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DETERMINATION OF OPTIMAL PARAMETERS FOR A PASSIVE RL-CIRCUIT BY SOLVING THE PROBLEM ON NATURAL VIBRATIONS OF ELECTROELASTIC BODIES

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Abstract. *The use of elements made of piezoelectric materials and connected to external passive electric RLC-circuits is one of the effective ways of damping the vibrations of structure elements. The key problem for this technique is identification of optimal parameters for the elements of the external circuits, which can provide maximal damping of a single mode or simultaneous damping of several modes. In this study, based on the mathematical formulation of the natural vibration problem for electroelastic bodies with external electric circuits we have developed an approach for evaluating the parameters of the external RL-circuit which can provide maximal damping of the particular vibration modes. A criterion for determining the external circuit parameters has been proposed based on the analysis of the obtained vibrational complex eigenfrequencies. For implementation of the proposed approach a finite-element algorithm has been constructed using the commercial ANSYS software package. A series of numerical experiments have been carried out for e few objects to test the performance of the developed technique.*

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1 INTRODUCTION

Damping capacity of materials, namely their ability to absorb vibrations, plays an important role in the dynamic behavior of structures [1]. A qualitative estimate of dissipative properties of structures generally relies on the results of solving two problems. The first problem deals with consideration of natural vibrations. Here the system dissipation manifests itself as vibration damping and the rate of damping qualitatively evaluates the dissipative properties of the system. The second problem is concerned with the forced steady-state vibrations. In this case, the dissipative properties of the system reveal themselves in the restriction of resonance amplitudes.

A search for optimal structures with maximal damping capacity using the numerical simulation methods requires heavy computation. On the one hand, it is necessary to investigate the influence of parameters on the damping properties of the system in the examined range of vibrations. On the other hand, one needs to analyze the behavior of the structure in a certain range of dynamic actions for each combination of these parameters. In the case of natural vibration this implies finding solution to the dynamic problem under different initial conditions, whereas in the forced vibration problem it is necessary to find solution to the examined problem for different loading regimes in the specified frequency range of vibrations.

An alternative to the above approaches, including the method of solving the optimization problem is the problem of natural vibrations allowing us to estimate damping properties of the system, irrespective of external force impacts, kinematic and other actions. In this case the eigenfrequencies are the complex quantities. Their real parts are the frequency and the imaginary parts are the damping ratio (the damping rate) of natural vibrations.

The object of investigations is a piece-wise homogeneous body composed of elastic and viscoelastic elements. The dissipative properties and resonance frequencies of the system can be controlled by the mechanical properties of the material, geometrical parameters of the system and boundary conditions. The employment of new materials in structure, for example functionally graded materials (smart-structures) makes it possible to extend the list of control parameters.

In the present paper, we consider complex structures, the elements of which are made of piezoelectric material. A distinctive feature of the structure is that the electroded surfaces of piezoelectric elements are located at certain points of the structure and connected via external electric circuits, comprising a resistor, (resistance), inductor (inductance) and capacitor (capacitance), to zero potential point.

The piezoelectric elements coupled with a shunt circuit and connected to a mechanical structure serve as the devices that provide energy dissipation provoking thus additional damping of vibrations [2].

The available literature gives a comprehensive description of the applications of shunt technique for damping vibrations of space structures and helicopter blades, suppressing the nonlinear supersonic panel flutter, controlling the mode shapes, damping the telescope vibrations, turbomachinery blade vibrations, reducing the level of noise in vehicles, suppressing the echo reflected from submarines and vibrations of computer disc drives, etc.

The use of shunt circuits offers some advantages over the traditional methods of damping the mechanical vibrations. Among these is the ease of arranging the external electric circuits generated by a set of standard electronic elements. Sometimes installation of additional electronic elements or power sources is not required at all and, in a few particular cases there is no need in feedback sensors. Furthermore, from the viewpoint of stability and reliability the proposed approach compares favorably with other techniques because it does not require the incorporation of complicated digital processors and in many situations the development of parametric models for designing new vibration damping systems is not necessary either [1].

The simplest shunt circuit is the connection of a resistive element (resistor) via the electrodes. Although the first reference to this method was made in work [3], the idea of vibration damping with the aid of shunting by external electric circuits was attributed to Hagood and von Flotow [4], who showed that a series -connected RL-circuit can essentially weaken a single vibration mode [5].

From a mechanical standpoint it can be said that the RLC-circuits play the part of external elastic and viscoelastic elements. In the context of such formulation the calculated complex eigenvalues determine the frequencies of natural vibrations and damping ratios. The complex frequencies and eigenmodes depend not only on the structural and engineering characteristics of shell systems but also on the parameters of piezoelectric elements, their arrangement schemes and characteristics of the external circuits comprising a resistors, inductor and capacitors. Targeted variation of these parameters allows one to optimize the resonance frequencies and damping ratios

2 MATHEMATICAL STATEMENT OF THE PROBLEM

A complete mathematical statement of the problem for electroviscous piecewise-homogeneous bodies is given in work [6].

The variational equation of motion for a body consisting of elastic and piezoelectric elements is formulated using the relations of the linear elasticity theory and quasi-static Maxwell equations [1, 7-8]:

$$\int_{V_1}\left(\sigma_{ij}\delta\varepsilon_{ij}+\rho\ddot{u}_i\delta u_i\right)dV+\int_{V_2}\left(\sigma_{ij}\delta\varepsilon_{ij}-D_i\delta E_i+\rho\ddot{u}_i\delta u_i\right)dV-\int_{\Omega_\sigma}\delta u_iP_id\Omega-\int_{\Omega_{el}}q_e\delta\varphi d\Omega=0 \quad (1)$$

Here, $\int_{\Omega_{el}} q_e\delta\varphi d\Omega$ - is the work done by the external electric circuit, D, E – are the vectors of the electric inductance and electric field strength; σ^{ij} – the components of a symmetric Cauchy stress tensor, $\varepsilon_{ij}=\frac{1}{2}\left(u_{i;j}+u_{j;i}\right)$ – are the components of the tensor of linear deformations, u_i – are the components of the displacement vector, P_i - are the components of the load vector, q_e and φ are the surface charge density and electric potential, V_1 is the volume of the elastic and viscoelastic parts of the body, Ω_{el} is the surface that bounds the body of volume V_2. Equation (1) is written in terms of the rectangular Cartesian coordinates.

For the electric field the condition of equipotentiality is fulfilled: $\varphi_{,i}=-E_i$.

For isothermal processes in linear electroelastic media the following physical relations hold true:

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl} \quad \text{- for elastic part of volume } V_1 \qquad (2)$$

$$\left.\begin{aligned}\sigma_{ij}&=C_{ijkl}\varepsilon_{kl}-\beta_{ijk}E_k\\ D_k&=\beta_{ijk}\varepsilon_{ij}+e_{ki}E_i\end{aligned}\right\} \text{- for piezoelement of volume } V_2 \qquad (3)$$

Here C_{ijkl} is the tensor of elastic constants, β_{ijk} and e_{ki} are the tensors of piezoelectric and dielectric coefficients. For viscoelastic materials the tensor of elastic constants C_{ijkl} in relation (2) can be replaced by the corresponding viscoelastic operator [9-11].

In the electroelastic problems the boundary conditions can be divided into two groups: mechanical and electrical conditions. In the natural vibration problem the mechanical boundary conditions are written as

$$S_\sigma: \quad \sigma_{ij} n_j = 0, \; S_n: \quad u_i = 0, \tag{4}$$

where $S = S_\sigma + S_n$ is the surface that bounds the volume V of the examined body.

The formulation of the physically realizable electrical boundary conditions depends on the way of electrical energy supply to a piezoelectic element. Both, the supply and release of energy from the deformed piezoelectric element is carried out via the electroded coatings applied to some parts of the body surface. They are assumed to be ideal conductors having negligible mass. Coating of the body surface Ω_{el} with a thin current conducting layer (electroding) ensures its equipotentiality.

$$\int_{\Omega_{el}} \delta\varphi \, q_e \, d\Omega = \delta\varphi \int_{\Omega_{el}} q_e \, d\Omega = \delta\varphi Q^{el} \tag{5}$$

Here Q^{el} is the total electrode charge.

The numerical implementation of the stated variational problem is carried out by the finite element method using the commercial ANSYS software package.

3 NUMERICAL RESULTS

The object of our investigation is a cantilevered plate, the surface of which is coupled with a piezoelement and external series-connected *RL*-circuit shunting its electroded surfaces (fig.1). The plate measures 210x26x0.6 mm and the piezoelectric element measures 50x20x0.36 mm. The piezoelement is located 12mm away from the edge of the plate and symmetrically about its axis of symmetry.

The plate is made of viscoelastic material with the following mechanical characteristics: bulk modulus – *B* =167.7 *GPa*, instantaneous shear modulus – *G* =76.923 *GPa*, loss tangent for the shear modulus *G* - $\delta = 0.01$, specific density - $\rho = 7800$ kG/m^3. The piezoelement is made from piezoceramics PZT-4-z [1] and polarized along the z-axis. The plate and piezoelement are perfectly bonded.

The numerical analysis was carried out for the following structures: the plate taken alone, the plate with the piezoelement connected to its surface, the plate with the piezoelement and external series-connected *RL*-circuit (fig.1,a), the pezoelement by itself with the external electric circuit connected to its electroded surfaces (fig.1,b).

Table 1 presents the values of the first eight eigenfrequencies of the plate with the piezoelement ω_R and the values of the damping ratio ω_I. Note that that column 2 includes the values of eigenfrequencies obtained in the open circuit (o/c) regime and the column 3 includes the values obtained in the short circuit ((s/c) regime.

The developed software package for computing the vibrational eigenfrequencies of electroviscoelastic structures connected to the external electric circuit was used to calculate eigenfrequencies of the examined objects at different values of the resistance R and inductance L of the external electric circuit.

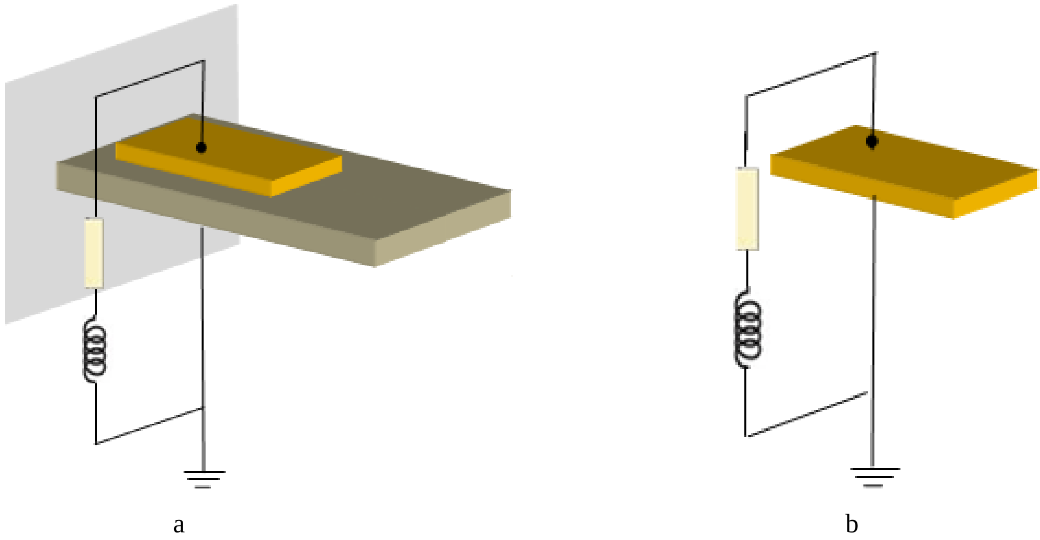

Figure1: Computational scheme; (a) the plate with piezoelement and external series-connected *RL*- circuit (system), (b) the peizoelement with external electric circuit connected to its electroded surfaces (electric circuit).

№	Eigenfrequencies of vibrations			
	Plate $\omega = \omega_R + i\omega_I$	Plate with piezoelement in o/c regime $\omega = \omega_R + i\omega_I$	Plate with piezoelement in s/c regime $\omega = \omega_R + i\omega_I$	Plate with piezoelement and external electric circuit (*R=100* ohm, *L=1000* (H) $\omega = \omega_R + i\omega_I$
	1	2	3	4
1	11.239 - 0.0473i	13.860 - 0.0490i	13.576 - 0.0467i	13.548 - 0.0464i
				47.347 - 0.04089i
2	70.396 - 0.297i	72.334 - 0. 290i	72.149 - 0.2895i	72.472 - 0.2889i
3	177.707 - 0.879i	198.325 - 0. 795i	197.899 - 0.7906i	198.351 - 0.7951i
4	197.271 - 0.830i	204.451 - 0. 874i	204.451 - 0. 8737i	204.451 - 0. 8737i
5	387.268 - 1.622i	404.159 - 1.542i	399.795 - 1.5119i	404.220 - 1.5421i
6	477.558 - 2.064i	489.406 - 2.021i	489.406 - 2.0215i	489.406 - 2.0214i
7	537.424 - 2.649i	584.108 - 2.472i	584.108 - 2.4717i	584.108 - 2.4717i
8	641.704 - 2.672i	674.968 - 2.509i	665.456 - 2.4730i	675.018 - 2.5088 i

Table 1: Spectrum of the system eigenfrequencies

The piezoelectric element possessing the capacitance properties form together with the external electric circuit the electric oscillatory circuit (fig. 1,b).

In the case when the external electric circuit is connected to the plate with piezoelement, this leads to the appearance of additional frequency in the spectrum of eigenfrequencies. The value of this frequency is determined by the values of *R* and *L*.

Since the piezoelectric elements are the reversible mechanical- to- electric transducers, i.e., the devices capable of converting the mechanical energy into the electrical energy and vice versa, a coincidence of the structure vibration frequency with the frequency of the electric oscillatory circuit causes a maximum amount of the mechanical vibration energy to convert into the electrical energy and dissipate in the external electric circuit. Therefore, to provide effective damping of the structure vibrations it is necessary to determine such values of the parameters of the external *RL*–circuit, at which the eigenfreqiecies of the structure and electric circuit coincide.

For our investigation we used the scanning method, which implies variation of the sought-for optimization parameters (the values of the inductance *L* and resistance *R*) with the aim to find the values, which could provide the maximum damping ratio of the plate with piezoelectric element and external electric circuit. This method allowed us to investigate the dependence of the normal eigenfrequencies of the examined structure (fig.1,a) first of al on the value of inductance of the external electric circuit *L*, and then on the value of the resistance of the external electric circuit *R* for the evaluated optimal (providing close coincidence of eigenfrequencies of the plate with piezoelement and electric oscillatory circuit) value of the inductance *L*.

Figure 2 shows the dependence of eigenfrequency of the plate with embedded piezoelement and external electric circuit in the vicinity of the first eigenfrequency of the plate with piezoelement on the value of the inductance *L* at a small constant value of the resistance, for example at *R=10* Ohms. In the figure, ω_{R1}, ω_{Re} denote the eigenfrequencies of the plate with the piezoelement and electric circuit, respectively, and ω_{I1}, ω_{Ie} denote the corresponding damping ratios.

Figure 2 demonstrates a highly pronounced effect of convergence of frequencies ω_{R1}, ω_{Re} in the range of the inductance L=10000 ÷ 20000 *H*. At some value of the inductance $L=L_r$ (in our calculations $L_r \sim 12000$ *H*) one can observe a maximal convergence of frequencies and a sharp change (in modulo) of the damping ratios: decrease of ω_{I1} and increase of ω_{Ie}.

The obtained results show that the inductance *L* has an essential effect on the value of eigenfrequency of the electric circuit ω_{Re} in the eigenfrequency spectrum of the system consisting of the plate with piezoelement and external electric circuit. It provides a shift and convergence of the eigenfrequency of the plate with piezoelement with the eigenfrequecy of the electric circuit.

Figure 3 shows how the resistance *R* affects the eigenfreqiencies of the system comprising the plate with piezoelement and external electric circuit in the vicinity of the first vibrational eigenfrequency of the plate with piezoelement at the evaluated constant optimal value of the inductance $L_r = 12000$ *H*.

As it is seen from fig.3, the frequencies of the electric circuit and the plate with piezoelement remain unchanged up to a certain value of the resistance *R*.

A convergence of the vibration frequencies ω_{R1}, ω_{Re} and a sharp increase (in modulo) of damping ratios ω_{I1}, ω_{Ie} take place in some range of resistance (*R*=300 ÷ 800 *kOhms*). In our calculations the closest coincidence of frequencies ω_{R1}, ω_{Re} occurs when the value of the optimal resistance is $R_r \sim 400$ *kOhm*.

Further, with increasing resistance the frequency of the electric circuit reduces up to zero, which suggests the absence of circuit vibrations and the eigenfrequecy of the plate with piezoelement changes to the value of frequency in the o/c regime, which implies the absence of shorting the electroded surfaces of the piezoelement.

Figure 4 illustrates the behavior of the imaginary parts of the first eigenfrequency of the plate with piezoelement in the vicinity of the values of parameters *L* and *R*, at which damping of the vivbrations occurs at the maximum rate. The values of the external circuit parameters *L* and *R*, at which the imaginary part of the eigenfrequencies of the plate with piezoelement reaches its maximum value are just the values, which are optimal for damping the vibrations of the system at these frequencies.

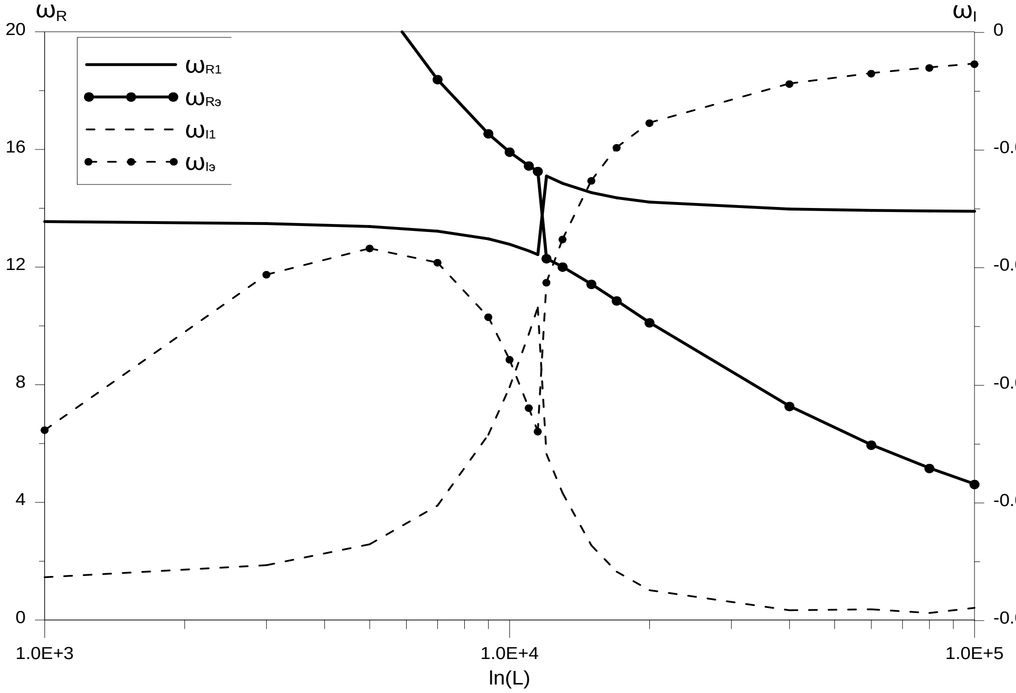

Figure 2: The influence of the value of inductance *L* (*R*=10 *Ohm*) on the first eigenfreqiency of the system consisting of the plate with piezoelement and external series-connected *RL*-circuit.

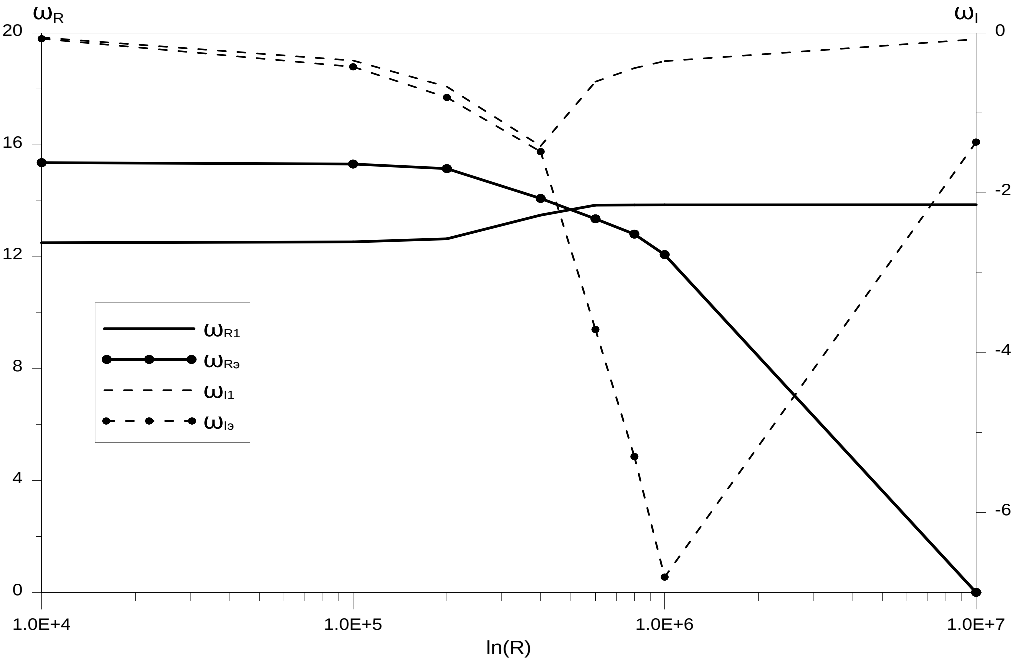

Figure 3: The influence of the value of resistance *R* (*L*=12000 *H*) on the first eigenfreqiency of the system consisting of the plate with piezoelement and external series-connected *RL*-circuit.

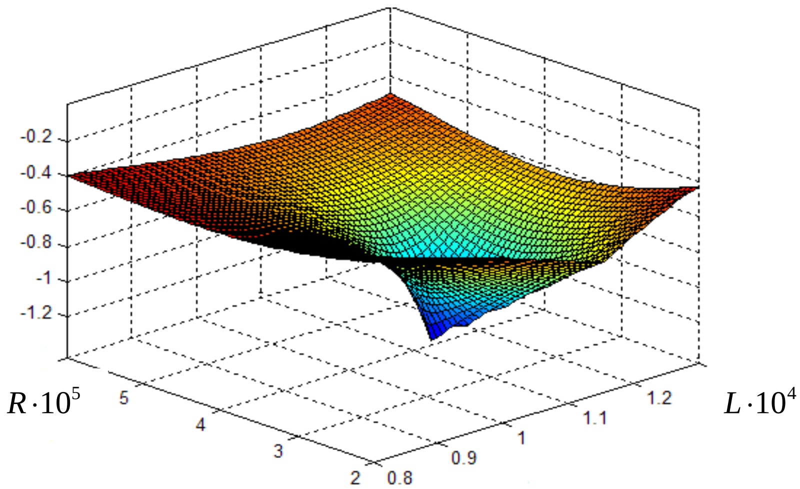

Figure 4: The behavior of the imaginary part of the first eigenfrequency of the plate with piezoelement

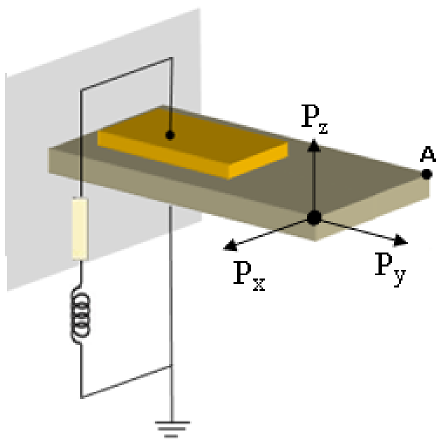

Figure 5: Computational scheme for determining the ampltude-frequency characteristics of the system.

Based on the optimal values of the parameters for the external electric circuit (L=11198 *H*, R=402493 *Ohm*) calculated for the first resonance frequency of vibrations we constructed the amplitude-frequency characteristics of the displacements U_s ($U_s = \sqrt{U_x^2 + U_y^2 + U_z^2}$) of point A on the cantilever part of the plate (fig.6) in the range of the first four resonance frequencies produced by the action of external axial harmonic forces $P_X = P_Y = P_Z$ =0.01 N applied to the angular point on the simply supported end of the plate (as shown in fig.5). Here the dashed line shows the AF characteristics of the plate with piezoelement and without the external electric circuit. The difference in the AF characteristics is observed only in the vicinity of the first resonance.

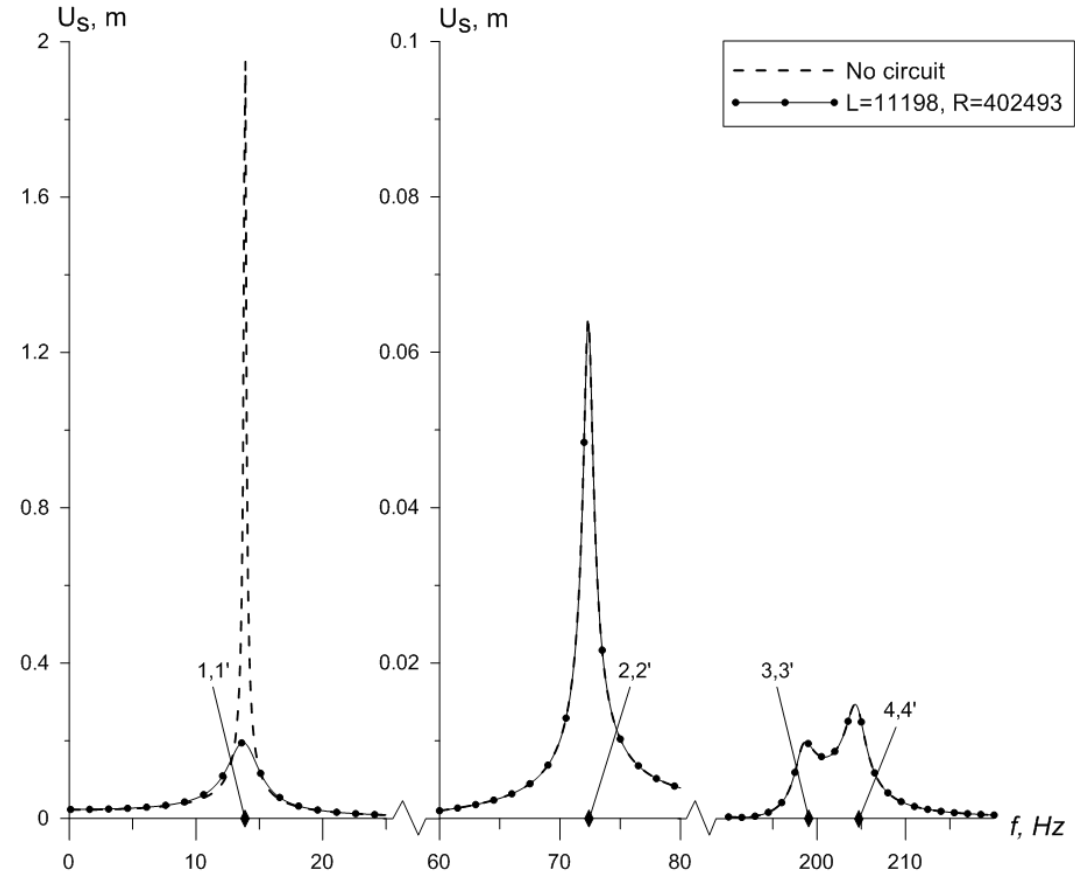

Figure 6: The amplitude-frequency characteristics of the plate with piezoelement and external electric circuit (fig. 5) in the range of the first 8 resonance frequencies in the case of tuning the electric circuit to the first resonance frequency.

As it is seen from the fig.6, a reduction of the displacement amplitude U_s at the free angular point A on the cantilever part of the plate at the first resonance frequency is - by a factor of 11.47. Note that in this case there is no change in the displacement amplitude U_s at other resonance frequencies (coincidence of the solid and dash lines in fig.6).

4 CONCLUSIONS

- The paper considers the problem of modeling and optimizing the dynamic characteristics, namely the resonance frequencies and the parameters characterizing the damping properties of the systems embedding piezoelectric elements in the presence of external electric circuits including resistance and inductance elements.
- A method for determining the optimal parameters of the external electric circuit, which will provide the maximal suppression of particular vibration frequencies has been developed based on the solution of the natural vibration problem for electro-viscoelastic bodies with external electric circuits.
- Numerical examples have been discussed to demonstrate the efficiency of using the problem of natural vibrations of electro-viscoelastic bodies with external electric circuits for optimization of damping properties of systems made on the basis of pizoelectric materials.

- It has been shown that the natural vibration problem is more effective than the problem of forced steady-state vibrations, since the obtained complex values of eigenfrequencies are the system characteristics, which do not depend on the type and method of external load application, which essentially reduces the computation time.
- It has been found that the imaginary part of the complex eigenfrequencies is the parameter, which characterizes the vibration damping ratio at the specified parameters of the external electric circuit.

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