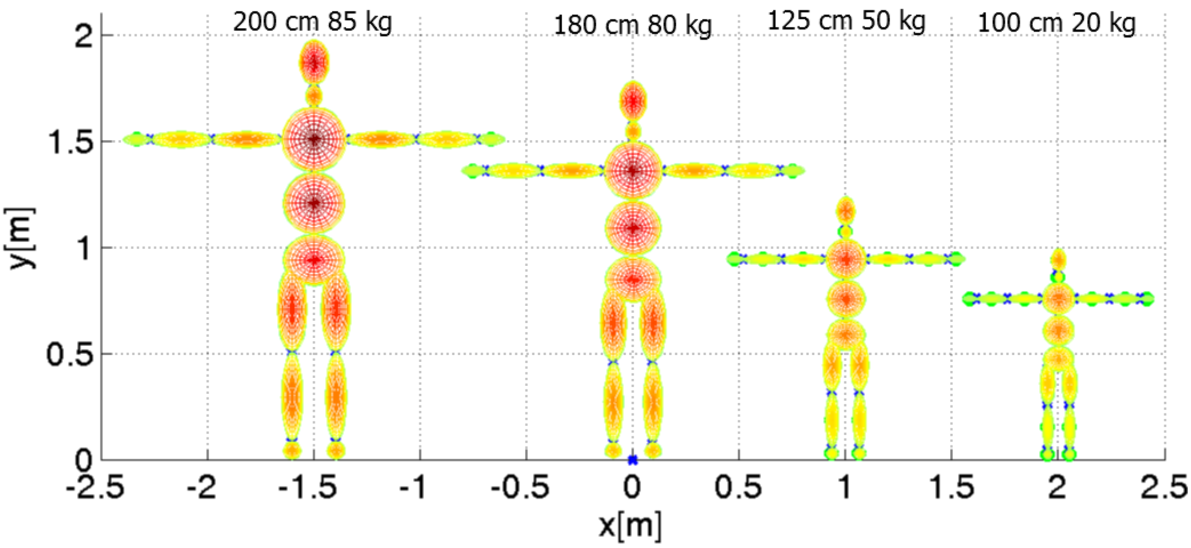

Figure 4: Human body model family.

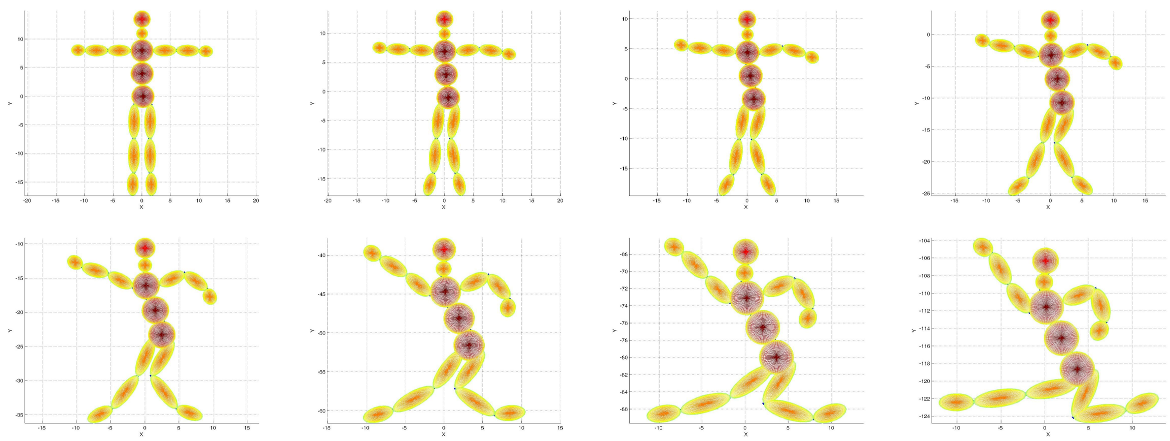

Figure 5: Illustrative motion of the human body model.

4 CONCLUSION

The presented work is focused on the investigation of the multibody principle in the modelling of the human body. Authors used Euler parameters set for the parametrization of the spatial rotations. This paper introduced such principle and pointed out the advantages and disadvantages of such approach. Generally, the Euler parameters can solve the problem with the singularities that occur with the Euler angles, however it brings an extra equation (constraint equation among Euler parameters) into the equation of motion for each body. Dynamics equations of motion for the case of unconstrained and constrained body, respectively, are defined here. Detection of the contact scenario is solved with the ellipsoid-to-plane contact algorithm and the consequent contact forces (normal and tangential) are calculate based on the continuous contact force model as a function of penetration. The definition of the forces consists of the virtual stiffness and virtual coefficients of restitution that will be further tuned up in order to correspond with the behaviour of the real human body. The presented model used the wide database of geometric properties of the human body, measured in 1970's in Czech Republic. The model is parametrized with the initial height and weight and the dimensions and particular masses depend on these values. The scaling process keeps this model scalable with respect to the particular human. This simple multibody model of human body was evaluated on the purpose of description of global behaviour of the human body. The goal was to evaluated the in-house software including contact scenario, scalable, easy-to-position model that can be easy modified for the further purposes, such as driving of the model or human cognitive behaviour.

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