ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
Crete Island, Greece, 5–10 June 2016

NONLINEAR FORCED RESPONSE OF A STATOR VANE WITH MULTIPLE FRICTION CONTACTS USING A COUPLED STATIC/DYNAMIC APPROACH

M. Lassalle[1], C. M. Firrone[2]

[1]Politecnico di Torino, Department of Mechanical and Aerospace Engineering, Aer-Mec Laboratory
Corso Duca degli Abruzzi, 24, Torino, Italy
email: marco.lassalle@polito.it
www.aermec-dimec.polito.it

[2]Politecnico di Torino, Department of Mechanical and Aerospace Engineering, Aer-Mec Laboratory
Corso Duca degli Abruzzi, 24, Torino, Italy
email: christian.firrone@polito.it
www.aermec-dimec.polito.it

Keywords: Vane-segment, non-linear dynamics, friction joint, MHBM, coupled static/dynamic approach, coupled/uncoupled comparison

Abstract. *The purpose of this paper is the nonlinear calculation of the forced response of a vane-segment (a sector of a row of stator blades) for aeronautical applications in the presence of multiple joints using a coupled static/dynamic method to calculate the contact forces between assembled components. The solution of the static/dynamic equilibrium equations is calculated using the Multi Harmonic Balance Method (MHBM) that is a technique widely accepted as an effective approach to calculate the steady-state forced response in the frequency domain in spite of the Direct Time Integration (DTI).The MHBM technique allows, in fact, a strong reduction of the computation in particular when non-linearities are introduced as in this case for a more realistic modeling of the physical behavior of the joints and system response in general.*

The non-linearity is introduced by means of contact elements that produce additional stiffening and damping in the system due the introduction of contact stiffness and friction forces based on Coulomb's law. Since the periodical nonlinear contact forces may depend on the history of the motion of the vibrating system, the MHBM steady-state solution may lead to a poor correlation with the DTI solution. To consider this dependence, in this paper, it is presented a coupled method which permits to solve together the static balance equations and the dynamic balance equations to have a better approximation of the real behavior of the system.

A sensitivity analysis is performed to investigate the effect of numerical, geometrical and contact parameters to optimize the design of the system. Calculated responses are finally compared with the uncoupled approach where the static equilibrium is calculated independently from the vibratory response calculation as usually done to investigate the difference of the two methods.

1. INTRODUCTION

The forced vibrations in resonance conditions shorten the life and reliability of aircraft engines and turbo-machinery in general. Although the sources of dynamic excitation producing oscillations of the whole structure are many, the time-varying forces loading gas turbines of aeronautic engines are caused by the considerable non-uniformity of the flow of the hot gases that pass through the turbine blades [1]. In a regular operating condition, the aerodynamic excitation pattern loading the blade array can be represented as a combination of multiple excitations having a harmonic pattern in space along the circumferential direction that travel with respect to the bladed disk. The circumference is divided into an integer number (called Engine Order EO [2]) of wavelengths for each travelling harmonic force. The single blade is therefore subjected to a wide excitation spectrum and multiple excitations at resonance condition may occur for a given rotating speed of the rotor. The same issue occurs for stator vane segments as well where travelling forces are generated by the wakes of front and aft-rotating disks. For this reason the designers of aircraft engines are called to introduce damping systems to reduce peak stress values during the vibratory phenomenon. Friction dampers are often used to dissipate energy in the system through friction, i.e. the amplitudes of oscillation are reduced increasing the fatigue life of the blade arrays. The major sources of friction damping in the bladed disk turbine are attributable to the blade-disc interfaces (blade root joint) [3,4], the contact between adjacent blades connected by interference at the tip (shrouds) or mid-span airfoil (snubber, [5]) and the presence of underplatform dampers [6, 7]. Typical joints for vane segments are the so-called interlocking joints [8] located at the inner radius of the sector connecting adjacent sectors.

In this paper, a particular excitation is considered to study the forced response of a stator vane segment subjected to Engine Order 1 travelling force. It is verified, in fact, that a mechanical excitation due to the unbalance of the rotor shaft in the engine may excite the vane segment according to the 1 per rev excitation frequency at not negligible levels. For this reason it is interesting to verify at what extent the interlocking contacts are able to limit the vibration amplitudes. Moreover, the study is extended assuming that also the joint connecting the vane segment to the casing (hook contacts) may act as a friction damper that is added to the interlocking contact. In the first part of the paper the vane segment is described and the rationale of the study is specifically introduced; in the second part of the paper the modelling of the vane segment and of the contact model are illustrated; in the third part the methodology to solve the equation of motions are explained; in the fourth part the sensitivity analysis and the comparison with the uncoupled method is shown. The uncoupled method provides the normal loads acting on the contacts before the vibratory response is calculated. Usually a nonlinear static calculation of the equilibrium is performed to achieve the set of normal loads that are applied on the contact and these values are kept constant through the forced response calculation. In this sense the normal load are a sort of contact pre-load in the system that is acceptable to simulate the assembly phase when stator vanes are mounted but not the operative phase. It was proven that in the presence of vibrations, when slip occurs, the normal pre-load may vary for a single contact when the tangential and normal direction of the contact are both involved in the vibrations. Furthermore, in case of multiple contacts in the vibrating system as in this case, if one of the contact is in slip state during vibrations, it may change the normal pre-load of other contacts in the system. These are the main reasons to adopt a coupled approach instead of the uncoupled approach. The conclusions are drawn in the final part of the paper.

2. TEST ARTICLE AND EXPERIMENTAL EVIDENCE

The problem studied in this paper is to evaluate the forced response of a stator vane segment shown in **Figure 1** in the presence of a predominant radial excitation due to the unbalance of the rotor shaft. The excitation is transferred from the unbalanced rotating shaft to the bearing outer race and then to the struts connecting the engine casing and finally to the vane segment that is connected to the casing by means of the hook joints (**Figure 2**). For this reason the load can be described as a radial force applied to the casing with unitary engine order (EO1, **Figure 3**) since the excitation frequency of a single vane segment can be directly connected to the 1 x rev excitation.

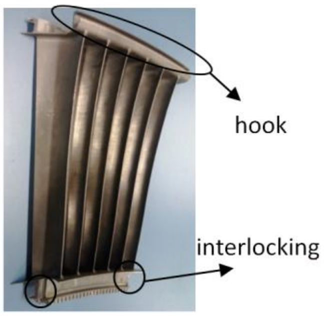

Figure 1: Vane-segment

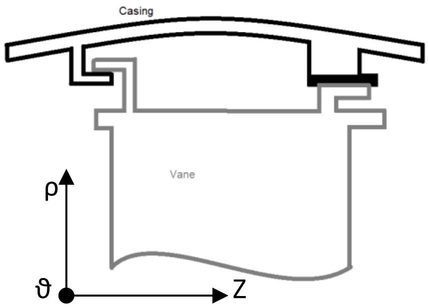

Figure 2: Hook joints

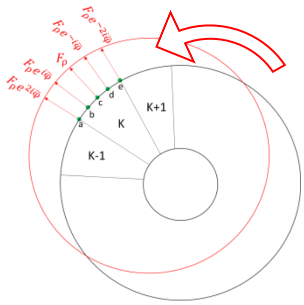

Figure 3: EO1 travelling excitation

It is important to study this phenomenon because it is predominant with respect to the aerodynamic forces due to the low engine order excitation.

For reason of confidentiality, in **Figure 4** is presented a sketch of an experimental Campbell diagram. Usually the values reported in the Campbell diagram are calculated after the measurement of strains obtained by means of a strain gauge placed on an airfoil of the vane segment. When vibrations are synchronous, i.e., when the vibration frequency is an integer number of times the rotation frequency of the rotor, the values of stress associated to the measured strain is produced by an engine order EO excitation, where EO is the integer number. A particular value of rotation speed exists where the natural frequency of a given mode shape matches the EO excitation. In this case, the second mode shape of the vane segment at Nodal Diameter ND = 1 is excited by an EO=1 excitation.

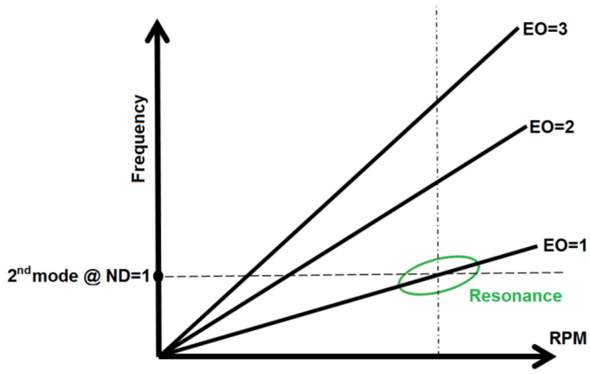

Figure 4: Campbell diagram

3. COMPONENT MODELING

The damping of vibrations through friction is a non-linear problem that is solved with iterative numerical methods. These methods require considerable computation time. Both the vane-segment and the casing have been modeled by FEM using a number of nodes so large that the resolution of the equations of motion in the time domain and in a direct form is not generally obtainable in an acceptable time. A reduction of the degrees of freedom (dofs) to solve is necessary in order to obtain a Reduced Order Model (ROM) without losing the critical information for the analysis. The reductions of the vane segment and casing stiffness and mass matrices have been done using three methods: for the vane segment only the Craig-Bampton Component Mode Synthesis (CB-CMS) [9] was used, while for the casing the CB-CMS technique, the Cyclic Symmetry (CS) technique [9], and the Tran technique [10] were sequentially used in the aforementioned order. The ROM techniques are applied to only one vane-segment of the stator and to a fundamental sector of the casing associated with the vane segment by means of the hook joints.

3.1. Craig Bampton - Component Mode Synthesis (CB-CMS)

The CB-CMS reduction technique is implemented in most FE softwares (Ansys, Nastran) and it is based on the definition of a set of master $\{x_m\}$ and slave $\{x_s\}$ degrees of freedom (dofs) of the physical model plus an additional set of slave modal dofs $\{\eta_s\}$. The first set is the set that remains explicit in the ROM while the remaining part of dofs is omitted as a function of the first set according to the Guyan static reduction technique formulation [11] where the static deflection shapes of the slave dofs are written by imposing a unitary displacement of each master dofs one by one. With this process the slave nodes are forced to move according to the deformation of the master nodes. Since the Guyan reduction technique is not sufficiently representative of the dynamics of the system, a number of slave modal shapes is added to the static deflection base obtained from a modal analysis where all the master dofs are constrained simultaneously.

Both the casing fundamental sectors and the vane-segment are first reduced according to the following transformation for each component:

$$\{x_{tot}\} = \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{bmatrix} I & 0 \\ G & \Phi_s \end{bmatrix} \begin{Bmatrix} x_m \\ \eta_s \end{Bmatrix} = [\Psi_{CMS}] \begin{Bmatrix} x_m \\ \eta_s \end{Bmatrix}$$

where I is the identity matrix, G is the Guyan matrix, Φ_s is a subset of the slave modal matrix and η_s is the vector of the modal slave coordinate.

For the vane-segment the master dofs are those belonging to the following nodes:

- 60 nodes on the hooks: 30 nodes on the front hook and 30 nodes on the rear hook arranged according to two bands of 15 nodes each;
- 20 nodes on the interlocking: 10 on the left interlocking and 10 on the right interlocking arranged according to two bands of 5 nodes each;
- 3 accessory nodes placed on one blade of the vane-segment.
- 10 Modal slave dofs

For the casing the master dofs are those belonging to the following nodes:

- 326 nodes for both the left and right interfaces. The interfaces are the cross sections obtained by selecting only the fundamental sector from the full casing;
- 60 nodes for the hooks mating the 60 nodes of the hooks of the vane;
- 16 nodes where the external force is applied along the circumferential direction of the casing simulating the EO1 unbalance mechanical force;
- 3 accessory nodes on the external face of the casing.
- 10 Modal slave dofs

Figure 5 and **Errore. L'origine riferimento non è stata trovata.** show the position of the master dofs for both components.

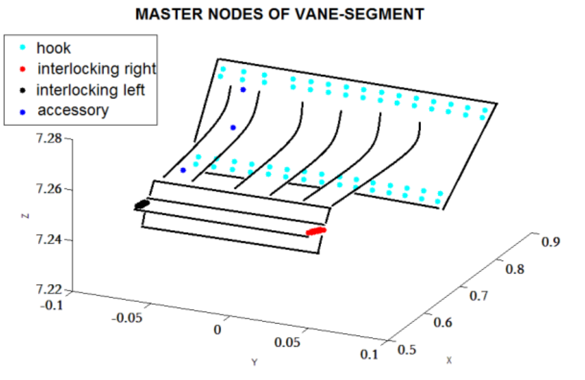

Figure 5: Master nodes on the vane

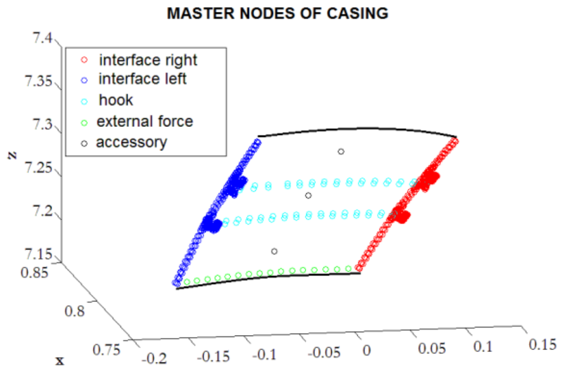

Figure 6: Master nodes on the casing

3.2. Cyclic symmetry

The casing is a cyclic symmetric component, therefore it shows the typical dynamic behavior and mode shapes of drums and rotors [9], in particular if a mode shape is excited by a travelling rotating force, each sector vibrates as the adjacent ones but with a phase lag given by the Inter Sector Phase Angle (ISPA) $\varphi = 2\pi ND/N$ with ND=Nodal Diameter and N=number of sector. The Nodal Diameter is a line passing from the center of the cyclic symmetric component where the modal displacements are null and its value is a characteristic of every mode shapes. A ND mode shape can be excited by an EO travelling force only if this relationship is verified:

$$EO = zN \pm ND, \quad z = 0,1,2, \dots$$

In this case we are interested in the response of the second resonance of the ND1 mode family excited by an EO1 travelling force (z=0).

Since the displacement of each sector is the same of the adjacent but with a phase lag, the full component can be reduced to a single sector by applying cyclic symmetry constraints to the left and right interfaces. The right interfaces dofs are written in function of the left interfaces dofs or vice-versa depending if the travelling force is clockwise or counterclockwise with respect to a defined cylindrical reference system. In this case for the casing:

$$\{x_l\} = \{x_r\}e^{i\varphi}$$

Where x_l are the dofs of the left interface while x_r are the dofs of the right interface of the casing. The set of dofs of the casing becomes then:

$$\begin{Bmatrix} x_m \\ \eta_s \end{Bmatrix} = \begin{Bmatrix} x_l \\ x_r \\ x_i \\ x_f \\ \eta_s \end{Bmatrix} = \begin{bmatrix} e^{i\varphi} & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} x_r \\ x_i \\ x_f \\ \eta_s \end{Bmatrix} = [\Psi_{CS}] \begin{Bmatrix} x_r \\ x_i \\ x_f \\ \eta_s \end{Bmatrix}.$$

where x_f is the sub-vector of the dofs where the external force is applied and x_i are the inner dofs (accessory and contact dofs).

3.3. Tran reduction technique

The Tran reduction technique [10] is applied on the dofs of the right interface because its number of dofs is still too large and consists in:

- Build an auxiliary ROM of the casing according to a Guyan static reduction technique where the master dofs are only the left and right interface dofs;
- Apply the cyclic symmetry reduction technique in order to write the left dofs with respect to the right dofs;
- Compute the right interface mode shape by means of a modal analysis.
- Of all the calculated mode shapes, substitute only few of them to the right dofs of the primary ROM model. in particular we considered 10 interface modal shape for the casing.

The array of the explicit dofs of the nodes becomes:

$$\begin{Bmatrix} x_r \\ x_i \\ x_f \\ \eta_s \end{Bmatrix} = \begin{bmatrix} \Phi_{cs} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \eta_{cs} \\ x_i \\ x_f \\ \eta_s \end{Bmatrix} = [\Psi_{TRAN}] \begin{Bmatrix} \eta_{cs} \\ x_i \\ x_f \\ \eta_s \end{Bmatrix}.$$

The whole transformation for the ROM of the casing is then:

$$\{x_{tot}\} = [\Psi_{CMS}][\Psi_{CS}][\Psi_{TRAN}] \begin{Bmatrix} \eta_{cs} \\ x_i \\ x_f \\ \eta_s \end{Bmatrix} = [\Psi_{TOT}] \begin{Bmatrix} \eta_{cs} \\ x_i \\ x_f \\ \eta_s \end{Bmatrix}$$

The stiffness and mass matrix of the two components can be obtained by pre-multiplication and post-multiplication of the transformation matrices:

$$[m] = [\Psi_{TOT}]^T [M][\Psi_{TOT}]$$
$$[k] = [\Psi_{TOT}]^T [K][\Psi_{TOT}]$$

The vane segment is kept in its position by means of a key/keyseat system close to the hooks that connect the vane segment to the casing. In order to model this kind of constraint, four contact nodes on the vane segment hooks close to the key are linked to the mating nodes of the casing hooks.

4. SOLUTION STRATEGY

In order to compute the forced response of a vane-segment with friction contacts the commercial finite element codes are not suitable since they are based on the time integration method of the non-linear differential balance equations which require very large calculations times. For this reason, ad hoc numerical codes must be developed in order to compute the forced response in the frequency domain. These codes are based on the Harmonic Balance Method (HBM) [12]. The periodic variables (displacements and forces) are written as a sum of harmonic terms by Fourier analysis and then the balance of each harmonic is imposed, turning the original nonlinear differential equations in a set of nonlinear algebraic equations.

The starting point in the forced response calculation of a mechanical system with friction contacts is writing the equations of motion in matrix form:

$$[M]\ddot{Q} + [C]\dot{Q} + [K]Q = F_e + F_{nl}, \tag{6}$$

where M, C and K are the mass, damping and stiffness matrices of the system, Q is the displacements array of degrees of freedom (dofs), F_e is the periodical external forces array acting on the system and F_{nl} is the non-linear array generated by friction contacts.

In order to reduce the calculation time of a numerical integration, the harmonic balance method (HBM) can be used to solve the equations of motion of the system [5,12,13, 14]. This is possible due to the periodicity of the external excitation that involves also the periodicity of the displacements Q and the non-linear forces F_{nl}. This property of periodicity permits to express the displacements (Q) and the forces (F_e and F_{nl}) in a truncated Fourier series of H orders, where H is the maximum number of harmonics considered. Consequently displacements and forces become:

$$Q = Q^{(0)} + \Re\left(\sum_{h=1}^{H} Q^{(h)} e^{ih\omega t}\right) \tag{2}$$

$$F_e = F_e^{(0)} + \Re\left(\sum_{h=1}^{H} F_e^{(h)} e^{ih\omega t}\right) \tag{3}$$

$$F_{nl} = F_{nl}^{(0)} + \Re\left(\sum_{h=1}^{H} F_{nl}^{(h)} e^{ih\omega t}\right) \tag{4}$$

where ω is the fundamental frequency of the excitation forces acting on the system, the 0th order represents the static components and the coefficients $Q^{(h)}$, $F_e^{(h)}$ e $F_{nl}^{(h)}$ are complex quantities. The Equations (2)-(4) are replaced into the equations of motion (1) obtaining two sets of algebraic equations, a set of static algebraic equations and a set of algebraic complex dynamic equations [14]:

$$[K_{stat}]Q^{(0)} = F_e^{(0)} + F_{nl}^{(0)} \tag{5}$$

$$\left[D^{(h)}\right]Q^{(h)} = F_e^{(h)} + F_{nl}^{(h)}, \qquad h = 1,2, \dots H \tag{6}$$

where $\left[D^{(h)}\right] = -(h\omega)^2[M] + ih\omega[C] + [K]$ is the hth dynamic stiffness matrix of the system. The equations (5) and (6) are non-linear because contact forces $F_{nl}^{(0)}$ and $F_{nl}^{(h)}$ depend on the relative static/dynamic displacements of the contact dofs that are a part of the total dofs Q.

In the classical approach, the static balance equations (5) are uncoupled from the dynamic balance equations, i.e. these equations are solved separately and then the 0th order coefficients of the displacements $Q^{(0)}$ are used as inputs for the solution of the dynamic equations. As a consequence, the value of the static contact forces, computed in the static balance equations is no more updated when the dynamic balance is imposed. In a more general approach, the static (5) and the dynamic (6) balance equations formulated by the HBM are solved together, i.e. the overall balance conditions (static and dynamic) are computed together.

This procedure will be referred as the '*coupled*' calculation of the forced response, where the term coupled means that the static and the dynamic quantities involved in the dynamic response of the vibrating system are updated at each iteration and may change for different excitation frequencies [14]. In order to separate the solution of the non-linear part of the system from its linear part it is convenient to rearrange the balance equations (5) and (6):

$$Q^{(h)} = \left[R^{(h)}\right]F_e^{(h)} + \left[R^{(h)}\right]F_{nl}^{(h)}, h = 0, \dots H \tag{7}$$

where $\left[R^{(h)}\right]$ is the receptance matrix (inverse of static or dynamic matrices: respectively $[K_{stat}]$ or $\left[D^{(h)}\right]$), $\left[R^{(h)}\right]F_e^{(h)} = Q_e^{(h)}$ is the linear response due to external excitation that does not change once the excitation frequency is fixed and $\left[R^{(h)}\right]F_{nl}^{(h)}$ is the non-linear response due to the contact forces that changes with the state of the contacts. Because the non-linear forces act only on the contact dofs, the vector of total displacements $Q^{(h)}$ can be split in two part: non-linear dofs $Q_{nl}^{(h)}$ and linear dofs $Q_l^{(h)}$; in this way the equation (7) becomes:

$$\begin{Bmatrix} Q_{nl}^{(h)} \\ Q_l^{(h)} \end{Bmatrix} = Q_e^{(h)} + \begin{bmatrix} R_{nl,nl}^{(h)} & R_{nl,l}^{(h)} \\ R_{l,nl}^{(h)} & R_{l,l}^{(h)} \end{bmatrix} \begin{Bmatrix} F_{nl}^{(h)} \\ 0 \end{Bmatrix}$$

where only the first equation $Q_{nl}^{(h)} = Q_{e}^{(h)} + \left[R_{nl,nl}^{(h)}\right]F_{nl}^{(h)}$ must be solved iteratively with a non-linear solver. Once the non-linear forces $F_{nl}^{(h)}$ are known it is possible to easily solve the second part of the equilibrium equations.

It must be observed that the h sets of equations $Q_{nl}^{(h)} = Q_{nl}^{(h)} + \left[R_{nl,nl}^{(h)}\right]F_{nl}^{(h)}$ are coupled to each other, because the number of harmonics that compose the nonlinear contact forces $F_{nl}^{(h)}$ depends on all the harmonics components of the displacements of the nonlinear dofs $Q_{nl}(t)$.

5. CONTACT MODEL

In order to solve the non-linear equations, described in the previous paragraph, it is necessary to introduce a contact model that computes the harmonic nodal contact force components $F_{nl}^{(h)}$ from a given set of harmonics contact displacements $Q_{nl}^{(0)}$ and $Q_{nl}^{(h)}$.

The contact model is based on the Coulomb friction law where the generic friction force $f_t(t)$ is equal to a normal preload $n(t)$ multiplied by the friction coefficient μ when sliding occurs:

$$f_t(t) = \mu n(t) \tag{8}$$

In this case the Coulomb friction law is implemented in a contact model [15] with a tangential k_t and normal k_n stiffness (**Figure 7**) in order to take into account a stick state of the contact that allows a relative motion between the two bodies in contact without sliding:

$$f_t(t) = f_0 + k_t(u(t) - u_0) \tag{9.a}$$

$$n(t) = k_n v(t) \tag{9.b}$$

(where f_0 and u_0 are respectively the value of the tangential force and tangential relative displacement at the end of the contact state preceeding the stick state) and the separation of the two parts in contact when $n(t) = k_n v(t)$ is less than zero, in that case:

$$\begin{cases} f_t(t) = 0 \\ n(t) = 0 \end{cases} \tag{10}$$

The couple static/dynamic approach, introduced to solve the balance equations of the system, involves that the tangential and normal relative displacements of the contact model include the correspondent static displacements, i.e.:

$$u(t) = u_0 + \bar{u}(t)$$
$$v(t) = v_0 + \bar{v}(t)$$

In this way the normal pre-load $n_0=k_n v_0$ of the contact model is no not more a design datum, but an information that comes by the iterative solution of the static and dynamic balance equation and it may change for different excitation frequencies. For this reason the normal relative static displacement v_0 (interference) is the necessary value to calculate the normal load of the contact model.

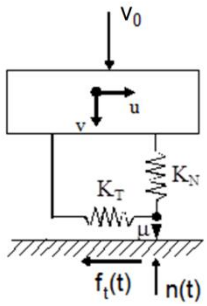

Figure 7: Contact model

The hysteresis loop can assume different shapes according to the relative phase lag of the relative tangential and normal displacements. In the **Figure** a, **Figure** b, **Figure** c three types of hysteresis loops are shown: the first when u(t) and v(t) are in-phase, the second when partial lift off occurs, the third when u(t) and v(t) do not vibrate in-phase.

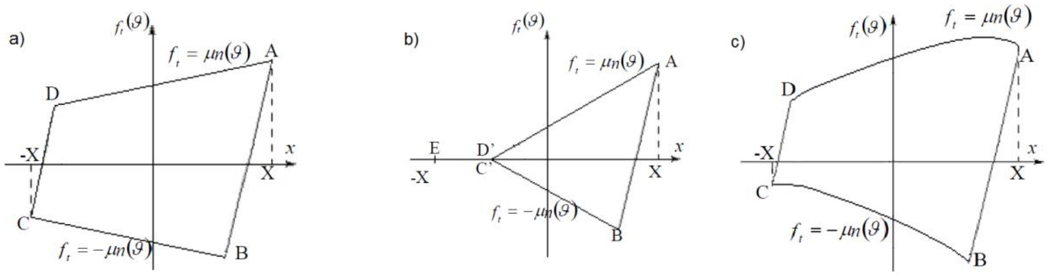

Figure 8: Hysteresis loops

The contact model is a 2-D model but it can be used to describe a 3-D behavior of the contact by using two orthogonal 2-D models that share the same normal relative static displacement v_0.

Since the equations of motion are formulated in the frequency domain the inputs of the contact model are the Fourier coefficients of the relative displacements $u^{(h)}$ and $v^{(h)}$ (with h=0,1,2,…,H) while the contact model operates in the time domain. For this reason for each iteration an Alternating Frequency Time (AFT) [15,16] also known as Hybrid Frequency Time (HFT) method [17] should be used in order to pass from the frequency domain to the time domain to calculate the hysteresis loop and then again to go back to the frequency domain by the Fourier series:

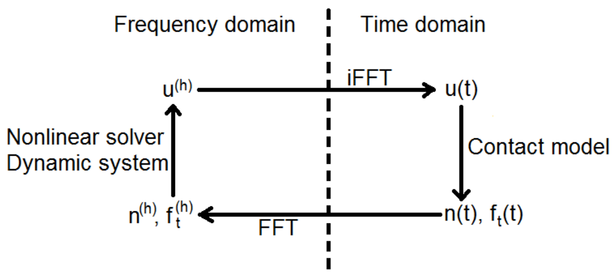

Figure 9: AFT/HFT method

The calculation procedure follows the following steps:

1. Periodical relative displacements $u(t)$ and $v(t)$ are computed from Fourier coefficients $u^{(h)}$ and $v^{(h)}$ (with h=0,1,2,…,H) by Inverse Fast Fourier Transform (IFFT);
2. Contact forces $n(t)$ and $f_t(t)$ are computed by means of equations (9) and (10);
3. Fourier coefficients $n^{(h)}$ and $f_t^{(h)}$ (with h=0,1,2,…,H) are computed from $n(t)$ and $f_t(t)$ respectively by Fast Fourier Transform (FFT).

With the Fourier coefficient of the contact forces it is possible to assemble the contact force vector F_{nl} that will be used in the resolution of the non-linear equations $Q_{nl}^{(h)} = Q_{nl}^{(h)} + \left[R_{nl,nl}^{(h)}\right]F_{nl}^{(h)}$ (with h=0,1,2,…,H).

6. SENSITIVITY ANALYSIS, TEST SIMULATION AND COMPARISON WITH MEASUREMENTS

The ROM assembly is used to compute the nonlinear forced response of a vane-segment. A sensitivity analysis has been performed changing the values of some parameters in the calculation code, in details:

- Number of joints (presence and absence of the interlocking contact);
- Contact parameters (friction coefficient and interlocking interference);
- Number of harmonics retained in the MHBM calculation.

These analysis have been then compared with the corresponding analyses made with the uncoupled code to show the difference between the two methods.

6.1. Variation of number of joints

The first analyses are performed by varying the number of joints in contact: (i) only the hook joints working and (ii) both hook and interlocking joints in contact. For these two types of analyses, the joints have been first considered in fully stick condition because the analyses, in this way, are linear (the contact points are linked together through the contact stiffness) and the two obtained forced responses are the limit linear ‘free’ and ‘stick’ interlocking conditions that the system can encounter.

The **Figure 10** shows the forced response with the hook joints working in the fully stick condition while the interlocking joints is free. Curves are normalized according to the resonant frequency and peak amplitude of the second mode that is the mode of interest. The dynamic load given by the unbalance is a traveling force characterized by EO=1 and fixed to the value assigned by industry data recovery as well as the interference at the hook contacts. In this conditions the response of the system obtained from the non-linear analysis should be coincident with the free linear response obtained linking together vane and casing through the contact stiffness of the hook joints. This happens because the major contribution to the damping of the system is given by the interlocking joints, while the hook joints give a very low contribution (see [18]). In fact, the nonlinear calculation is actually linear because slip does not occur and this is the case where the static equilibrium of the system is independent from the dynamic equilibrium.

The forced response with both hook and interlocking joints working in the stick condition is presented in **Figure 11**. Curves are obtained by decreasing the value of the excitation amplitude at the casing in order to have no slip at the contacts. In this condition, it is possible to see the increase of the resonance frequency of interest due to the full sticking of all the joints (the structure is more rigid). On the same figure, it is shown the forced response obtained from the uncoupled method setting the same parameters used in the coupled code (value of contact stiffness, value of forcing and same frequency range) and the value of the pre-load on the interlocking contacts corresponding to the interlocking interference. The value of pre-load is obtained from a static non-linear analysis of the vane in which the equation (7) is solved at the zero-order to obtain the static displacements of the interlocking. These values are then multiplied for the normal contact stiffness to get the pre-load on the interlocking. This process is necessary to compare the coupled method with the uncoupled method in order to verify differences in the results obtained from the coupled method. In **Figure 11** the two forced responses, couple and uncouple, are coincident and this is right because in the fully stick condition the static and dynamic calculations are not coupled.

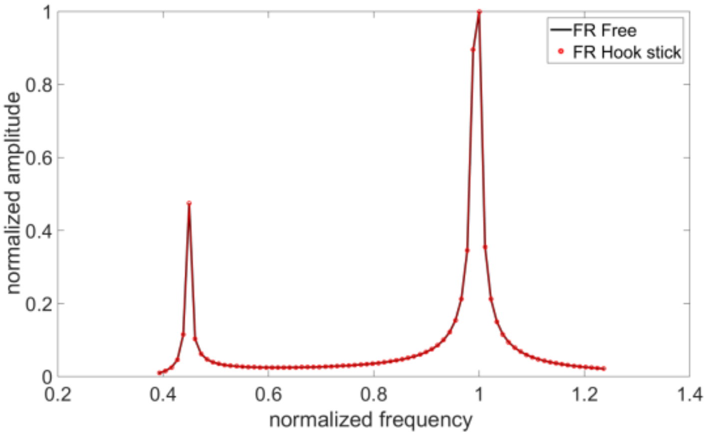

Figure 4: FR with hook joints in stick condition

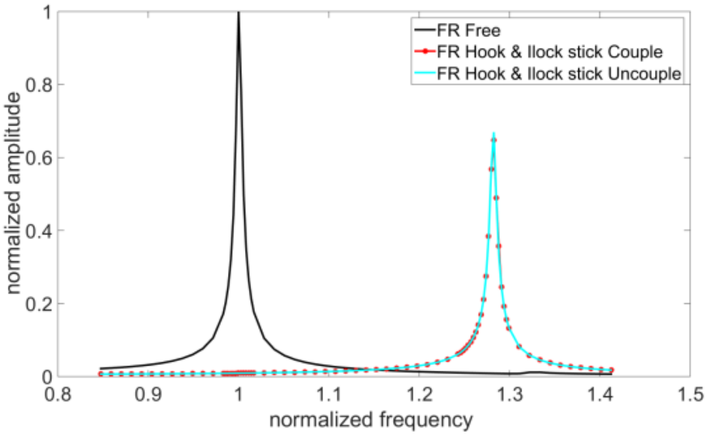

Figure 5: FR with hook & Ilock in stick condition

Figure 12 shows the comparison between different nonlinear responses by varying the friction coefficient μ. These preliminary analyses have been made using only the first harmonic of the Fourier series fixing the design value of interference in the interlocking contact. It is possible to see that the nonlinear responses amplitudes are lower than the fully stick response of **Figure 11**, moreover the amplitude decreases if the friction coefficient increases. This effect is described in [8], basically when the friction coefficient value is low, friction forces are low as well and the dissipated energy is not effective to limit the vibration amplitudes because the contact tends to a 'frictionless' contact.

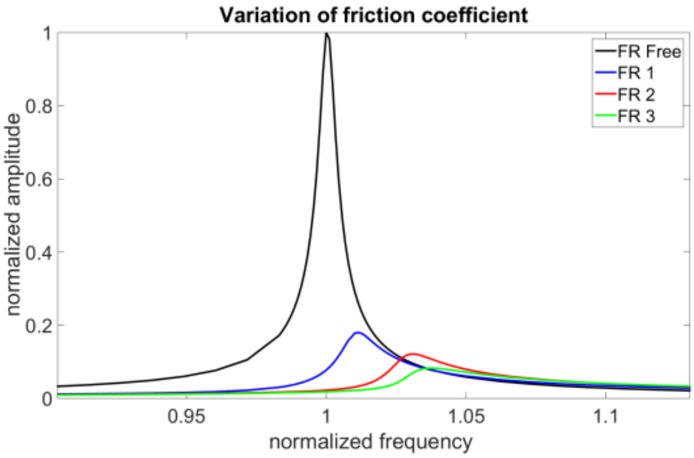

Figure 6: Variation of μ

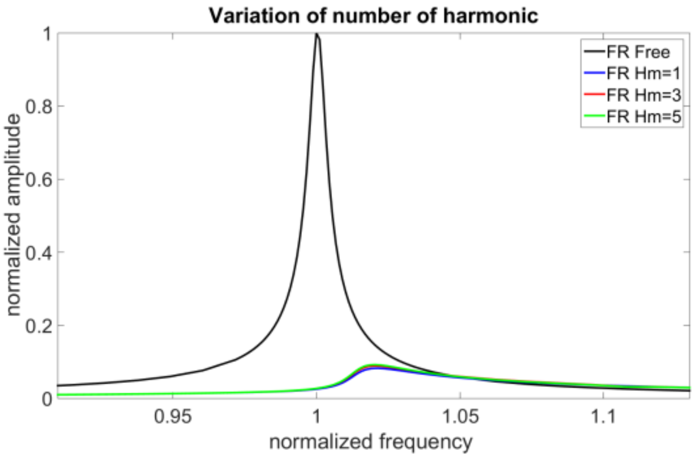

Figure 7: Variation of harmonics

We now take as a reference the value of friction coefficient of the response 3 (the value is not given for reasons of confidentiality). With this value, we have evaluated the effect of the variation of the number of harmonic taken into account during the resolution of the equations of motion. It is possible to see (**Figure 13**) a slight increase of the forced response when the number of harmonic is increased to three but no further increase is found if the number of harmonics is five. However, the increase of the number of harmonic do not produce a significant improvement of the approximation of the response. This means that is not necessary to add harmonics higher than one for a good approximation of the nonlinearities using the HBM produced by the friction contacts.

Figure 14 shows the comparison between different nonlinear response by varying the value of the interlocking interference. Also these analyses have been made using only the first harmonic of the Fourier series (to contain the calculation time which increases fast with the number of harmonics taken into account) fixing the highest value of friction coefficient. Similarly to what happened in the case of variation of friction coefficient, the nonlinear responses amplitudes are lower than the fully stick response of **Figure 11,** moreover there is a decrease of the amplitude of resonance when the value of interference increases.

The variation of interlocking interference plays a similar role to the variation of friction coefficient, i.e. small interferences produce low friction forces and, as a consequence, the dissipated energy is not effective to limit the vibration amplitudes because the contact tends to a 'frictionless' contact.

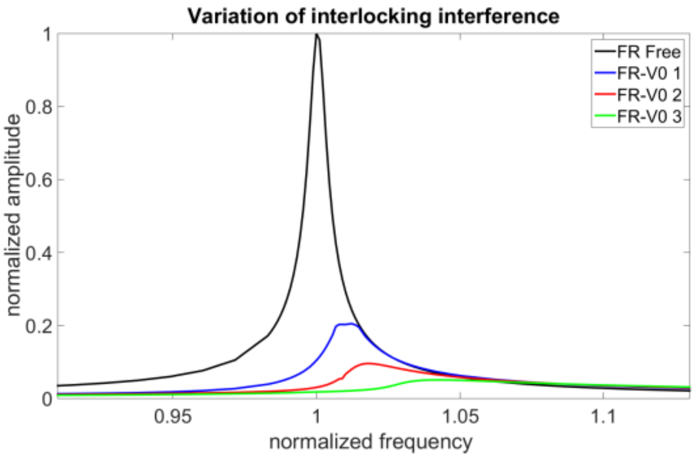

Figure 8: Variation of interference

6.2. Couple/uncouple comparison

In this section the difference between the two types of approaches are investigated. For this reason two conditions are analyzed: forced responses varying the friction coefficient (**Figure 15**) and forced responses varying the interlocking interference (**Figure 16**). The way to realize this comparison is explained in the section 6.1 where the response in the fully stick condition is shown for the two methods.

Figure 15 shows the comparison of the forced response varying the friction coefficient between the coupled methods (continuous lines) and the uncoupled method (dashed lines). It is possible to see how the qualitative response of the two approaches is similar, i.e. the response has a similar form in the two methods varying the frequency of the exciting force. However, the uncoupled responses, fixed the contact parameters, show a shift of the resonance peak to higher frequency than the frequency found using the uncoupled method with a stronger decrease of the amplitude. This behavior is due to the fact that, in the uncouple method, the pre-load acting on the contact elements of the interlocking is set to a fixed value for the whole simulation and it is independent from the dynamic behavior of the system while in the coupled approach the static behavior is influenced by the dynamic one and vice-versa. **Figure 16** shows a similar result as the **Figure 15** and this is correct because the effect of the variation of the interference is the same of the variation of the friction coefficients (see **Figure 14**).

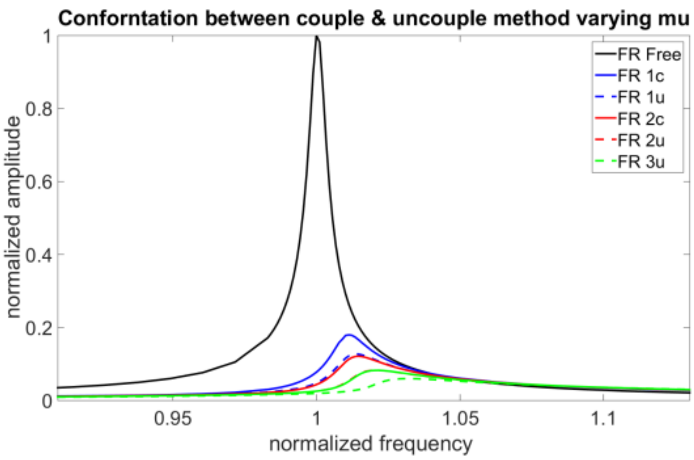

Figure 15: Couple/uncouple comparison varying μ

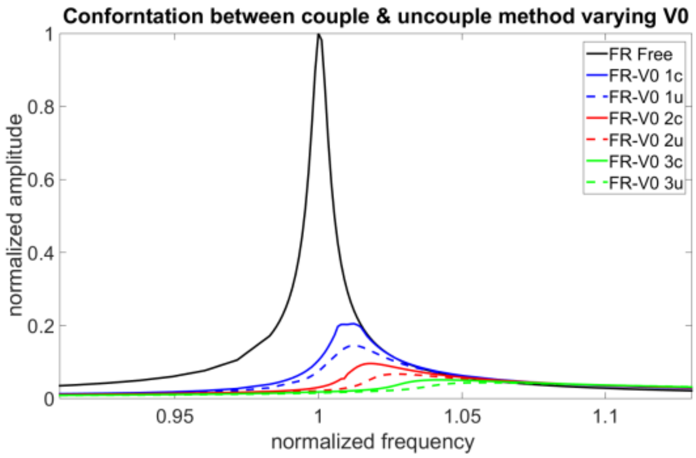

Figure 16: Couple/uncouple comparison varying interference

7. CONCLUSION

The calculation code for the study of the nonlinear dynamics of a vane-segment in the presence of friction damping is very complex because it introduces many numerical methods that try to reduce and simplify the system without reducing the level of quality of the approximation.

A new approach has been implemented to increase the level of approximation in the numerical simulations. To validate this new approach the simulated responses of the model have been compared with the responses obtained from an uncoupled method based on MHBM that is widely used to calculate the forced responses of the rotor and static blades in the aeronautic engines.

The comparison between the coupled and the uncoupled approaches shows that the coupled method results more conservative than the uncouple one, at the same time an increase of the calculation time has to be expected. However, if the velocity of calculation is not a priority, it would be better to use a more accurate model as the one that is realized with the coupled method.

ACKNOWLEDGMENTS

The work described in this paper has been developed within the MIUR CLUSTER ‘Greening the Propulsion’ project funded by the Italian Government in collaboration with Avio Aero within the Politecnico di Torino-GE Avio Aero partnership ‘Greatlab’ (GReen Engine for the AirTransport LABoratory).

REFERENCE

[1]Petrov E.P., Ewins D.J. (2004). “State-of-the-art dynamic analysis for non-linear gas turbine structures”, *J Aerospace Eng. In: Proceedings of the IMechE*, Part G, vol 218, pp 199-211

[2]Ewins D.J. (2010). “Control of vibration and resonance in aero engines and rotating machinery – An overview”, *International Journal of Pressure Vessels and Piping*, vol 87, pp 504-510

[3]Petrov E.P. & Ewins D.J. (2006). “Effects of damping and varying contact area at blade-disc joints in forced response analysis of bladed disk assemblies”, *Journal of Turbomachinery*, Vol.128, No.2, (April 2006), pp. 403-410, ISSN 0889-504X

[4] Zucca, S. , Firrone, C.M. , Gola, M.M., “Numerical assessment of friction damping at turbine blade root joints by simultaneous calculation of the static and dynamic contact loads”, Nonlinear Dynamics Volume 67, Issue 3, February 2012, Pages 1943-1955.

[5]Petrov E.P. & Ewins D.J. (2003). "Analytical formulation of friction interface elements for analysis of nonlinear multiharmonic vibrations of bladed discs", *Transactions of ASME Journal of Turbomachinery*, Vol.125, No.2, (April 2003), pp.364-371, ISSN 0889-504X
[6]Firrone C.M.; Zucca S. & Gola M.M. (2011). "The effect of underplatform dampers on the forced response of bladed disks by a coupled static/dynamic harmonic balance method", *International Journal of Non-Linear Mechanics*, Vol.46, No.2, (March 2011), pp.363-375, ISSN 0020-7462
[7] Firrone, C.M. "Measurement of the kinematics of two underplatform dampers with different geometry and comparison with numerical simulation" Journal of Sound and Vibration Volume 323, Issue 1-2, 5 June 2009, Pages 313-333
[8]Zucca S., Filippi S, Droetti F, Gola M.M, "Forced Response of Interlocked Vane Segments: Numerical Prediction and Experimental Results", *8th Biennal ASME Conference on Engineering Systems Design and Analysis*, Torino, Italy July 4-7, 2006, Vol. 3, pag 413- 420, ISBN: 0791842509, DOI: 10.1115/ESDA2006-95551
[9]Tran D.M.; "Reduced models of multi-stage cyclic structures using cyclic symmetry reduction and component mode synthesis", *Journal of Sound and Vibration*, Volume 333, Issue 21, 13 October 2014, Pages 5443–5463
[10]Tran D.M., "Component mode synthesis methods using interface modes. Application to structures with cyclic symmetry", *Computers and Structures*, Vol. 79, 2001, pp. 209-222
[11]Guyan R.J., "Reduction of stiffness and mass matrices", *AIAA Journal*, Vol. 3 (2), 1965, pp. 380.
[12]Cardona, A.; Lerusse, A. & Geradin, M. (1998). "Fast Fourier Nonlinear Vibration Analysis", *Computational Mechanics*, Vol.22, No.2, (August 1998), pp. 128-142, ISSN 0178-7675
[13]Griffin J.H. (1980). "Friction damping of resonant stresses in gas turbine engine airfoils", *Journal of Engineering for Power*, Vol.102, No.2, pp.329-333, ISSN 0022-0825
[14] Zucca S.; Firrone C.M. (2014). "Nonlinear dynamics of mechanical systems with friction contacts: Harmonic Balance Method and multiple solutions", *Journal of Sound and Vibration*, Volume 333, Issue 3, 3 February 2014, Pages 916-926
[15]Siewert C.; Panning L.; Wallaschek J. & Richter C. (2009). "Multi-harmonic forced response analysis of a turbine blading coupled by nonlinear contact forces", *Proceedings of ASME Turbo Expo 2009: Power for Land, Sea and Air*, June 8-12, 2009, Orlando, Florida, USA
[16]Cameron, T. M., Griffin, J. H. (1989). "An Alternating Frequency/Time Domain Method for Calculating the Steady-State Response of Nonlinear Systems", *Journal of Applied Mechanics*, Vol. 56, pp. 149-154, 1989
[17]Poudou, O., Pierre, C.: Hybrid (2003). "Frequency-Time Domain Methods for the Analysis of Complex Structural Systems with Friction Damping", *Collection of Technical Papers - AIAA / ASME / ASCE / AHS / ASC Structural Dynamics and Materials Conference*, Norfolk (USA), Vol. 1, pp. 111-124, 2003
[18]Lassalle M.; Firrone C.M. (2015). "Nonlinear forced response of stator vanes with multiple friction contacts", *AIAS Conference 2015*, Messina, Italy September 2-5, 2015.