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GREEN'S FUNCTIONS FOR THE EVALUATION OF ANCHOR LOSSES IN MEMS

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Abstract. *The issue of dissipation has a peculiar importance in micro-electro-mechanical-structures (MEMS). Among the sources of damping that affect their performance, the most relevant are [1]: thermoelastic coupling, air damping, intrinsic material losses, electrical loading due to electrode routing, anchor losses. Moreover, recent experimental results indicate the presence of additional temperature dependent dissipation mechanisms which are not yet fully understood (see e.g. [2, 12]). In a resonating structure the quality factor Q is defined as:*

$$Q = 2\pi W/\Delta W \tag{1}$$

where ΔW and W are the energy lost per cycle and the maximum value of energy stored in the resonator, respectively. According to eq.(1), the magnitude of Q ultimately depends on the level of energy loss (or damping) in a resonator. The focus of the present contribution is set on anchor losses and the impact they have in the presence of axial loads. Anchor losses are due to the scattering of elastic waves from the resonator into the substrate. Since the latter is typically much larger than the resonator itself, it is assumed that all the elastic energy entering the substrate through the anchors is eventually dissipated. The semi-analytical evaluation of anchor losses has been addressed in several papers with different levels of accuracy [3, 6]. These contributions consider a resonator resting on elastic half-spaces and assume a weak coupling, in the sense that the mechanical mode, as well as the mechanical actions transmitted to the substrate, are those of a rigidly clamped resonator. The displacements and rotations induced in the half-space are provided by suitable Green's functions. Photiadis, Judge et al. [7] studied analytically the case of a 3D cantilever beam attached either to a semi-infinite space or to a semi-infinite plate of finite thickness. Their results are based on the semi-exact Green's functions established in [4]. More recently Wilson-Rae et al. [9, 10] generalized all these approaches using the involved framework of radiation tunnelling in photonics. Unfortunately, these contributions provide estimates of quality factors that differ quantitatively. In this paper we revisit the procedure of [7], which rests on simple mechanical principles, but starting from the exact Green's functions for the half space studied by Pak [14]. Through a careful analysis utilizing the theory of residues and inspired by the work of Achenbach [15], we show that the results obtained coincide exactly with those of [9], but for the case of torsion.

1 INTRODUCTION: ANALYTICAL ESTIMATE OF DISSIPATION

Following a rather standard procedure [6, 7, 8], in this Section we describe the simplest possible analytical (or semi-analytical) approach based on a decoupling assumption. Let us consider a structure, like the beam of Figure 1, attached to semi-infinite elastic spaces and vibrating in one of its fundamental modes with angular frequency ω. The number of anchor points is irrelevant and the procedure must be identically repeated for all of them.

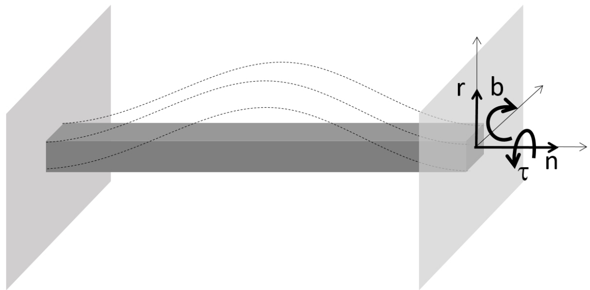

Figure 1: Sketch of a bistable doubly clamped beam

As a consequence we focus on a specific attachment point and start considering it as perfectly rigid. Standard theories of structural mechanics permit to express concentrated forces and couples exerted by the structure on the support. These generally include a constant component (due for instance to pre-stresses or initial deformation) and a sinusoidal varying contribution (see Figure 1 for the notation):

$$\text{axial force}: \quad n(t) = n_0 + Ne^{i\omega t} \tag{2}$$
$$\text{shear force}: \quad r(t) = r_0 + Re^{i\omega t} \tag{3}$$
$$\text{bending couple}: \quad b(t) = b_0 + Be^{i\omega t} \tag{4}$$
$$\text{torque}: \quad \tau(t) = \tau_0 + Te^{i\omega t} \tag{5}$$

The shear force and bending couple have in general two components which are treated in the same manner.

We now introduce the decoupling assumption, according to which frequencies, forces and couples are not significantly altered if the rigid support is replaced with a deformable half space. These concentrated actions induce displacements and rotations:

$$n(t) \quad \rightarrow \quad d(t) = d_0 + De^{i\omega t} \tag{6}$$
$$r(t) \quad \rightarrow \quad v(t) = v_0 + Ve^{i\omega t} \tag{7}$$
$$b(t) \quad \rightarrow \quad \phi(t) = \phi_0 + \Phi e^{i\omega t} \tag{8}$$
$$\tau(t) \quad \rightarrow \quad \psi(t) = \psi_0 + \Psi e^{i\omega t} \tag{9}$$

where D, V, Φ, Ψ are in general complex variables and denote the amplitude of the time dependent part of, respectively, axial and tangential displacements, bending and torsional rotations. These are known as Green's functions for the half elastic space and it is worth stressing that the real part of these kinematic quantities is in general unbounded.

However, the dissipation over one cycle due to the scatter of elastic waves in the infinite half-space is:

$$\Delta W = -\pi \left(\text{Im}[D]N + \text{Im}[V]R + \text{Im}[\Phi]B + \text{Im}[\Psi]T \right)$$

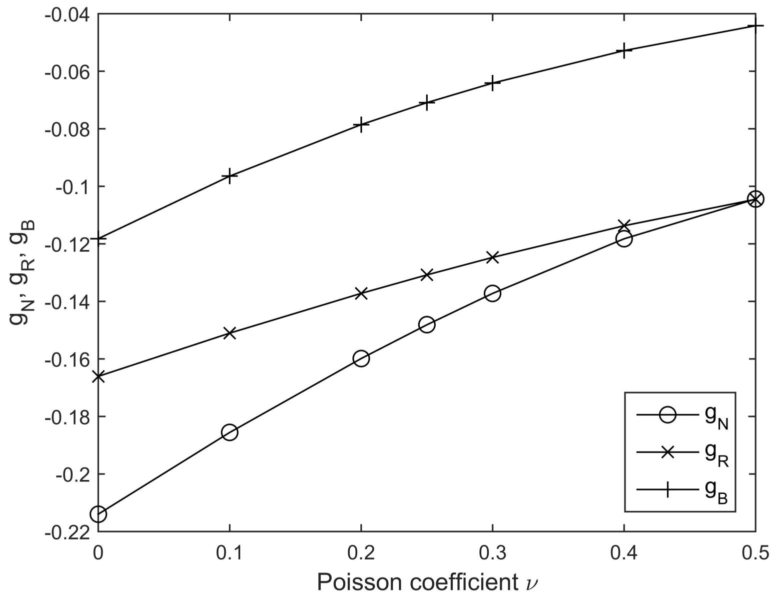

Figure 2: Functions g_N (eq.21), g_R (eq.28) and g_B (eq.24)

and only depends of the bounded imaginary part of the Green's functions.

In particular, setting $k_{\rm T} = \omega/c_{\rm T}$, $c_{\rm T} = \sqrt{\mu/\rho}$, in Section 2 we show that

$$\text{Im}[D] = N\frac{k_{\rm T}}{\mu}g_N(\nu) \qquad \text{Im}[V] = R\frac{k_{\rm T}}{\mu}g_R(\nu) \tag{10}$$

$$\text{Im}[\Phi] = B\frac{k_{\rm T}^3}{\mu}g_B(\nu) \qquad \text{Im}[\Psi] = -T\frac{k_{\rm T}^3}{\mu}\frac{1}{12\pi} \tag{11}$$

where g_N, g_R and g_B are plotted versus the Poisson coefficient ν in Figure 2. Most of these functions, rigorously established starting from the work by Pak [14], are similar to analogous results published in [7], but differ quantitatively. On the contrary, the numerical values of the g functions coincide, but for Im[Ψ], with the expressions given in [9] starting from a totally different perspective.

Finally, summing over all the anchor points:

$$\Delta W = -\pi\sum_i\left(N_i^2\frac{k_{\rm T}}{\mu}g_N(\nu) + R_i^2\frac{k_{\rm T}}{\mu}g_R(\nu) + B_i^2\frac{k_{\rm T}^3}{\mu}g_B(\nu) - T_i^2\frac{k_{\rm T}^3}{\mu}\frac{1}{12\pi}\right) \tag{12}$$

In order to apply eq.(12) only the expressions of R, N, B, T are required. In many cases fully analytical estimates are available, like for a cantilever beam in axial or bending vibrations. More in general, a numerical tool is required, as for the case of the buckled beam of interest in this paper, discussed in the following section.

2 DISPLACEMENTS AND ROTATIONS DUE TO POINT LOADS ON AN ELASTIC HALF-SPACE

The general procedure proposed by Pak [14] is here employed to derive the surface displacements and rotations induced by harmonic point forces and couples exerted on the surface of an elastic half-space of outward normal $\mathbf{e}_z$. In [14] forces, represented as stress discontinuities, are distributed on a circle of radius a. By suitable specifying their form and taking the limit $a \to 0$, all the concentrated loads of interest herein can be recovered. Starting from formulas (18) and (25) provided by [14], the Green's functions for bending and torque couples are also obtained. Next, focusing on the imaginary part of interest for the dissipation, free terms are evaluated using the theory of residua and results are provided in the form of at most weakly singular integrals, in general functions of the Poisson coefficient. All the results are presented graphically in Figure 2.

Here we set $k_\mathrm{T} = \omega/c_\mathrm{T}, k_\mathrm{L} = \omega/c_\mathrm{L}, \eta = k/k_\mathrm{T}$ (c_T and c_L are shear and longitudinal wave velocities) and

$$\alpha^2 = \frac{c_T^2}{c_L^2} = \frac{1-2\nu}{2(1-\nu)} \tag{13}$$

$$q^2 = k^2 - \omega^2/c_\mathrm{T}^2 = k^2 - k_\mathrm{T}^2 = k_\mathrm{T}^2(\eta^2 - 1) \tag{14}$$

$$p^2 = k^2 - \omega^2/c_\mathrm{L}^2 = k^2 - k_\mathrm{L}^2 = k_\mathrm{T}^2(\eta^2 - \alpha^2) \tag{15}$$

$$G = (k^2+q^2)^2 + 4k^2pq = k_\mathrm{T}^4\left((2\eta^2-1)^2 + 4\eta^2\sqrt{\eta^2-\alpha^2}\,\sqrt{\eta^2-1}\right) \tag{16}$$

$$F = (k^2+q^2)^2 - 4k^2pq = k_\mathrm{T}^4\left((2\eta^2-1)^2 - 4\eta^2\sqrt{\eta^2-\alpha^2}\,\sqrt{\eta^2-1}\right) = k_\mathrm{T}^4 f(\eta) \tag{17}$$

Vertical displacement due to a vertical force. The distribution of surface stresses:

$$t_z = \frac{1}{\pi a^2}$$

over the circle of radius a centered at the origin induces a unit vertical force. From [14], the resulting displacement w along the z axis is:

$$w(r) = \frac{1}{\pi\mu a}\int_0^\infty \Omega(k)J_1(ak)J_0(kr)\mathrm{d}k \tag{18}$$

where J_m is the m-th order Bessel function, $r = |\boldsymbol{y} - \boldsymbol{x}|$ and

$$\Omega = \frac{1}{2k_T^2}\left(-p + \frac{k^2}{q} - \frac{G}{F}\Big(p + \frac{k^2}{q}\Big) + \frac{8k^2p(k^2+q^2)}{F}\right) = -\frac{1}{k_T}\frac{\sqrt{\eta^2-\alpha^2}}{f(\eta)} \tag{19}$$

where $f(\eta)$ has been defined in eq.(17). Equation (18) can be rewritten:

$$w(r) = -\frac{1}{\pi\mu a}\int_0^\infty \frac{\sqrt{\eta^2-\alpha^2}}{f(\eta)}J_0(k_T\eta r)J_1(k_T\eta a)\mathrm{d}\eta \tag{20}$$

It is worth stressing that the real part has a potential singular behavior at the origin, as expected. However we are interested only in the imaginary part of $D = \lim_{r\to 0} w(r)$ which is smooth, since the integrand in eq.(18) is real for $\eta > 1$. Hence the limits $r \to 0$, $a \to 0$ can be safely taken.

The integral has a pole in $\eta = \eta_R$ such that $f(\eta_R) = 0$, which corresponds to Rayleigh waves, with $\eta_R > 1$. Following [15] this gives the free term:

$$D^R(r) = -\frac{k_T}{2\pi\mu}\left(-\mathrm{i}\pi\frac{\eta_R\sqrt{\eta_R^2-\alpha^2}}{f'(\eta_R)}\right) = \mathrm{i}\frac{k_T}{\mu}\left(\frac{\eta_R\sqrt{\eta_R^2-\alpha^2}}{2f'(\eta_R)}\right)$$

The integral in eq.(20) has to interpreted in the Cauchy principal value sense. Globally:

$$\mathrm{Im}[D] = N\frac{k_{\mathrm{T}}}{\mu}g_N(\nu) \tag{21}$$

with:

$$g_N(\nu) = -\frac{1}{2\pi}\mathrm{Im}\left[\int_0^1 \frac{\eta\sqrt{\eta^2-\alpha^2}}{f(\eta)}\mathrm{d}\eta\right] + \frac{\eta_R\sqrt{\eta_R^2-\alpha^2}}{2f'(\eta_R)}$$

It is worth stressing that the final results coincides with that provided by Achenbach ([15], eq.77) using a different procedure.

Rotation due to bending moment. The distribution of surface stresses

$$t_z = \frac{4}{\pi a^4}r\cos\theta = \frac{2}{\pi a^4}re^{i\theta} + \frac{2}{\pi a^4}re^{-i\theta}$$

over the circle of radius a is equivalent to a unit bending couple around the $\theta = \pi/2$ axis. Indeed, in order to apply the procedure of [14], t_z needs to be expressed in polar coordinates over the circle as:

$$t_z = \sum_m t_{z,m}(r)e^{im\theta}$$

The induced radial displacement on the surface is:

$$w(r) = \frac{4\cos\theta}{\pi\mu a^2}\int_0^\infty \Omega(k)J_2(ak)J_1(kr)\mathrm{d}k \tag{22}$$

which, by differentiation, yields the rotation ϕ around the $\theta = \pi/2$ axis:

$$\phi = \left.\frac{\partial w(r)}{\partial r}\right|_{\theta=0} = \frac{2}{\pi\mu a^2}\int_0^\infty \Omega J_2(ak)k(J_0(kr)-J_2(kr))\mathrm{d}k \tag{23}$$

Since we are only interested in the imaginary part of Φ and the integrand is real for $\eta > 1$, the rotation for $r \to 0$, $a \to 0$, is bounded and:

$$\mathrm{Im}[\Phi] = B\frac{k_{\mathrm{T}}^3}{\mu}g_B(\nu) \tag{24}$$

with:

$$g_B(\nu) = -\frac{1}{4\pi}\mathrm{Im}\left[\int_0^1 \frac{\eta^3\sqrt{\eta^2-\alpha^2}}{f(\eta)}\mathrm{d}\eta\right] + \frac{\eta_R^3\sqrt{\eta_R^2-\alpha^2}}{4f'(\eta_R)}$$

where the second term represents the contribution of Rayleigh waves at $f(\eta_R) = 0$.

Tangential displacement due to tangential force. Like the vertical force also this case is already treated in [14]. The surface stress distribution

$$t_\theta = -\frac{1}{\pi a^2}\sin\theta \qquad t_r = \frac{1}{\pi a^2}\cos\theta$$

over the circle of radius a, which is equivalent to a unit horizontal force, generates the radial displacement u_r:

$$u_r = \frac{\cos\theta}{2\pi\mu a}\left[\int_0^\infty (\gamma_1+\gamma_2)J_0(kr)J_1(ak)\mathrm{d}k + \int_0^\infty (\gamma_2-\gamma_1)J_2(kr)J_1(ak)\mathrm{d}k\right] \tag{25}$$

with

$$\gamma_1 = \frac{1}{2k_T^2}\left(\frac{k^2}{p} - q - \frac{G}{F}\Big(\frac{k^2}{p}+q\Big) + \frac{8k^2q(k^2+p^2)}{F}\right) = -\frac{1}{k_T}\frac{\sqrt{\eta^2-1}}{f(\eta)} \tag{26}$$

$$\gamma_2 = \frac{1}{q} = \frac{1}{k_T}\frac{1}{\sqrt{\eta^2-1}} \tag{27}$$

The displacement V at the point of application of the load and $\theta = 0$ is the limit of u_r for $r \to 0$, $a \to 0$, and

$$\mathrm{Im}[V] = R\frac{k_\mathrm{T}}{\mu}g_R(\nu) \tag{28}$$

having set:

$$g_R(\nu) = -\frac{1}{4\pi}\int_0^1 \frac{\eta h_1(\eta)}{f(\eta)\sqrt{\eta^2-1}}\mathrm{d}\eta + \frac{\eta_R h_1(\eta_R)}{4f'(\eta_R)\sqrt{\eta_R^2-1}} \tag{29}$$

with:

$$h_1(\eta) = -2 - 4\eta^4 + \eta^2\Big(5 + 4\sqrt{\eta^2-1}\sqrt{\eta^2-\alpha^2}\Big) \tag{30}$$

Rotation due to torque. Finally, the distribution of surface stresses

$$t_\theta = \frac{2}{\pi a^4}r$$

over the circle of radius a is equivalent to a unit torque and induces the circumferential displacement

$$u_\theta = \frac{2}{\pi\mu a^2}\int_0^\infty \gamma_2(k)J_2(ak)J_1(kr)\mathrm{d}k \tag{31}$$

and the torque angle

$$\psi = \frac{\partial u_\theta}{\partial r} = \frac{2}{\pi\mu a^2}\int_0^\infty \gamma_2(k)J_2(ak)(J_0(kr)-J_2(kr))k\mathrm{d}k \tag{32}$$

with γ_2 defined in eq.(27). In this case the free term due to Rayleigh waves is absent and, all the limits taken, the imaginary part of the rotation Ψ at the origin is

$$\mathrm{Im}[\Psi] = T\frac{k_\mathrm{T}^3}{\mu}g_T \tag{33}$$

with:

$$g_T = \frac{1}{8\pi}\mathrm{Im}\left[\int_0^1 \frac{\eta^3}{\sqrt{\eta^2-1}}\mathrm{d}\eta\right] = -\frac{1}{12\pi}$$

It is worth stressing that g_T is independent of the Poisson coefficient, like in [7], but differs from the results of [9].

3 APPLICATIONS: AXIAL AND BENDING MODES

Simple applications of these formulas give estimates of the quality factors of cantilever beams of length L and cross section area A, resting on an elastic half space. For simplicity, the half-space is assumed to be made of the same isotropic material as the beam. In the case of axial vibrations for a cantilever on a rigid support the axial displacement reads:

$$u = U \sin(k_{\mathrm{B}} x) e^{i\omega t}$$

with:

$$k_{\mathrm{B}} = (1+2m)\frac{\pi}{2L}, \quad \omega = \sqrt{\frac{E}{\rho}}\, k_{\mathrm{B}}$$

The maximum value of the stored elastic energy is:

$$W = \frac{1}{2} U^2 E A k_{\mathrm{B}}^2 \frac{L}{2}$$

and the force exerted on the support $N = EAk_{\mathrm{B}}U$. Assuming that ω, N, W are not significantly altered if the rigid support is replaced with a deformable half space, N induces a displacement D of the half space given by eq.(21) and the dissipation is:

$$\Delta W = -\pi N\, \mathrm{Im}[D] = \pi E^2 A^2 k_{\mathrm{B}}^2 U^2 \frac{1}{\mu} k_{\mathrm{T}}\, g_N(\nu)$$

leading to:

$$Q = \frac{L}{EA} \frac{\mu c_{\mathrm{T}}}{g_N(\nu)} \frac{1}{\omega} \tag{34}$$

Similarly, for a bending mode $\psi(x)U$ characterized by a given wave number k_B (e.g. $k_B L = 1.875$ in the first mode) and normalised such that

$$\int_0^L \psi^2 \mathrm{d}s = L,$$

the maximum stored energy is $W = (1/2)EILk_B^4 U^2$, while the bending couple and shear force read

$$B = 2EIk_B^2 U, \qquad R = 2EIk_B^3 \beta U, \quad \text{with} \quad \beta = \frac{\sin k_B L - \sinh k_B L}{\cos k_B L + \cosh k_B L}$$

If only the shear force is considered (bending dissipation is usually negligible):

$$Q = \frac{L}{4EJk_B^2 \beta} \frac{\mu c_{\mathrm{T}}}{g_R(\nu)} \frac{1}{\omega} \tag{35}$$

where $\omega = \sqrt{EI/(\rho A)} k_B^2$. Formulas (34) and (35) coincide with the ones given by [9], Table 1.

4 APPLICATION: ANALYSIS OF A BISTABLE BEAM

The motion of a beam of length L subjected to a compressive force P and undergoing large displacements is governed, as a first approximation, by the non-linear equation:

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + \left[P - \frac{EA}{2L} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \right] \frac{\partial^2 w}{\partial x^2} = 0 \tag{36}$$

where w is the beam deflection, A is the cross-section area, I is the inertia modulus and E is the Young modulus. In eq.(36) the classical equation of slender beams has been corrected for the presence of the uniform axial force

$$n(t) = -P + \frac{EA}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \mathrm{d}x$$

In particular we start considering the quasi-static post-buckling response of a clamped-clamped beam, i.e. the evolution beyond the critical load $P_c = 4\pi^2 EI/(L^2)$ as P is slowly increased.

Let ϕ denote the buckling mode:

$$\phi = \frac{1}{2} \left(1 - \cos \frac{2\pi x}{L} \right)$$

Assuming that the beam deflection has the expression $w(x) = b\phi(x)$, one obtains a non-linear relationship between P and b:

$$P = P_c + b^2 \frac{EA}{2L} \int_0^L \frac{\mathrm{d}^2 \phi}{\mathrm{d}x^2} \mathrm{d}x$$

We now study the small vibrations around a postbuckling state, characterized by a given b:

$$w(x,t) = b\phi(x) + u(x,t), \qquad |u(x,t| << b|\phi(x)| \tag{37}$$

A linearization of eq.(36) yields:

$$\rho A \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} + P_c \frac{\partial^2 u}{\partial x^2} - \frac{EAb^2}{L} \frac{\mathrm{d}^2 \phi}{\mathrm{d}x^2} \int_0^L \frac{\mathrm{d}\phi}{\mathrm{d}x} \frac{\partial u}{\partial x} \mathrm{d}x = 0 \tag{38}$$

An explicit and simple expression for the quality factor can be obtained with very good approximation for small values of b/V. In this case it is reasonable to assume that $\Phi(x)$ is the buckling mode. Assuming that the greatest contribution to dissipation is due to axial loads:

$$N = \alpha EA \frac{b}{L} \frac{\pi^2}{2} \quad \rightarrow \quad \Delta W = N^2 \frac{b}{\mu L^2} \left(4\pi^3 \sqrt{\frac{1+\nu}{3}} g_N(\nu) \right)$$

and the quality factor is:

$$Q \frac{HV^2}{L^3} = \frac{1}{\tilde{b}} \left(\frac{1}{8\pi^3} \frac{1}{g_N(\nu)} \frac{1}{1+\nu} \sqrt{\frac{3}{1+\nu}} \right) \tag{39}$$

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