ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

LAYOUT OPTIMIZATION OF PIEZOELECTRIC ELEMENTS WITH EXTERNAL ELECTRIC CIRCUITS IN SMART CONSTRUCTIONS BASED ON SOLUTION OF THE NATURAL VIBRATIONS PROBLEM

Nataliya A. Iurlova [1], Valerii P. Matveenko[2], Dmitriy A. Oshmarin[2], Nataliya V. Sevodina[2], Maksim A. Yurlov [2]

[1] Institute of Continuous Media Mechanics of the Ural Branch of RAS
Acad. Korolev str., 1, Perm, 613013 Russia
e-mail: yurlova@icmm.ru

[2] Institute of Continuous Media Mechanics of the Ural Branch of RAS
Acad. Korolev str., 1, Perm, 613013 Russia
e-mail: mvp@icmm.ru, oshmarin@icmm.ru, natsev@icmm.ru, yurlovm@icmm.ru

Keywords: Electroviscoelesticity, Piezoelectric element, Natural vibrations, Optimal placement, Electric potential

Abstract. *The presence of the elements, which work as sensors or actuators, is one of the main features of smart-constructions. These elements are commonly made of piezoelectric materials, the main feature of which is a direct and inverse piezoelectric effect, which allows its functioning as a sensor and actuator. In addition, one and the same element can combine both functions. Moreover electroding of surfaces of the piezoelectric elements provides an additional means of controlling the dynamic processes due to application of different electric circuits connected to piezoelectric elements. The location of the piezoelectric element, its size and geometry play a significant role in damping the vibration of structures containing elements made of piezoelectric materials. In this paper, a comparative analysis of different possible options of smart-function realization is made by comparing the patterns of strains and electric potentials generated on the piezoelectric element surface. The numerical results of determining the optimal piezoelectric element location, which will provide the generation of maximal electric potential at certain resonant frequencies, are presented. The numerical implementation is carried out by the finite element method using the commercial ANSYS software package.*

1 INTRODUCTION

In the last few years the method of damping the structure vibrations based on converting the energy of mechanical vibrations into the electrical energy via piezoelectric elements embedded in the structure and connected to the external electric circuits has been the area of extensive research in mechanics. The piezoeletric effect provides a transformation of part of the mechanical vibration energy into the electrical energy, which is then dissipated through the external electric shunt circuit operating as a mechanism of passive vibration damping [1]. Changing the parameters of the external circuit one can suppress the vibrations at the specified frequency.

Piezoelectric elements should provide the best execution of the ultimate purpose of their application in smart-structures, that is, damping of structure vibrations, controlling the shape of structures subject to external impacts of different nature, detecting the structural defects, etc. The efficiency of peizoelement performance depends not only on the control parameters (the corresponding schemes of external electric circuit connection) but also on the arrangement of piezoelectric elements serving as sensors and actuators [2].

Depending on the criterion, the optimal schemes for location of sensors (actuators) can differ from one another. The optimization criterion can use as a basis different factors: maximization of forces, moments or deformations transferred by a peizoelement playing the role of the actuator, slight variation of dynamic characteristics of the original structure, selection of desired dynamic characteristics for the original structure, minimization of control function, maximal damping of original structure vibrations, etc.

Traditionally, the passive vibration dampers are located in the regions of the structure, in which the strain energy is maximal [3]. As a rule, these zones are found based on the analysis of structure deformation patterns. Nevertheless, the problem of optimal arrangement of piezoelectric elements is in the center of attention of many researchers, whose studies in this area are devoted to the development of both the optimization algorithms and criteria for optimal arrangement of piezoelectric elements.

It should be noted that thus far no universal approach to the examined problem has been developed due to the necessity of taking into account a variety of factors: characteristics and geometry of piezoelements, their arrangement in the structure, the number of piezoelements, etc. In work [4], which is a survey of more than 100 papers and in work [5], which makes reference to about 50 papers devoted to the problem of piezoelement arrangement in structures, the authors advance an opinion that despite the abundance of methods and approaches to this problem, the appropriate location of a sensitive element has long been a matter of skill that relies on researcher's intuition. Such an approach contradicts the intention of design engineers to reduce the number of intuitive solutions. In view of this fact, work [4] presents and systemizes all previously developed criteria for optimal arrangement of peizoelements in from the viewpoint of minimization of intuitive approaches to this problem.

As mentioned in [4], an advanced approach to the problem of appropriate arrangement of sensors (actuators) in the structure is based on the six optimization criteria: maximization of modal forces or moment transferred through the actuators; maximization of strains in the original structure; minimization of control forces or maximization of dissipated energy; maximization of the degree of controllability; maximization of the degree of observability and minimization of side effects. The most general recommendations on the optimal location obtained on the basis of these criteria are systemized in the review paper [4] for beam and plate smart-structures.

The application of voltage to the piezoelectric element placed on the structure surface causes transverse deformations (deflection) of the structure. The transverse deformation of the initial structure is thus the function of the sensor (actuator) position. Hence, as it was shown in works [6-8], the transverse deformation of the initial structure can be used as a criterion for selection of the optimal location of sensors or actuators.

In work [9] the authors offer the strategy, which is based on the mathematical analysis of the deformation fields under structure vibrations and pursues the aim of finding the regions of maximal deformation which are equal in size to a piezoelement. This strategy was applied to a structure of complex geometry made of anisotropic material with locally varying properties and a small value of the internal material damping ratio.

In what follows, we restrict the discussion to the problem of piezoelement location only as a means of damping the vibration in the original structure.

The distribution of damping energy over the volume of the examined structure gives a rather clear idea of the best location for a passive damping element. However, it should be noted that the modal analysis of the electroviscoelastic bodies composed of structural materials with the embedded piezoelectric element allows us to obtain information on the relative electric potential generated on the electroded surfaces of piezoelements, which can be also used for identification of optimal variants of piezoelement location.

In works [10-12] it has been noted that the location of piezoelements operating as sensors in the zones, which are unsuitable for generation of considerable electric potential, can even impair the rate of vibration damping.

In paper [3], the optimal location of a piezoelement is determined by taking into account both the electromechanical coupling coefficient and the value of the electric potential generated on the piezoelement surface. However, the authors of this paper restrict their consideration to the case when the selected location of the piezoelement is optimal only for suppressing the first mode of beam vibration.

The objective of this paper is to construct an algorithm, which will allow us to determine the piezoelement location such that provides generation of the maximal relative electric potential in the structure experiencing deformation at the prescribed frequency. This is essential for the purpose of providing maximal damping of vibrations at this particular frequency using the external shunt circuits. A search for an optimal location of the piezoelement is carried out in the framework of the natural vibration problem, which is formulated for electrovicoelastic body with the external electric circuits.

2 MATHEMATICAL STATEMENT OF THE NATURAL VIBRATION PROBLEM

Consider a piecewise-homogeneous body of volume $V = V_1 + V_2$, where the volume V_1 consists of homogeneous elastic parts and the volume, the volume V_2 - consists of piezoelectric elements. The piezoelectric elements are connected through the electroded surfaces to RLC-circuit of arbitrary structure comprising a resistance, capacitance and inductance elements.

For the problem of natural vibration we seek a solution written as

$$u_i\left(x,t\right)=\overline{u}_i\left(\vec{x}\right)e^{-i\omega t},\varphi_i\left(x,t\right)=\overline{\varphi}_i\left(\vec{x}\right)e^{-i\omega t} \tag{1}$$

where ω - is the eigenfrequency of vibrations, $\overline{u}_i\left(\vec{x}\right),\overline{\varphi}_i\left(\vec{x}\right)$ - are the eigenmodes of displacements and electric potential for the elastic sub-regions V_1 and V_2.

A detailed mathematical formulation of the problem is given in works [13-14].

The numerical implementation of the stated variational problem was carried out by the finite element method using the commercial ANSYS software package.

In the finite element formulation the variational problem reduces to the algebraic eigenvalue problem

$$\left(\left[K\right]-\omega^2\left[M\right]\right)\{u\}=0 \tag{2}$$

The condition of the existence of nontrivial solution to equation (2) is

$$D(\omega)=\det\left(-\omega^2\left[M\right]+\left[K\right]\right)=0 \tag{3}$$

Here, we accepted the following notation: $\{u\}$ - is the vector of nodal displacements, ω - is the eigenfrequency, $\left[K\right]$ – is the stiffness matrix, $\left[M\right]$ – is the mass matrix.

In the problem of electroviscoous elasticity each point of the body is associated with the vector of state $\{u\}=\{u_1,u_2,u_3,\varphi\}^T$, where u_i are the components of the displacement vector, φ is the electric potential.

Out of other factors, determining the effectiveness of piezoelement performance in smart structures, we used the value of the electric potential generated on the electroded surfaces of piezoelements during the dynamic processes.

The results obtained from the solution of the natural vibration problem formulated for electroelastic bodies allow us to estimate the value of the electric potential generated on the surface of the piezoelement undergoing deformation.

The relative value of the electric potential can be estimated using the relative value of the strain energy in the zone of piezoelement location. The optimal values of the relative strain energy can be also obtained from the modal analysis.

At small dimensions of piezoelements, the attachment of which to the structure does not cause essential changes in the character of natural vibrations, the relative values of the strain energy in the zone of piezoelement location can be estimated using the eigenmodes of the structure including no piezoelements. i.e., based on the results of solving the problem of natural vibrations of elastic or viscoelastic bodies.

The value of the relative strain energy J in the zone of piezoelement location is defined by the strain distribution ε_r over the structure surface under the piezoelement

$$J=\int_{S_n}\varepsilon_r ds \tag{4}$$

where S_n is the area of the piezoelement, ε_r is the deformation in the direction perpendicular to the structure surface.

The application of voltage to a piezoelement attached to the surface of the structure induces a transverse deformation (deflection) of the structure. Note that the arising transverse deformation of the original structure depends on the location of the piezoelement. Hence, as was stated in review paper[4], the transverse deformation of the original structure can be used as a criterion for the optimal location of the piezoelement.

In work [14], it was stressed that from the viewpoint of computational costs the problem of natural vibrations is most efficient for determining the relative values of the electric potential generated on the surface of the piezoelement. The described approach was applied to plane objects in the form of plates. It has been shown that in this case, to provide a proper work of the piezoelement, it is necessary to locate it asymmetrically about the structure axis (if there is any).

From the analysis of literature it follows that the efficiency of peiezoelement performance in the smart-structures is often estimated in terms of the electromechanical coupling coefficient, which reflects the efficiency of converting the mechanical energy of vibrations into the electrical energy [1, 3, 15].

The coefficient of electromechanical coupling can be determined based on the eigenfrequencies of the structure with piezoelement in the open circuit regime (termed as o/c regime) and in the short-circuit regime (s/c regime). The o/c regime is realized in the case when the lower surface of the piezoelement is grounded (the lower surface potential is equal to zero) and the upper surface is free of load. In the s/c regime the potential prescribed on the electroded surfaces is equal to zero. As a result, the electromechanical coupling coefficient is evaluated by formula (5) [1]

$$K = \sqrt{\frac{\omega_{o/c}^2 - \omega_{s/c}^2}{\omega_{s/c}^2}} \tag{5}$$

The larger is the coefficient, the more effective is the energy conversion and thus the larger is the quantity of energy that can be dissipated by the elements of the external circuit.

As it was shown in works [1, 3, 15], the location of the piezoelement can be considered optimal in the case when the electromechanical coupling coefficient determined by formula (5) takes a maximum value.

At the same time, the larger is the value of the relative electric potential generated on the surface of the piezoelement undergoing deformation, the more effective is its performance in the structure. Hence, the location of the piezoelement on the surface of the structure, providing the generation of the maximal electric potential, is considered to be optimal.

In turn, as the value of the relative electric potential is specified by the degree of the piezoelement deformation, it should correlate with the value of the relative strain energy in the zone of piezoelement location. This observation has been supported in work [14] for plane objects in the form of plates.

In the present work, we have calculated the electromechanical coupling coefficient and determined the value of the relative electric potential and the value of the integral J, which characterizes the damping energy in the zone of peizoelement location defined by the distribution of the strain ε_r over the surface of the structure under the piezoelement.

The coefficient of the electromechanical coupling is the absolute quantity in contrast to the electric potential and strain energy, which are the relative quantities. This allows us to make a qualitative comparison of the obtained results.

3 NUMERICAL RESULTS

In this study we investigate a thin-walled shell in the form of a half cylinder having the following dimensions: $R = 76mm, L = 300mm, h = 0.25mm$ (fig.1). The shell was made of the elastic isotropic material displaying the following physic-mechanical characteristics: $E = 1.96 \cdot 10^{11} Pa; \nu = 0.3; \rho = 7700 kg/m^3$. The piezoelement was made of PZT-4 ceramics in the form of a ring segment, the physic-mechanical characteristics of which are given below in the cylindrical coordinates

$$C_{11} = 13.5 \cdot 10^{10}\, N/m^2 \qquad C_{12} = C_{13} = 7.43 \cdot 10^{10}\, N/m^2 \qquad C_{23} = 7.78 \cdot 10^{10}\, N/m^2$$
$$C_{22} = C_{33} = 13.9 \cdot 10^{10}\, N/m^2 \qquad C_{44} = C_{66} = 2.56 \cdot 10^{10}\, N/m^2 \qquad C_{55} = 3.06 \cdot 10^{10}\, N/m^2$$
$$\beta_{21} = \beta_{31} = -5.2\, C/m^2 \qquad \beta_{11} = 15.2\, C/m^2 \qquad \beta_{42} = \beta_{63} = 12.7\, C/m^2$$
$$э_{22} = э_{33} = 6.45 \cdot 10^{-9}\, F/m \qquad э_{11} = 5.62 \cdot 10^{-9}\, F/m \qquad \rho = 7500\, Kg/m^3$$

and has dimensions: r =*76.25 mm,* φ =*15.08*^{0}*, h=1.2 mm.*

The upper and lower surfaces of the piezoelement are electroded and the polarization axis is aligned with the r -axis. The shell is rigidly clamped at the ends and freely supported at the generating line. (fig.1).

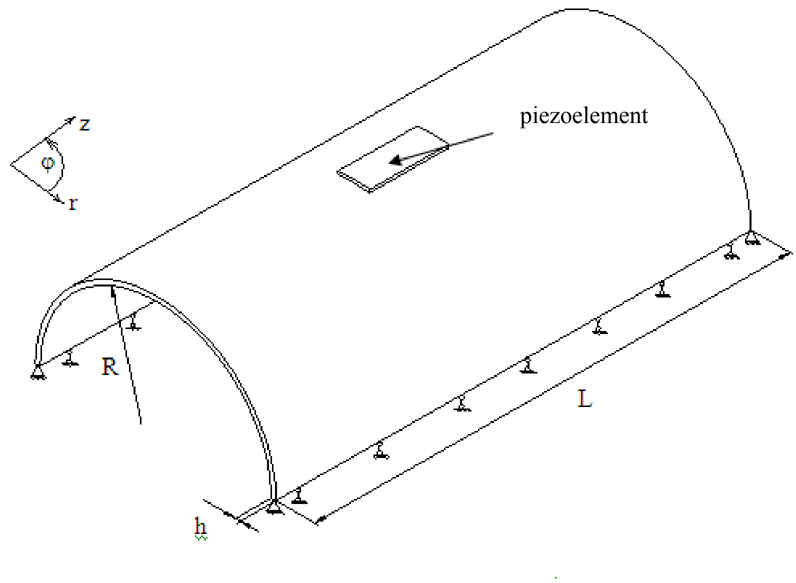


Figure 1: Schematics of the connected piezoelement

Let us estimate the efficiency of piezoelement performance using only two parameters: the value of the relative electric potential generated on the surface of the piezoelement under deformation and the value of the electromechanical coupling coefficient.

The optimal location of the piezoelement was determined by the scanning method using the previously proposed approach [13-14], which is based on solving the natural vibration problem.

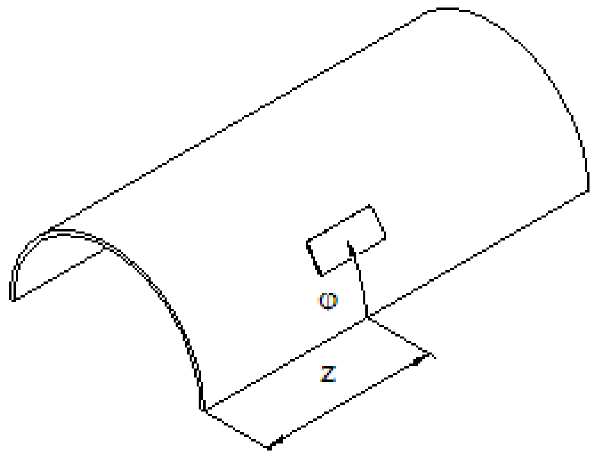
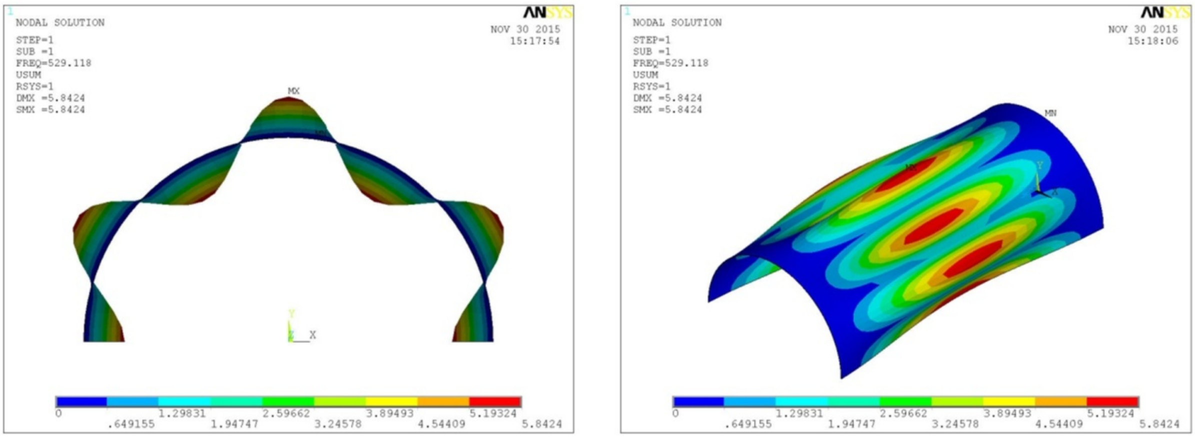


Figure 2: The directions of angular (φ) and length-wise (z.) variations in the coordinates of the piezoelement mass center.

To this end, a series of computations was performed for different variants of piezoelement location, which was determined by the angular and transverse coordinates of its mass center (fig.2). The range of angular variation of the coordinate is $\varphi \in [10°,170°]$ with a step of

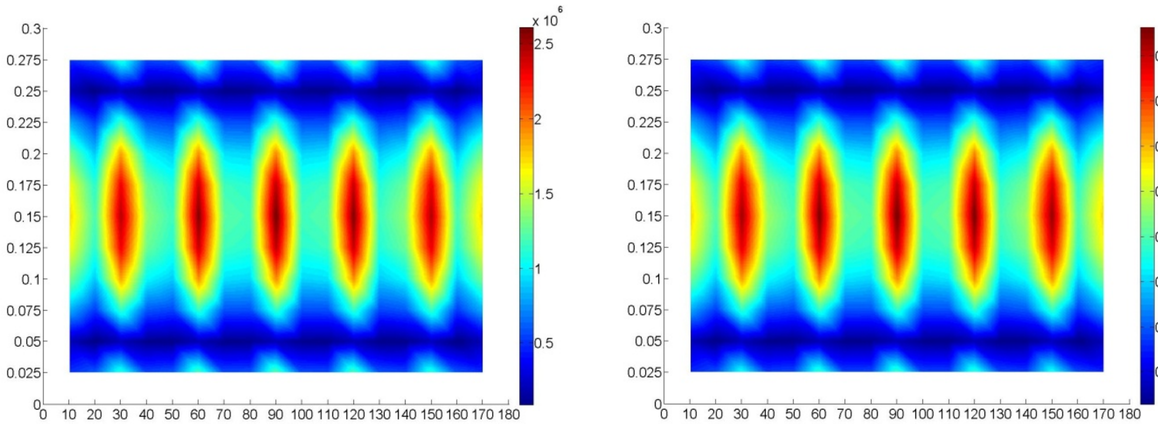
10˚.The range of length-wise variation of the coordinate is $z \in [25, 275]$ mm with a step of 25mm.

Figure 3 presents the shape of the first eigenmode of the system.



(a) (b)

Figure 3: The first vibrational eigenmode.



(a) (b)

Figure 4: Distribution of the values of the relative electric potential values (a) and electromechanical coupling coefficient (b) for the first vibrational mode of the shell.

Figure 4 presents images of the distributions of the computed relative electric potential (fig.4, a) and electromechanical coupling coefficient (fig.4,b) for the first vibration mode, which are used as the criteria of finding regions for the optimal location of the piezoelement on the shell surface.

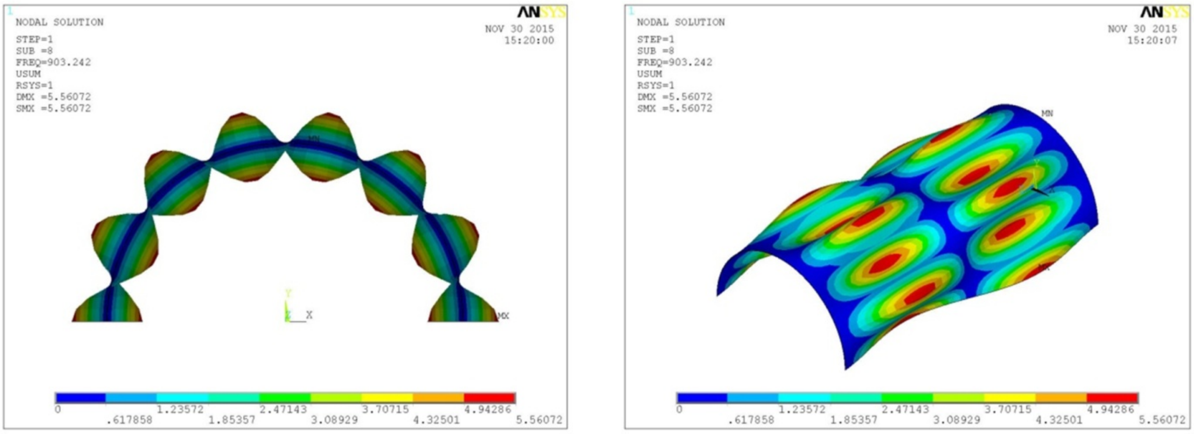
To demonstrate this more clearly, the surface of the shell is unrolled onto a plane- the angular coordinate φ is laid off along the x-axis, and longitudinal coordinate z is laid off along the y-axis.

The analysis of the relative electric potential distribution (fig.4,a) allows us to conclude that the piezoelement performs at its best in the case when its mass center is located at the point with coordinates $\varphi = 90^\circ$, $z = 150$ *mm*.

Figures 4(a) and 4(b) demonstrate a complete qualitative agreement between the results of calculation with respect to two different parameters - electric potential and electromechanical coupling coefficient, which allows us to draw a conclusion about the equivalency of both parameters for deciding the question of optimal location of the piezoelement on the structure surface.

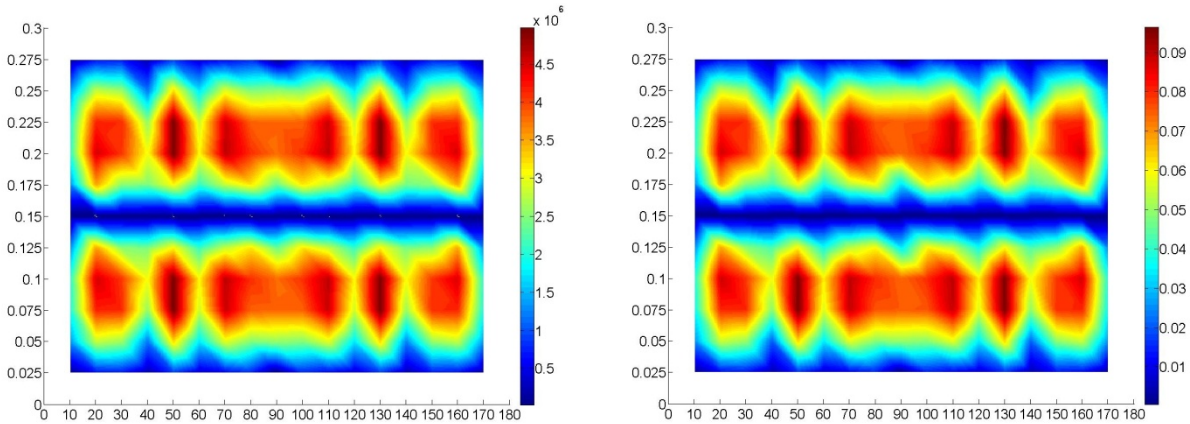
It should also be emphasized that the images of the distribution of the above parameters (the relative electric potential and electromechanical coupling coefficient) qualitatively correlate with the vibration mode: the points, at which the relative electric potential has the highest values coincide with the convexity points (points of maximal displacement amplitudes) of the vibration mode.

Note that for the shell frequencies from the second to the fourth order one can observe the same picture of distributions of the relative electric potential and electromechanical coupling coefficient, which determine the regions of optimal location of the piezoelement on the shell surface.



(a) (b)

Figure 5: The eighth vibrational eigenmode of the shell.



(a) (b)

Figure 6: The distribution of the values of the relative electric potential (a) and the electromechanical coupling coefficient (b) for the eighth vibrational eigenmode of the shell.

As another constitutive parameter we used the eighth mode of the shell, which is shown in fig.5. It also presents the results of computation of the relative electric potential (fig.6,a) and the electromechanical coupling coefficient (fig.6,b) for the mode of vibration which specifies the regions of optimal location of the piezoelement. As is evident from the figure, the correlation between the distributions of the above parameters and vibrational mode of the shell is rather good.

According to the results displayed in fig. 6 (a, b) for the eighth vibrational mode, there are 4 optimal zones for the location of the piezoelement. For these zones the coordinates of the piezoelement mass center are as follows: $\varphi=50^\circ$, $z=87.5mm$; $\varphi=50^\circ$, $z=212.5$ mm; $\varphi=130^\circ$, $z=87.5mm$; $\varphi=130^\circ$, $z=212.5$ mm. At the same time the situation, similar to the one in fig. 6, is observed for the vibration frequencies of the shell from the fifth to the eighth one.

From the comparison of the results of computation presented in fig. 4and 6 it can be inferred that the location of the piezoelement such that its subsequent connection to the external electric circuit can provide effective damping of the vibrations at the first frequency, proves to be improper for damping vibrations at the eighth frequency and vice versa, the piezoelement location being optimal for damping of the vibration at the eighth frequency will fail to provide good performance of the piezoelement at the first frequency.

4 CONCLUSIONS

- In this paper, we have considered and verified the possibility of applying the natural vibration problem for electroviscoelastic bodies for optimization of smart structures based on the identification of appropriate zones for piesoelement location such that ensures the generation of the maximal relative electric potential at the corresponding vibrational mode.
- It has been shown that the results of determining the best location of the piezoelement on the structure surface providing a maximum increase in the efficiency of its performance based on the values of the relative electric potential generated at the electroded surfaces of the piezoelement and the electromechnical coupling coefficient are equally applicable for this particular optimization problem. The advantage of using the electromechanical coupling coefficient is that the computation yields its true values compared to the electric potential, the values of which are evaluated up to a factor. However, the latter criterion has one essential disadvantage - the necessity of performing two times as many calculations because of the need to determine the eigenfrequencies for two regimes - open circuit and short circuit regimes. In contrast, the value of the electric potential for each location can be obtained from a single computation run, namely in the open circuit regime.

The work supported by RFBR (projects № 15-01-03976, 14-01-96003 *r-ural*_a,) and UB RAS complex program (project 15-10-1-18).

REFERENCES

[1] N.W. Hagood, A. von Flotow, Damping of structural vibrations with piezoelectric materials and passive electrical networks. *Journal of Sound and Vibration* **146**(2), 243–268, (1991).

[2] Crawley E.F., de Luis J. Use of Piezoelectric Actuators as Elements of Intelligent Structures// *AIAA Journal*, **25**(10), 1373–1385, 1985.

[3] S. Venna, Y.-J. Lin. An Effective Approach for Optimal PZT Vibration Absorber Placement on Composite Structures, *Modern Mechanical Engineering* **3**, 21-26, 2013.

[4] V. Gupta, M. Sharma, N. Thakur. Optimization Criteria for Optimal Placement of Piezoelectric Sensors and Actuators on a Smart Structure: A Technical Review, *Journal of Intelligent Material Systems and Structures*, **21**(8), 1227-1243, 2010.

[5] F. Bachmann, A.E. Bergamini, P. Ermanni. Optimal piezoelectric positioning a strain-energy based finite element approach, *Journal of Intelligent Material Systems and Structures*, **23**(14), 1575–1591, 2012.

[6] J.C. Bruch Jr., J.M. Sloss, S. Adali and I.S. Sadek, Optimal piezo-actuator locations/lengths and applied voltage for shape control of beams. *Smart Mater. Struct*. **9**, 205-211, 2000.

[7] V.M.F. Correia, M.A.A Gomes, A. Suleman, C.M.M. Soares, C.A.M. Soares, Modelling and design of adaptive composite structures. *Computer Methods in Applied Mechanics and Engineering,* 185(2-4), 325-346, 2000.

[8] V.M.F. Correia, C.M.M. Soares, C.A.M. Soares, Refined models for the optimal design of adaptive structures using simulated annealing. *Composite Structures*, 54(2-3), 161-167, 2000.

[9] P. Kostka, K. Holeczek, W. Hufenbach, Structure-integrated Active Damping System: Integral Strain-based Design Strategy for the Optimal Placement of Functional Elements. *International Journal of Composite Materials*. **3**(6B), 53-58, 2013.

[10] R.E. Holman, S.M. Spencer, E.M. Austin, C.D. Johnson, Passive damping technology demonstration, *SPIE*, **3045**(1), 51-59, 1997.

[11] J.J. Hollkamp, R.W. Gordon, An experimental comparison of piezoelectric and constrained layer damping, *Smart Mater. Struct*., **5**, 715–722, 1996.

[12] S. Wu, T.L. Turner, S.A. Rizzi, Piezoelectric shunt vibration damping of F-iS panel under high acoustic excitation, *SPIE*, **3989**(2), 276-287, 2010.

[13] V.P. Matveenko, E.P.Kligman, N.A. Yurlova, M.A. Yurlov, Simulation and optimization of dynamic characteristics of piezoelectric smart structures. *Physical Mesomechanics,* **15**(3–4), 190-199, 2012.

[14] N.V. Sevodina, N.A. Yurlova, D.A. Oshmarin, The optimal placement of the piezoelectric element in a structure based on the solution of the problem of natural vibrations. *Solid State Phenomena*, **243**, 67-74, 2015.

[15] M. A. Trinidade, A. Benjeddou, Effective electromechanical coupling coefficients of piezoelectric adaptive structures: critical evaluation and optimization. Mechanics of Advanced Materials and Structures, **16**(3), 210-223, 2009.