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# NUMERICAL STUDY OF TWO OPTIMIZED COUPLING INTERFACE TREATMENTS FOR STEADY CONJUGATE HEAT TRANSFER PROBLEMS

**M-P Errera[1] and R. El Khoury[2]**

[1] ONERA - The French Aerospace Lab - DMFN -
29 Avenue de Division Leclerc 92322 Châtillon Cedex, France
e-mail: marc.errera@onera.fr

[2] Centre for energy efficiency of systems (CES), MINES ParisTech, PSL Research University,
5 rue Léon Blum, 91120 Palaiseau, France
e-mail: roch.khoury@gmail.com

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**Abstract.** *This paper presents two transmission interface treatments, Dirichlet-Robin and Neumann-Robin procedures, that may be employed for conjugate heat transfer problems. These conditions are analyzed on the basis of a 1D simplified model problem. In the first part of the paper, the Dirichlet-Robin procedure is presented. This interface treatment is the most widely employed in the literature. The same analysis is then performed with a Neumann-Robin procedure. On the basis of the model problem, the general expression of the amplification factor, the stability bounds and the optimal coefficients are provided. It is shown that the two interface treatments are opposite and complementary. Moreover, the so-called optimal coefficient provides the best results in terms of stability and convergence in the Dirichlet-Robin procedure. A criterion is expressed to choose the most appropriate transmission procedure and its importance is underlined by a test case.*

# 1 INTRODUCTION

Conjugate heat transfer (CHT) analysis refers to the ability to address the thermal interaction between a body and a fluid flowing over or through it. The objective of CHT modeling is the accurate prediction of temperature and heat flux distribution in space and time in a body and on its boundaries. Conjugate heat transfer problems occur whenever the fluid convection and the solid material conduction are taken into account simultaneously. This mutual interaction is becoming increasingly important because in many numerical simulations it is no longer acceptable to consider heat transport in the fluid only, with some ideal conditions at the fluid-solid interface.

Even if coupled procedures are the most correct and reasonable way to use when accurate heat transfer predictions are needed, in most cases, arbitrary relaxation parameters are used to stabilize the coupling procedure. This may have a significant negative impact on the numerical properties (low convergence rate, oscillations, and instabilities). Our goal in this paper is to present two interface conditions derived from a stability analysis and to present their main numerical characteristics.

Recently, we have shown [1] using a 1D thermal model problem that in a coupled system, a numerical transition can be identified. This fundamental result has been derived from a normal mode stability analysis based on the theory of Godunov-Ryabenkii [2][3]. This transition can be regarded as an optimal choice in terms of stability and convergence. Ideally, a Robin condition on each side of the interface should be considered. But in this case, a general Robin-Robin interface condition leads to a too large family of schemes. Thus, in this paper we will confine ourselves to two commonly used conditions :

- Dirichlet-Robin procedure : the temperature coming from the solid is applied on the fluid side and a "relaxed heat flux" is in turn used as a boundary condition for the solid.

- Neumann-Robin procedure : the heat flux coming from the solid is applied on the fluid side and a Robin condition is in turn used as a boundary condition for the solid.

These conditions have been chosen in order to deal with most situations likely to arise. They will be analysed on the basis of a 1D coupled aerothermal model problem. In the first part of the paper, Dirichlet-Robin interface treatments are presented. This transmission interface treatment is the most widely employed in the literature. The same analysis is then performed in the second part with a Neumann-Robin procedure. These two treatments will then be compared.

# 2 MODEL EQUATIONS

## 2.1 Model problem

It has been often been stressed that the nature of instabilities derived from a 1D model can give insight into the potential instabilities in 2D/3D computations. As a result, the behavior of interface conditions in CHT is often studied using a normal mode analysis. This is because one may reasonably assume that the modes that may be unstable are those whose variation is in the direction normal to the coupled interface. A 1D model is composed of two partitions with a shared interface. Partitioned techniques [4][5] are very popular, because they allow the direct use of specifically designed solvers for different fields and may offer significant benefits in terms of efficiency over the monolithic techniques. In this study, the fluid-solid system will be decomposed into two partitions: the (+) subdomain **(**$\Omega_+$, index $j \geq 0$ ; $x \geq 0$) and the (–) subdomain **(**$\Omega_-$, index $j \leq 0$ ; $x \leq 0$) as illustrated in Figure 1.

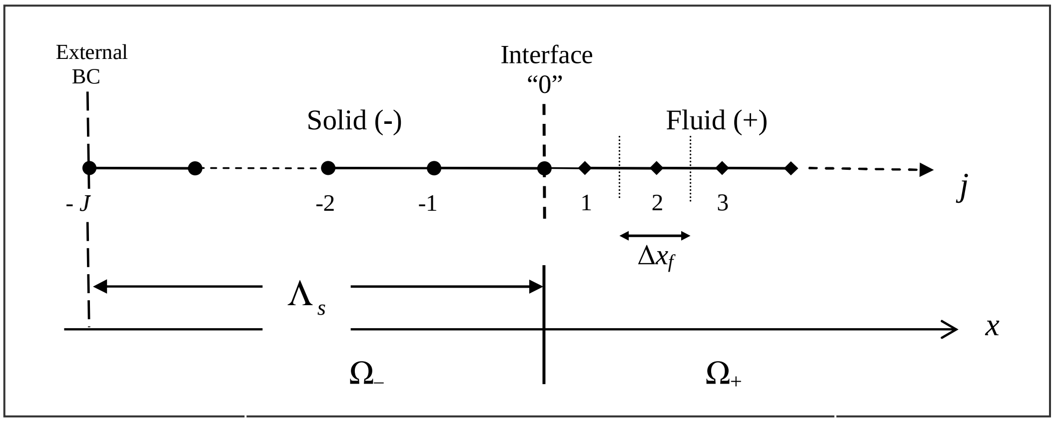


Fig.1 : Schematic of the fluid & solid domains

## 2.2 Interface conditions on the solid side

Interface conditions are needed on either side of the shared interface ($x$ = 0), where coupling conditions are applied. Our goal is to ensure a stable CHT process and to avoid destabilizing effects. It is well known that Robin conditions have many attractive features since such a condition introduces an interface stiffness forcing the boundary to behave in the same way as the boundary of the other domain resulting in much better stability properties. A Robin condition on the solid side is written simply as

$$\left[\hat{Q}_s + \alpha_f \hat{T}_s\right] = \left[Q_f + \alpha_f T_f\right] \qquad (1)$$

The subscripts $f$ and $s$ denote the fluid and solid domain respectively and the (^) notation indicates the sought values. $Q$ is the interfacial heat flux (W.m$^{-2}$) and $T$ is the interface temperature. The general Robin condition (1) introduces the numerical coupling parameter $\alpha_f$ (W.m$^{-2}$.K$^{-1}$) the choice of which directly influences the stability of the coupling process.

## 2.3 Interface conditions on the fluid side

A similar expression could be used on the fluid side

$$\left[\hat{Q}_f + \alpha_s \hat{T}_f\right] = \left[Q_s + \alpha_s T_s\right] \qquad (2)$$

Condition (2) introduces another coefficient $\alpha_s$ computed in the solid side and implemented in the fluid side. The simultaneous consideration of two coefficients could be complicated and thus it has been decided in this paper to focus on two complementary treatments :

A Dirichlet condition on the fluid side, obtained easily by setting $\alpha_s = \infty$ in (2)

$$\hat{T}_f = T_s \qquad (3)$$

and a Neumann condition on the fluid side, obtained by setting $\alpha_s = 0$ in (2). We readily obtain

$$\hat{Q}_f = Q_s \qquad (4)$$

### 2.4 Equations in the fluid and solid domains

The CHT strategy used to obtain rapidly a fluid-structure steady solution relies on the significant discrepancy of the characteristic physical times of the two domains, namely a fast transient process in the fluid, a very slow one in the structure. In the fluid subsystem, the Navier-Stokes (NS) equations are solved generally to steady-state by a time marching scheme. As a consequence, a time marching scheme will be employed here in the fluid domain in the CHT model.

On the contrary, in the solid domain, if we are only interested in computing the steady-state solution, then a second order ordinary differential equation can be solved directly and coupled to the unsteady fluid domain. Thus, only a Laplace equation is considered in the solid.

### 2.5 Characteristic factors

Three dimensionless numbers play a major role in CHT problems and appear in the stability analysis performed in this paper.

The first one is the mesh Fourier number defined as follows

$$D_f = a_f \Delta t_f \big/ \Delta x_f^2 \tag{5}$$

where $a_f$ is the fluid diffusivity (m$^2$.s$^{-1}$). $D_f$ is the mesh Fourier number that characterizes transient heat diffusion in the fluid domain.

The second one is a conventional Biot number defined by

$$Bi = h \big/ K_s \tag{6}$$

where $h$ (W.m$^{-2}$.K$^{-1}$) is the heat transfer coefficient and $K_s$ is the thermal conductance of the solid. $Bi$ determines whether or not the temperatures inside a solid body vary significantly in space, while the body heats or cools over time, from a thermal gradient applied to its surface.

The third number, a local Biot number, naturally appears in the stability analysis performed hereafter and takes the following form

$$Bi^{(\Delta)} = K_f \big/ K_s \tag{7}$$

$Bi^{(\Delta)}$ is a "new" parameter that can be regarded as the ratio of the thermal conductance of the 1$^{st}$ fluid cell over the thermal conductance of the whole solid domain. This number represents the balance at each coupling time between the solid resistance of the entire solid body (only a steady-state is considered in this domain) and the resistance of the transient fluid domain represented by the diffusion in the first near wall interface.

It is also interesting to introduce a normalized Fourier number $\overline{D}_f$ (see [6] for more details) defined by

$$\overline{D}_f = \frac{D_f}{1 + D_f + \sqrt{1 + 2D_f}} \tag{8}$$

The domain of $\overline{D}_f$ is $D_f \geq 0$ and the range is $0 \leq \overline{D}_f < 1$.

### 2.6 Normal mode analysis

We have performed a stability analysis by considering normal mode solutions in the form

$$T_j^n = \begin{cases} z^{n-1}\kappa_f^j, & j > 0 \\ z^n \kappa_s^j, & j \le 0 \end{cases} \tag{9}$$

where $z$ and $\kappa$ are two complex functions. $z$ is the "temporal amplification factor" and $\kappa$ is the "spatial amplification factor". The stability analysis is very similar to the standard Fourier stability method except that the Fourier analysis ignores boundary conditions and as these may affect stability, the theory of Godunov & Ryabenkii [2][3] is preferable. After elementary transformations not reported in this paper (see details in [1]), we find that the temporal amplification factor $z$ (i.e. each mode increases in amplitude by a ratio $|z|$) takes the following forms in the two interface conditions considered in this paper :

(1) Dirichlet-Robin interface condition : $(\alpha_f, \alpha_s) = (\alpha_f \ge 0, \infty)$

The temporal amplification factor can be written as

$$z = g(z, \alpha_f) = \frac{1}{K_s + \alpha_f}\left[\kappa_f(z, \alpha_f) - (K_f - \alpha_f)\right] \tag{10}$$

(2) Neumann-Robin interface condition : $(\alpha_f, \alpha_s) = (\alpha_f > 0, 0)$

The temporal amplification factor can be written as

$$z = g(z, \alpha_f) = \frac{1}{K_s + \alpha_f}\left[\alpha_f . \kappa_f(z, \alpha_f) + \frac{K_f - \alpha_f}{Bi^{(\Delta)}}\right] \tag{11}$$

### 2.7 A fundamental transition

The complex function $g$ is a rather complicated nonlinear equation for $z$. After the change of variable $z \to 1/z$, this function becomes holomorphic on the open set $|z| < 1$. As a result, on the basis of the maximum modulus principle in complex analysis, the maximum value of $|g|$ is achieved on the boundary, i.e., at some point on the unit circle $|z| = 1$.

Actually, the maximum is obtained either at $z = +1$ or $z = -1$. But, under certain conditions, there is a sudden transition from one point to another resulting in an amplification factor composed of two half-lines with a singular point at the intersection of these two lines. At this intersection the amplification factor turns back and attains its absolute minimum always located in the stable zone $|z| < 1$.

The point where the maximum is transferred from $z = -1$ to $z = +1$ is a fundamental transition in the aerothermal coupling.

# 3 DIRICHLET-ROBIN INTERFACE CONDITIONS

## 3.1 Stability bounds

The stability condition $\left|g(z,\alpha_f)\right|<1$ applied to Eq.(10) leads, after some basic calculus manipulations, to a lower stability bound $\alpha_f^{\min}$

$$\alpha_f > \alpha_f^{\min} = \frac{K_f}{2}(1-\overline{D}_f) - \frac{K_s}{2} \tag{12}$$

## 3.2 A dynamical Biot number

Substituting the definition of $Bi^{(\Delta)}$ (Eq.(7)), $\alpha_f^{\min}$ becomes

$$\alpha_f^{\min} = \frac{K_s}{2}\left[Bi^{(\Delta)}(1-\overline{D}_f)-1\right] \tag{13}$$

As the coupling coefficient is always positive, two zones are clearly identified :

- $Bi^{(\Delta)}(1-\overline{D}_f) \le 1$ : the coupling process is stable $\forall \alpha_f \ge 0$
- $Bi^{(\Delta)}(1-\overline{D}_f) \ge 1$ : the coupling procedure exhibits a lower stability bound $\alpha_f^{\min}$

This demonstrates how stability depends mainly on the ratio of thermal resistances, but also on the dynamics of the transient fluid system. The higher the local Biot number, the more difficult it will be to stabilize the coupling. Of course, the transient effects of the fluid system also play a major role and can stabilize any process via the term $(1-\overline{D}_f)$, but at a significant cost.

## 3.3 Optimal procedure

It is very noteworthy that the modulus of the amplification factor does not have a monotonic variation (in terms of $\alpha_f$) but goes through an absolute minimum, denoted $\alpha_f^{opt}$. In other words, the existence of a transition value for $\alpha_f$ can be identified. At this transition value, the shape of the curve of the amplification factor switches and turns back as can be seen in Figure 1. In this Figure, the two curves have been plotted using the following characteristics :

- Local Biot number $Bi^{(\Delta)} = 1.25\ 10^2$
- Mesh Fourier numbers : (1) $D_f = 40 \Rightarrow \overline{D}_f = 0.8$ (2) $D_f = 40{,}000 \Rightarrow \overline{D}_f = 0.992$

Thus, the stabilizing effect of the term $(1-\overline{D}_f)$ is noticeable. The most striking point is that at the remarkable coefficient $\alpha_f = \alpha_f^{opt}$, the modulus of the amplification attains its absolute minimum value, always less than one.

Low values of $\alpha_f$ ($\alpha_f < \alpha_f^{opt}$) will result in a rapid convergence, but in this case however $\overline{D}_f$ must be sufficiently large to allow heat diffusion on the fluid side. Otherwise, a low Fourier number will soak up a lot of heat. It will be then necessary to enhance stability by increasing the coupling parameter.

But, likewise, large values of $\alpha_f$ $(\alpha_f >> \alpha_f^{opt})$ will always lead to an extremely slow convergence since this corresponds to a slow diffusion of heat through the fluid subsystem. But it should be pointed out that relatively large Fourier numbers ($\overline{D}_f \approx 1$) indicate fast propagation and energy in this case will be unnecessarily frozen by $\alpha_f$.

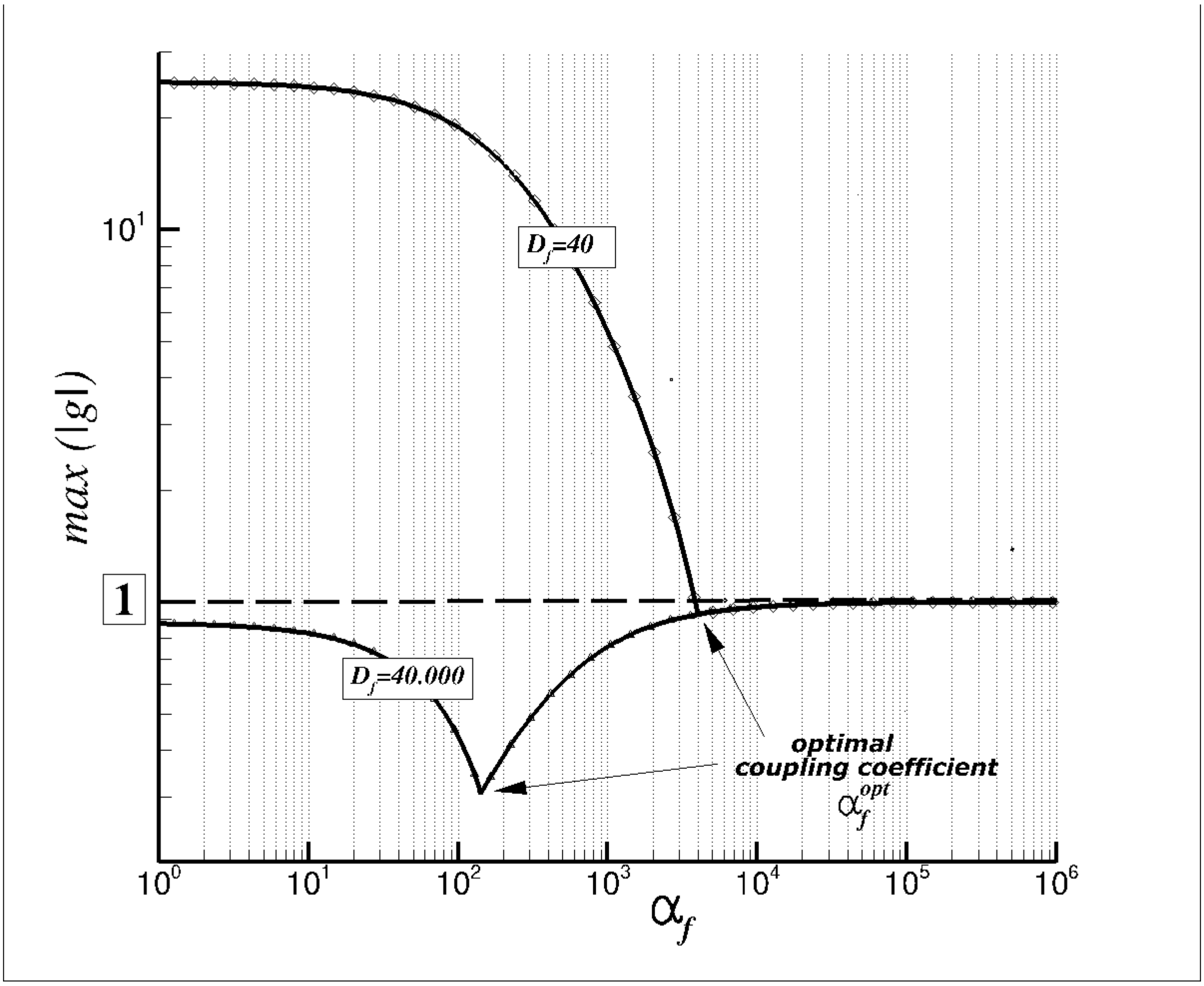


Figure 1: Amplification factor for $D_f = 40$ & $D_f = 40.000$
Dirichlet-Robin procedure

It should be noted that these situations might occur in the same coupled computation or even in the same fluid-structure interface. It is the reason why it is crucial to use local coupling coefficients and it has been shown that $\alpha_f = \alpha_f^{opt}$ is the optimal choice in the case of the model equation adopted herein.

### 3.4 Summary

The general behavior of the Dirichlet-Robin coupling procedure for steady CHT, in terms of $\alpha_f$ is illustrated in Table 1.

| $\alpha_f$ | 0 | | $\alpha_f^{\min}$ | | $\alpha_f^{opt}$ | | $\infty$ | condition |
|---|---|---|---|---|---|---|---|---|
| $\lvert g\rvert$ | | UNSTABLE | 1 | ↘ | $g^{opt}$ | ↗ | 1 | $Bi^{(\Delta)}(1-\overline{D}_f) \geq 1$ |
| $\lvert g\rvert$ | $g^0$ | ↘ | | | $g^{opt}$ | ↗ | 1 | $Bi^{(\Delta)}(1-\overline{D}_f) \leq 1$ |

Table 1: Numerical behavior of Dirichlet-Robin procedure vs $\alpha_f$ $(\alpha_s = \infty ; \alpha_f \geq 0)$

## 4 NEUMANN-ROBIN INTERFACE CONDITIONS

### 4.1 Stability bounds

The stability condition $\lvert g(z,\alpha_f)\rvert < 1$ applied to Eq.(10) leads, after some basic calculus manipulations, to an upper stability bound $\alpha_f^{\max}$

$$\alpha_f < \alpha_f^{\max} = \frac{2K_s K_f}{K_s - K_f(1+\overline{D}_f)} \tag{14}$$

Substituting the definition of $Bi^{(\Delta)}$ (Eq.(7)), $\alpha_f^{\max}$ becomes

$$\alpha_f^{\max} = \frac{2K_f}{1 - Bi^{(\Delta)}(1+\overline{D}_f)} \tag{15}$$

As the coupling coefficient is always positive, two stability regions are highlighted :

- $Bi^{(\Delta)}(1+\overline{D}_f) \geq 1$ : the coupling process is stable $\forall \alpha_f \geq 0$
- $Bi^{(\Delta)}(1+\overline{D}_f) \leq 1$ : the coupling procedure exhibits an upper stability bound $\alpha_f^{\max}$

### 4.2 Optimal procedure

This time, in the framework of the Neumann-Robin procedure, three zones can be identified. The first zone exhibits an upper stability limit as just mentioned. There is also in this zone an optimal coefficient for which the modulus of the amplification factor attains an absolute minimum.

The second zone is very narrow. It also presents an optimal coefficient, but in contrast to the previous case, there is no stability bound.

The third zone is also unconditionally stable but no optimal coefficient can be defined. That is to say the minimum of the amplification factor is obtained for $\alpha_f = \infty$, i.e. for the Neumann-Dirichlet procedure (heat flux imposed on the fluid side & temperature imposed on the solid side) with no relaxation. Another way of explaining it, is that the amplification factor is a monotonic function.

### 4.3 Summary

The general behavior of the Neumann-Robin coupling procedure for steady CHT, in terms of $\alpha_f$ is illustrated in Table 2.

| $\alpha_f$ | 0 | | $\alpha_f^{opt}$ | | $\alpha_f^{max}$ | | $\infty$ | **c o n d i t i o n** |
|---|---|---|---|---|---|---|---|---|
| $\lvert g \rvert$ | 1 | ↘ | $g^{opt}$ | ↗ | 1 | **U N S T A B L E** | | $Bi^{(\Delta)}(1+\overline{D}_f) \le 1$ |
| $\lvert g \rvert$ | 1 | ↘ | $g^{opt}$ | → | | | $\lvert g(\infty) \rvert$ | $1 \le Bi^{(\Delta)}(1+\overline{D}_f) \le 2$ |
| $\lvert g \rvert$ | 1 | ↘ | | | | | $\lvert g(\infty) \rvert$ | $Bi^{(\Delta)}(1+\overline{D}_f) \ge 2$ |

Table 2: Numerical behavior of Neumann-Robin procedure vs $\alpha_f$ ($\alpha_s = 0\,; \alpha_f \ge 0$)

## 5 COMPARISON OF THE TWO INTERFACE CONDITIONS

### 5.1 Stabilizing effect of the Fourier number

As just stated, the two interface conditions considered in this paper may be regarded as two complementary conditions, and it seems that we can go as far as to say that they are almost opposite. The first condition is unconditionally stable at small Biot number while the second one is unconditionally stable at large Biot numbers. But there is one major difference. In the case of the Dirichlet-Robin procedure, the term $(1-\overline{D}_f)$ may become as small as necessary and as a result the unconditional stability condition $Bi^{(\Delta)}(1-\overline{D}_f) \le 1$ ($2^{nd}$ row of Table 1) can always be satisfied. In other words, the lower stability bound can always be removed by an appropriate choice of $\overline{D}_f$. Conversely, in the condition $Bi^{(\Delta)}(1+\overline{D}_f) \le 1$ ($1^{st}$ row of Table 2), the term $(1+\overline{D}_f)$ has no stabilizing effect.

### 5.2 Overlapping zone

It is not quite accurate to say that the two interface conditions are opposite. Actually they exhibit an overlapping zone where both may be considered as stable. On the basis of the stability limits presented in the preceding sections, it is seen that there is an overlapping zone where both procedures are unconditionally stable and defined by

$$\frac{1}{1+\overline{D}_f} \le Bi^{(\Delta)} \le \frac{1}{1-\overline{D}_f} \tag{16}$$

This overlapping zone gets narrower as the normalized Fourier number $\overline{D}_f$ gets smaller. On the contrary, this zone is significantly extended for large Fourier numbers.

### 5.3 Existence and non-existence of optimal procedures

The two interface conditions presented in this paper exhibit 5 different zones (2 in Table 1 & 3 in Table 2). In one of them ($3^{rd}$ row of Table 2), there is no optimal coefficient. This does however not mean that the coupling procedure will converge slowly but simply that the condition $\alpha_f = \infty$ is a relevant choice. In other words the Neumann-Dirichlet transmission is recommended. In the first procedure, the optimal choice requires a certain amount of relaxation and this time a Dirichlet-Robin procedure is recommended in conjunction with the use of the optimal coefficient.

Actually, there exist two types of optimal procedures. The first one is not subject to any condition and always exhibits an optimal coefficient. The second one presents a conditional optimal coefficient. Clearly, the transient effects of the fluid system play a major role in the stability analysis.

### 5.4 Criterion for a relevant interface treatment

We have just seen how to optimize a specific transmission condition procedure. But this study suggests that it can be interesting to select adequately the most relevant procedure and not necessarily to optimize a particular one, more or less arbitrarily chosen. Two options might be considered :

(1) A "dynamic" option

The dynamic option is supported by the fact that the optimal coefficient of the Dirichlet-Robin condition is able to stabilize any procedure.

- $Bi^{(\Delta)}(1-\overline{D}_f) \leq 1$ **:** Dirichlet-Robin procedure with $\alpha_f = \alpha_f^{opt}$
- $Bi^{(\Delta)}(1-\overline{D}_f) > 1$ :

  if $(\alpha_f^{opt} >> K_s)$ **:** Neumann-Robin procedure with $\alpha_f$ sufficiently great
  else
  Dirichlet-Robin procedure with $\alpha_f = \alpha_f^{opt}$
  endif

Note that in the overlapping zone, the procedure with the term $(1-\overline{D}_f)$ has been retained.

(2) A "static" option

The other option does not take into consideration the dynamics of the fluid system $(\overline{D}_f = 0)$. The preceding conditions thus become :

- $Bi^{(\Delta)} \leq 1$ : Dirichlet-Robin procedure with $\alpha_f = \alpha_f^{opt}$
- $Bi^{(\Delta)} > 1$ :
  if $(K_f >> K_s)$ : Neumann-Robin procedure with $\alpha_f$ sufficiently great
  else
  Dirichlet-Robin procedure with $\alpha_f = \alpha_f^{opt}$
  endif

The first option takes advantage of the stabilizing effect of the mesh Fourier number in the transient fluid domain. The second option is more secure especially when the ratio $Bi^{(\Delta)}$ is very high, for instance in CFD computations in which a high wall resolution is adopted. In this case, it is always possible to exploit to advantage the term $(1-\overline{D}_f)$ such that $Bi^{(\Delta)}(1-\overline{D}_f)\leq 1$ but this can have a substantial cost.

## 5.5 Link between the local and the conventional Biot numbers

This local Biot number introduced by Eq. (7) takes directly into account the thermal and dynamic response of the boundary layer and directly participates in the stability of the coupled process. If a transient fluid flow is employed during the CHT computation it is this number that will drive and control the stability of the CHT problem.

On the other hand, the conventional Biot defined by Eq.(6) is a measure of the resistance to heat flow within the solid relative to the resistance presented by the convection processes at the surface. This parameter is also a key parameter, but only at steady state since it determines the stability of the fluid-solid equilibrium. However, it is clear that this conventional Biot number, not easily defined during the transients, cannot be used to set up a numerical CHT procedure, as long as a transient fluid state is involved in this procedure.

At steady state, the thermal fluid resistance is equal to $1/h$ and thus the various stability criteria and stability bounds presented in this paper may easily be extended to steady state by simply imposing $\overline{D}_f = 0$ and $K_f = h$.

## 5.6 Numerical example

A complex CHT computation has been performed in a 3D test case representing an industrial reheat furnace with heteroclite fluid-structure interface (details can be found in [7]). As a first step, the Dirichlet-Robin condition has been used and a systematic and comparative study of various coupling coefficients has been undertaken. This first study showed that the optimal coupling coefficient presented in this paper was by far, the best-performing procedure.

But by looking more closely at the fluid-solid interfaces of the furnace, high local Biot number can be found, for instance

$$Bi^{(\Delta)} = \frac{K_f}{K_s} = 1.25\ 10^2 \tag{17}$$

This ratio strongly suggests the use of a Neumann-Dirichlet procedure as mentioned in § 5.4. Figure 2 presents the convergence history of these CHT tests. All the tests performed are not indicated in this figure. It is sufficient to mention that the so-called, optimal Dirichlet-Robin procedure is always stable and oscillation-free and clearly the most efficient coupling method.

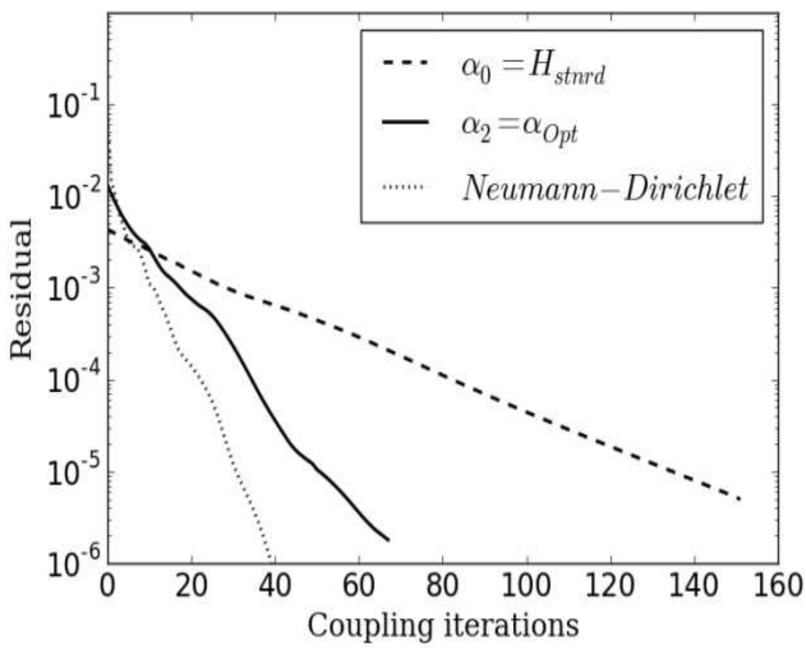


Figure 2 : Convergence history and comparison between Dirichlet and Neumann procedures

But, as suggested by the criterion proposed in this study, it is instructive to see that a relevant procedure, even with no relaxation parameter can be even more performing. This means that when the local Biot number $Bi^{(\Delta)}$ is high, stability can be obtained with the Dirichlet-Robin condition but at the expense of computational efficiency (CPU time). In this case, a natural physics-based approach is to impose a Neumann condition.

## 6 CONCLUSIONS

On the basis of a simplified model problem, the general expression of the amplification factor, the stability bounds, the optimal coefficient and the general numerical behavior have been presented for two different and complementary interface treatments. It was shown that the numerical properties depend on the ratio of fluid and thermal resistances and that the mesh Fourier number plays a crucial role. Furthermore, a local Biot number has been introduced and it was shown that this number "drives" the overall coupling process.

The two interface treatments have been compared and it has been highlighted that these interface treatments are almost opposite and complementary. When the first method is unconditionally stable, the second one exhibits an upper stability bound. When the second method is unconditionally stable, the first one exhibits a lower stability bound. These two interface schemes present an overlapping area where both of them are stable. It is also shown that the so-called optimal coefficient provides the best results in terms of stability and convergence in the Dirichlet-Robin procedure. The numerical criteria establishing the nature and character of the most relevant interface treatment have been identified and expressed. A comparison in a CHT test case has emphasized the importance of a physics-based numerical approach.

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