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HOMOGENIZATION TECHNIQUES FOR ACCURATE AND APPROXIMATE ESTIMATES FOR OVERALL PROPERTIES OF MICROCRACKED VISCOELASTIC MASONRIES

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Abstract. *Based on the coupling between brittle fracture mechanics and homogenization techniques, either analytical or finite elements homogenization, this work provides accurate (up to numerical errors) and approximate overall estimates for masonries accounting for their creep behaviour and a certain level of damage as it is the case for instance for refractory linings serving at high temperatures or middle-ages masonry building.*

1 INTRODUCTION

It is well known that creep phenomenon has a noticeable effects not only on new structures but also on historical monuments [20, 3, 17]. To model the creep behaviour of traditional mortar, various rheological models namely the USBR, Feng, Ross, typical and modified versions of the Burgers and Modified-Maxwell models may be investigated [5, 11]. On the other hand, there exist several approaches accounting for damage in viscoelastic materials [11, 12]. Indeed, the approach presented in [12] is based on a coupling between continuum damage mechanics and viscoelasticity through the generalized Kelvin-Voigt model. Accordingly, a three-dimensional phenomenological model was developed to describe the long-term creep of gypsum rock. The main disadvantage of this model is that it requires experimental investigation [11] or computational efforts to resolve nonlinear equation [12] function of internal damage variables.

In the literature, little attention is devoted to the prediction of the macroscopic creep behaviour of masonry. In this context, Brooks, Cecchi & Tralli, Cecchi & Taliercio and Taliercio [4, 7, 8, 22] proposed models to predict creep coefficients according to the properties of each masonry constituent. These models are based on analytical or numerical homogenization using the finite elements method (FEM) in order to deduce the macroscopic creep of undamaged (without cracks) masonry.

The objective of this study is to evaluate the effective and local behaviour of masonries exhibiting nonlinear behaviours mainly viscoelastic at short and/or long times especially when they are subjected to severe or long term loading such as historical monuments or refractory masonry linings working under high temperatures. More details about this analysis are provided in the references [13, 18].

2 METHODOLOGY AND HYPOTHESIS

In the present study, the coupling between the Griffith's theory and the dilute scheme [2] will be applied to provide the effective behaviour of a micro-cracked mortar.

As a first approach and for the sake of simplicity, only the mortar is assumed to be viscoelastic and microcracked. The distribution of microcracks is assumed to be isotropic. The overall creep behaviour of the mortar is obtained due to the coupling between the Griffiths theory [1] and mean-field homogenization [2]. At short and long terms, this procedure allows fast and easy approximation of elastic and viscous properties of the mortar assumed to follow the Generalized Maxwell rheological model. These properties which are explicit function of time and crack density parameter are determined without recourse to a heavy numerical inversion of the Laplace-Carson transform.

The second step of this work consists to carry out either a 'direct' (analytical or numerical) homogenization modeling of the periodic masonry or 'indirect' method based on two steps: (i) the differentiating of the mortar's constitutive law defining thus a thermo-elastic incremental behaviour and (ii) finite elements homogenization of the masonry periodic cell. The last step, as the 'direct' homogenization method, relies on the computation of the localization strain tensors in each brick and mortar in order to define overall stiffness and prestress in the periodic cell. More details about principles and results of these models are provided in [13, 18, 19].

For concision reason, this paper deals only with direct homogenization model coupling between brittle fracture theory in order to assess the effective microcracked mortar's behaviour and periodic homogenization. Basic steps followed by this model are summarized in Figure (2).

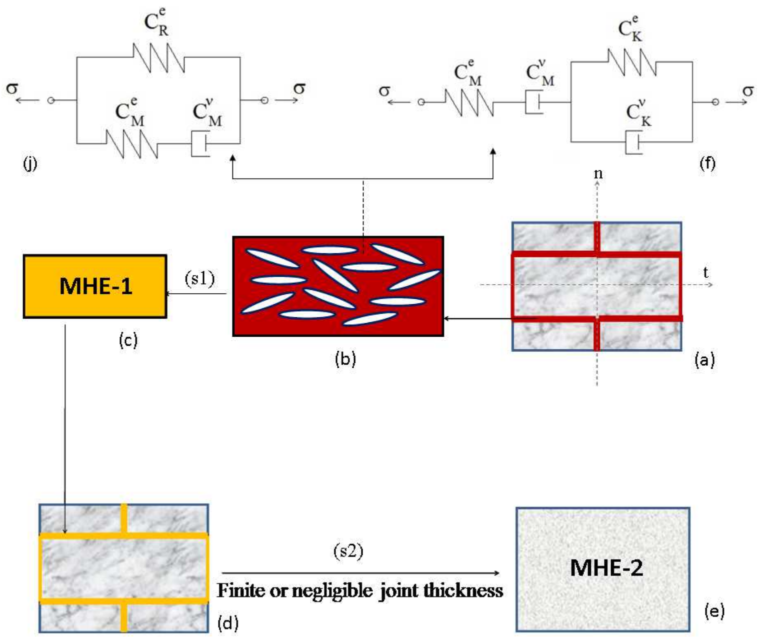

Figure 1: Main steps of the proposed model.

2.1 Microcracked mortar's effective behaviour

Let us firstly recall that the effective stiffness of an elastic porous medium with a homogeneous solid phase tensor C^s is:

$$\tilde{C} = C^s : (I - \phi A^p) \tag{1}$$

where A^p is the average of the strain localisation tensor $A(z)$ over the pore space Ω^p and ϕ is the porosity volume fraction. For a flat spheroid - usual $3D$ crack model - of aspect ratio $w << 1$, the Eshelby tensor S^E is a function of w. Accordingly the components A^p_{iikl} and A^p_{ikik} of the dilute localization tensor $A^p = (I - S^E)^{-1}$ in the pore are of the order of $1/w$ and therefore is the ratio of the normal strain ε_{nn} to the macroscopic strain E. Possible none negligible variations of $1/w$ is in contradiction with the assumption of linearity of the localization relationship. To overcome this difficulty, it was proposed [9] to consider the rate-type formulation of the problem i.e. the strain localization tensor should be replaced by a strain rate localization as in the following $\dot{\varepsilon}(z) = A(z) : \dot{E}$. Similarly, the rate-type formulation of the Eshelby problem for a spheroidal cavity reads:

$$\dot{\varepsilon}^p = (I - S^E(w))^{-1} : \dot{E} \tag{2}$$

Such hypothesis implies that the use of A^p in the homogenized stiffness (1) leads to an estimate of the tangent effective stiffness. Moreover, since the crack porosity ϕ is proportional to w [10], the tangent effective stiffness is mathematically independent of w. This renders the effective behaviour linear elastic. Note that the rate-type hypothesis is indispensable [10] to also avoid troubles related to possible large strain in the direction normal to the crack.
On the other hand, the extension of the linear homogenization schemes to non-aging viscoelasticity is based on the Laplace-Carson (LC) transform [21]. The effective stiffness $\tilde{C} =< C : A >$ becomes

$$\tilde{C}^* =< C^* : A > \tag{3}$$

in the LC space where C^* is the apparent elastic stiffness. The presence of microcracks implies the existence of nonlinearity at the local scale in the relationship between the crack strain and the macroscopic strain. Accordingly the homogenization of a viscoelastic cracked medium is not as straightforward as (3). The basic idea consists in anticipating that both the microscopic strain field and the displacement discontinuity vectors $[u]_i$ linearly depend on the macroscopic stress. Such hypothesis, confirmed by [15], justifies the use of the LC transform which can be applied to the macroscopic strain related to the microscopic strain and the displacement discontinuity vectors $[u]_i$ between two lips of crack. In the framework of the stress-based dilute scheme [10], the displacement jump $[u]_i$ is linearly related to Σ. Indeed, the normal displacement jump (mode I) at the lips of a crack in an infinite matrix submitted to an isotropic asymptotic macroscopic stress $\Sigma^* = \Sigma^*\, i$ can be written:

$$[u_n]^* = \frac{4(1-\nu_s^*)}{\pi}\frac{\Sigma^*}{\mu_s^*}\sqrt{l^2-\rho^2}. \tag{4}$$

where l is the crack's radius and ρ is the position of a point M pertaining to the crack's plane. The tangential displacement jump (mode II) under an asymptotic shear stress $\Sigma^* = \Sigma^*\, n \otimes^s t$ where t is parallel to the crack's plane reads:

$$[u_t]^* = \frac{4(1-\nu_s^*)}{\pi(2-\nu_s^*)}\frac{\Sigma^*}{\mu_s^*}\sqrt{l^2-\rho^2}. \tag{5}$$

Under respectively an isotropic and deviatoric loading and assuming that all cracks have the same radius l, an integration over all orientations on the unit sphere yields the total crack contribution and allows the determination of the apparent effective bulk's and shear moduli as follows

$$\begin{aligned} \frac{1}{\tilde{k}^*_{DL}} &= \frac{1+d_cQ^*}{k_s^*} \quad \text{where} \quad Q^* = \frac{16}{9}\frac{(1-\nu_s^{*2})}{(1-2\nu_s^*)} \\ \frac{1}{\tilde{\mu}^*_{DL}} &= \frac{1+d_cM^*}{2\mu_s^*} \quad \text{where} \quad M^* = \frac{32}{45}\frac{(1-\nu_s^*)(5-\nu_s^*)}{2-\nu_s^*} \end{aligned} \tag{6}$$

in which the crack density parameter and the symbolic Poisson's ratio of the safe matrix reads respectively: $d_c = Nl^3$ and $\nu_s^* = \frac{3k_s^*-2\mu_s^*}{6k_s^*+2\mu_s^*}$. Since the expressions (6) can not be satisfied rigorously, it was proposed in [15] to identify the best approximation of the effective behaviour in the class of Burgers' (Modified Maxwell's) model if the mortar in its safe state follows the Burgers' (Modified Maxwell's) model. The idea is to satisfy the series expansion of the dilute estimates of the bulk's (6)-a and shear (6)-b moduli to the first order at $p = 0$ and $p \to \infty$ such that

$$\lim_{t\to\infty} f(t) = \lim_{p\to 0} f^*(p) \quad \text{et} \quad \lim_{t\to 0} f(t) = \lim_{p\to\infty} f^*(p) \tag{7}$$

The dilute symbolic moduli of a non-aging linear viscoelastic (n.a.l.v.) microcracked mortar following the MM's model can be approached by expressions similar respectively to those available for a safe mortar following the MM's rheological model

$$k^*_{MM} = k_R(d_c) + \frac{pk_M(d_c)\eta^s_M(d_c)/3}{k_M(d_c)+p\eta^s_M(d_c)/3} \quad \mu^*_{MM} = \mu_R(d_c) + \frac{p\mu_M(d_c)\eta^d_M(d_c)/2}{\mu_M(d_c)+p\eta^d_M(d_c)/2} \tag{8}$$

Following idea (7), it is possible to identify the MM's six parameters [14] and accordingly to determine an approximate creep function of a microcracked mortar which matrix follows the MM's model reads

$$\begin{aligned} J^{app}_{MM}(t,dc) &= \tfrac{1}{9k_R(d_c)}\left(1-\tfrac{k_M(d_c)}{(k_R(d_c)+k_M(d_c))}e^{-t/\tau^s_{MM}(d_c)}\right) \\ &+\tfrac{1}{3\mu_R(d_c)}\left(1-\tfrac{\mu_M(d_c)}{(\mu_R(d_c)+\mu_M(d_c))}e^{-t/\tau^d_{MM}(d_c)}\right) \end{aligned} \tag{9}$$

in which the characteristic times of the spherical and deviatoric parts are respectively

$$\tau^s_{MM}(d_c) = \frac{\eta^s_M(d_c)(k_R(d_c) + k_M(d_c))}{3k_R(d_c)k_M(d_c)} \quad \tau^d_{MM} = \frac{\eta^d_M(d_c)(\mu_R(d_c) + \mu_M(d_c))}{2\mu_R(d_c)\mu_M(d_c)}. \tag{10}$$

2.2 Masonry's periodic cell global behaviour

For the viscoelastic periodic problem, the auxiliary problem reads

$$\begin{cases} \dot{E} + \varepsilon(\dot{u}^{per}) = \varepsilon(\dot{u}) \\ div(\sigma(t)) = 0 \\ \sigma(t).n \quad \text{anti-periodic on} \quad \partial Y \\ u^{per} \quad \text{periodic on} \quad \partial Y \\ \langle \sigma(t) \rangle = \Sigma(t) \end{cases} \tag{11}$$

where $\sigma(t)$ is the microscopic stress tensor state, $\varepsilon(\dot{u}(y))$ is the microscopic strain tensor state, u^{per} is the periodic displacement field and $\dot{E}$ is the macroscopic in-plane strain tensor rate. Following the procedure of Cecchi and Tralli [7] accounting for the joint thickness, the expression of the homogenized tensor $\tilde{A}^R$ of the periodic cell is provided in the same quoted reference. If only the mortar is assumed to be viscoelastic such that its stiffness tensor reads $A^m(t) = A^m(1 + \phi_m(t))$, the main idea consists to substitute the Young's modulus E^m by $J_m^{-1}(t) = E^m(1 + \phi_m(t))$.
Recently, based on the works of Cecchi $\&$ Sab, Cecchi $\&$ Barbieri [6] and Cecchi $\&$ Tralli, Cecchi $\&$ Taliercio [8] have proposed a homogenized compliance for viscoelastic undamaged masonry with mortar joints assimilated to interfaces. In this paper we propose to extend also this model to microcracked masonry with finite dimensions of microcracked mortar joints. The effective compliance of the damaged viscoelastic masonry reads then:

$$\begin{array}{l} \tilde{S}^R_{1111}(t,dc) = \frac{S^b_{1111}\frac{e_v}{a+e_h}J'_v(t,dc) + 4S^b_{1111}\frac{e_h}{b+e_v}J'_h(t,dc) + 4\frac{e_v e_h}{(b+e_v)^2}J'_v(t,dc)J''_h(t,dc)}{4\frac{e_h}{b+e_v}J''_h(t,dc) + \frac{e_h}{a+e_h}J'_v(t,dc)} \\ \tilde{S}^R_{1212}(t,dc) = S^b_{1212} + \frac{e_h}{a+e_h}J''_h(t,d_c) + \frac{e_v}{b+e_v}J''_v(t,dc) - \frac{e_v^2(b+e_v)(a+e_h)J''^2_v(t,dc)}{4\frac{e_h}{b+e_v}J'_h(t,dc) + \frac{e_v}{a+e_h}J''_v(t,dc)} \\ \tilde{S}^R_{2222}(t,dc) = S^b_{2222} + e_h a + e_h J'_h(t,dc), \\ \tilde{S}^R_{1122}(t,dc) = S^b_{1122} \end{array} \tag{12}$$

The damaged masonry's orthotropic effective properties are then the following: $\tilde{E}_{11}(t,d_c) = \frac{1}{\tilde{S}^R_{1111}(t,d_c)}$, $\tilde{E}_{22}(t,d_c) = \frac{1}{\tilde{S}^R_{2222}(t,d_c)}$, $\tilde{\mu}_{12}(t,d_c) = \frac{1}{\tilde{S}^R_{1212}(t,d_c)}$, $\tilde{\nu}_{ij}(t,d_c) = -\tilde{E}_{ii}(t,d_c)\tilde{S}^R_{iijj}(t,d_c)$ where $(i,j) \in \{1,2\}$. These moduli are explicit function of the crack density - damage parameter -, time and ratios $\frac{e_h}{a+e_h}$, $\frac{e_v}{b+e_v}$ instead of respectively $\frac{e_h}{a}$ and $\frac{e_v}{b}$. It is worth noting, that for the case of undamaged mortar interfaces, Cecchi $\&$ Taliercio have proven that this solution is more consistent with a numerically homogenized solution based on the finite elements method for ratios $E_b/E_m \geq 20$ when the mortar follows a Generalized Maxwell model. In the following, we assume that this condition ensuring the accuracy of the Cecchi $\&$ Taliercio's analytical model with the additional assumption of finite joints dimensions is also available for damaged mortars.

3 Results

The proposed model is applied to the case of a masonry with hybrid mortar's joints which material's properties are given in table 1. The properties of the safe bricks are the following:

E_M (MPa)	τ_M (s)	E_R (MPa)	τ_R (s)	ν_m
4038	46490	2112	90866	0.22

Table 1: Elastic and viscous moduli of hybrid mortar [11]

$\nu_b = \nu_m$ and $E_b = 100E_m$ with dimensions 55x250 mm^2. Mortar's joints thicknesses are $e_h = e_v = 10\,mm$. Note that the characteristic times for the spherical and deviatoric hybrid mortar's behaviour are assumed to be equal $\tau^s = \tau^d = \tau$ for the MM's and Burgers' models. The Young's modulus E_R and Poisson's ratio ν_R coincide respectively with E_K and ν_K, properties of the Kelvin-Voigt's spring.

3.1 Mortar's effective creep function

Figure (2) illustrates the evolutions of normalized inverse dilute creep functions of respectively the Burgers' and MM's models with respect to the damage parameter d_c and the time t.

For a given crack density parameter $d_c = 0,\ 0.1$ or 0.2, Figure (2)-b demonstrates that the MM's model yields to a constant function $J^{-1}(t, d_c)/J^{-1}(0, 0)$ with variation of the time beyond $t = 10^6$(s) (i.e. almost 11 days). As expected, the increase of d_c decreases the level of the asymptotic limits reached by this function. The difference between the MM's curves for different d_c is not negligible (around 15%) unlike that observed for the Burgers curves which are very close especially at short and long terms. Figure (2)-a shows that the Burgers model leads to vanishing inverse creep functions for $t \geq 310^8$ (s) i.e. 1157 days for every crack density value $d_c \geq 0$.

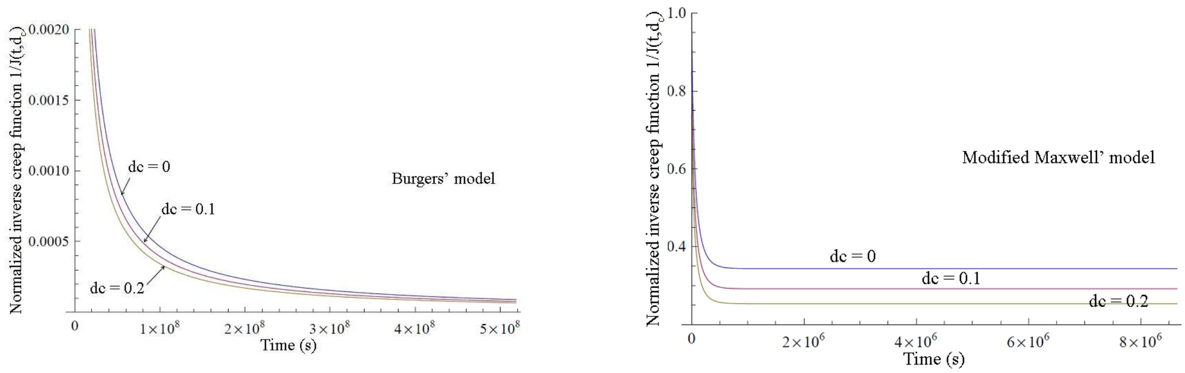

Figure 2: Variation of $J^{-1}_{MM}(t, d_c)/J^{-1}_{MM}(0, 0)$ versus time for safe mortar ($d_c = 0$) and microcracked mortars ($d_c = 0.1,\ d_c = 0.2$). Mortar following (a) the Burgers's and (b) MM's model.

3.2 Case of a compressed masonry panel

We study the case of a masonry panel of dimensions $L = 1560\,mm$ (length) and $H = 1040\,mm$ (height) studied in [7] subjected to boundary conditions BC-1 with three distributed loads at the top and two lateral edges (see Figure (3)). Bricks are assumed to be elastic or rigid. The mortar inside the joints is assumed to be microcracked with a matrix which obeys to linear viscoelastic behaviour following the Burgers or Modified Maxwell's model. As the arrangement of the bricks is regular, the effective behaviour of the panel is assumed to be well estimated by that of a periodic cell (see Figure 2-a). The panel can then be modeled as a homogeneous material which properties coincide with those of the equivalent material MHE-2 (see Figure (1)-e). For the sake of simplicity, it is assumed that the crack density d_c is set equal to zero at the initial time $t_0 = 0$ and evolves linearly with time t as follows

$$d_c = \dot{d}_c\, t \tag{13}$$

where, here, the rate of the crack density $\dot{d}_c$ is assumed to be a positive constant lower than 0.002/day (i.e. if $t = 100$ days, then d_c reaches a maximum of 0.2). Indeed, beyond this limit, the dilute estimate will not be appropriate. Of course, as well known, the increase of the damage rate reduces the stiffness of the masonry cell. Under boundary conditions BC-1, the stress

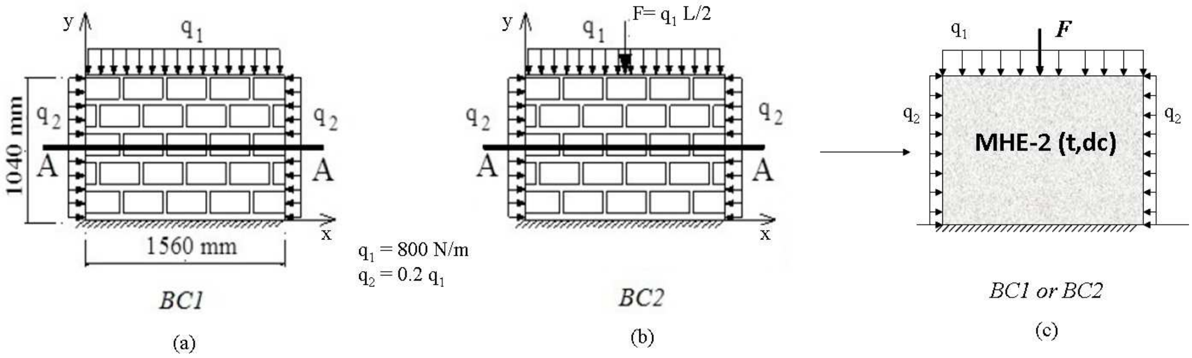

Figure 3: Boundary conditions BC-1 applied to the masonry panel [7]

(strain) fields σ_{yy} and σ_{xy} (ε_{yy} and ε_{xy}) are concentrated at both left and right superior corners of the compressed wall. The magnitude of the stress field σ_{yy} is similar for both the MM's and Burgers' models. However, the Burgers estimates of the stress fields σ_{xx} and σ_{xy} are superior to those predicted by the MM's model. These trends are confirmed by the evolutions of the stress

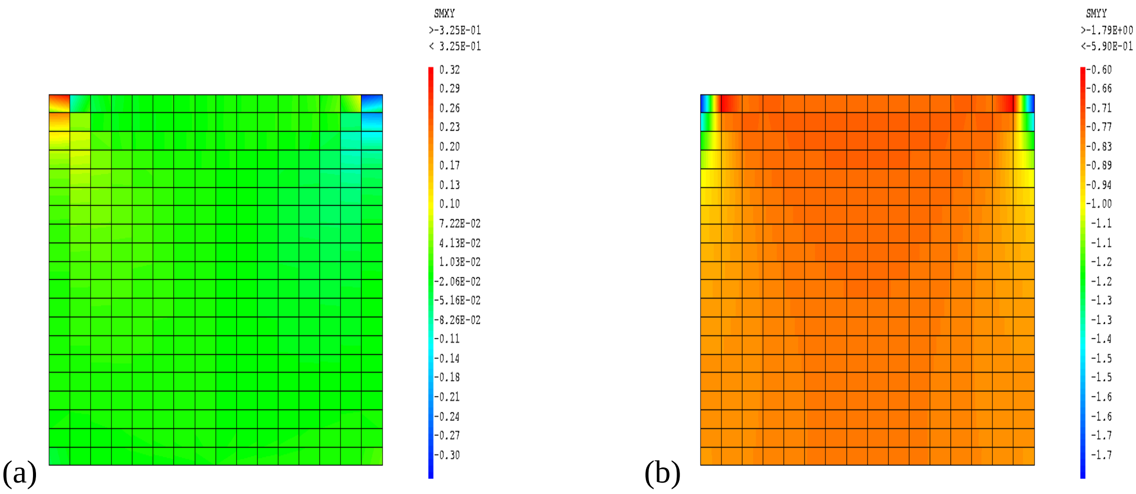

Figure 4: *BC-1 boundary conditions: MM's model predictions of σ_{xy} (a) and σ_{yy} (b) maps for $d_c = 0.15$ at $t = 1000$ days. Here crack density $d_c = \dot{d}_c t$ where $\dot{d}_c = 1.5\ 10^{-4}$/day.*

components with respect to the axis x as shown on figure (6). Note that unlike the stress (strain) fields σ_{yy} and σ_{xx} (ε_{yy} and ε_{xx}) which are symmetric by reference to the axis of symmetry of the panel ($x = L/2$), the shear stress σ_{xy} (shear strain ε_{xy}) is anti-symmetric. For the MM's model,

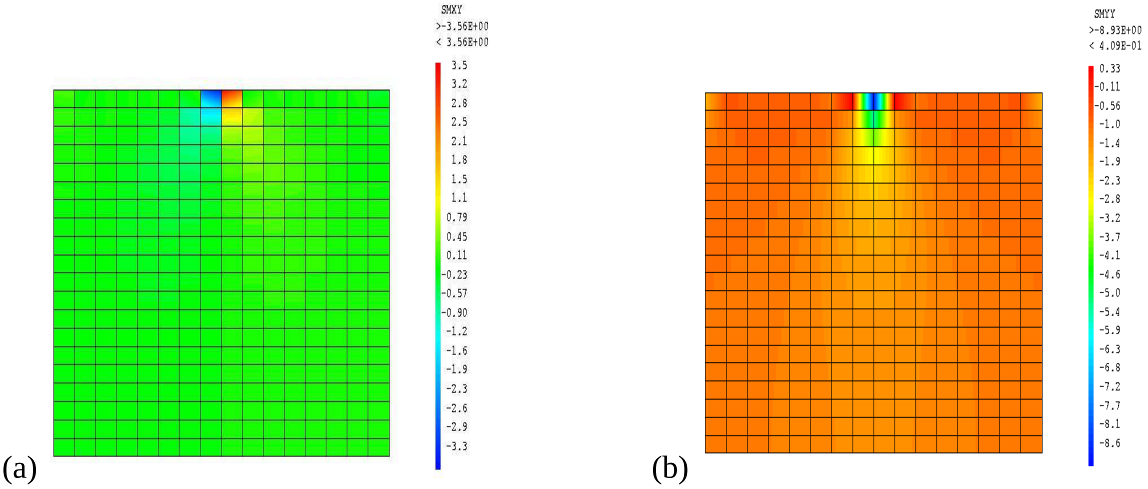

Figure 5: *BC−2 boundary conditions: MM's model predictions for the maps of stresses σ_{xy} (a) and σ_{yy} (b) for(a) $d_c = 0$ at $t = 0$ and (b) $d_c = 0.15$ at $t = 1000$ days. Here crack density $d_c = \dot{d}_c t$ where $\dot{d}_c = 1.5\ 10^{-4}/$day.*

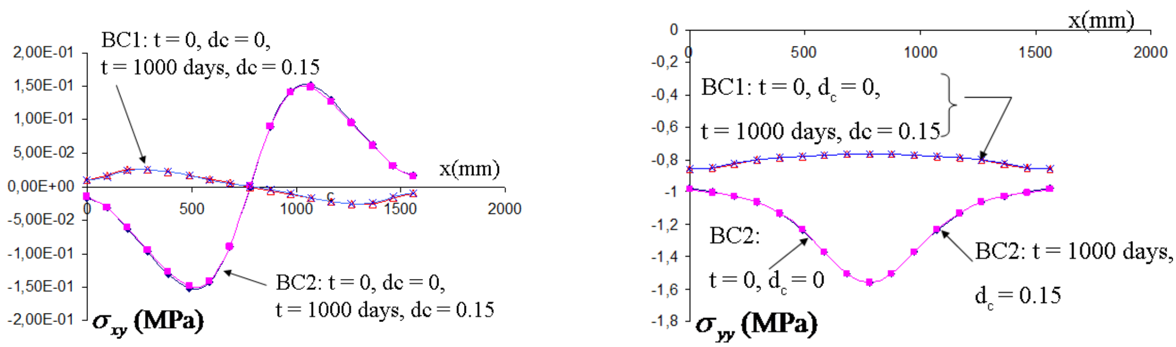

Figure 6: Boundary condition $BC-1$ or BC−2: trends in σ_{xy} (a) and σ_{yy} (b) at the panel section A-A for the Burgers' (a) and Modified Maxwell's model at time $t = 0$ with $d_c = 0$ and $t = 1000$ days with $d_c = 0.15$. Here crack density $d_c = \dot{d}_c t$ where $\dot{d}_c = 1.5\ 10^{-4}/$day.

it is observed [18] that the strain fields increase slightly with time and damage. Moreover, this model predicts small strains unlike the Burgers model. This rheological model predicts for the masonry large strains similarly to the case of polymer materials. This quantitative aspect for the Burgers predictions is available at $t \geq 25$ days in a safe or damaged state. This result violates the hypothesis of small strains adopted in this study. These results motivate to avoid the use of a mortar following the Burgers model in masonries since such a model leads to vanishing masonry's stiffnesses and large strains increasing thus the risk of failure. Such results motivate to avoid modeling traditional mortars with short-term gathered properties using the Burgers model since it leads to vanishing masonry's stiffnesses and large strains increasing thus the risk of failure. These trends for the Burgers model are not valid if the Maxwell's relaxation time of the mortar is too high ($\tau_M \geq 10^7(s)$) as it is the case f hybrid mortar with long-term characteristic times (Table-12). In the later case, the Burgers model is expected to provide local fields predictions similar to the MM's estimates. The boundary conditions BC-1 are preferable to BC-2 since the later increases the stress and strain levels throughout the wall. Owing to Figures (4) and (5), at $t = 1000$ days, while failure occurs in the wall's area located around the application's point of the concentrated load F under BC-2, there is no failure in the wall submitted to conditions BC-1.

4 CONCLUSION

This paper proposes 'direct' (analytical and numerical) homogenization models for non-aging viscoelastic microcracked masonries based on the extensions of the Cecchi $\&$ Barbieri [6], Cecchi $\&$ Tralli [7] and Cecchi $\&$ Taliercio [8] models which are available for uncracked masonries. The herein proposed models rely on the coupling between the Griffith's theory and the periodic homogenization (analytical and numerical) schemes in order to derive easily and with low computational effort - without recourse to numerical inversion of the Laplace transform - the effective creep function of a microcracked non-aging viscoelastic mortar. Notice that this 'direct' homogenization model is preffered to the 'indirect' incremental homogenization which requires the additional computation of prestress in the masonry and is dependent of the time increment. The main result of this study dealing with microcracked mortars following the Modified Maxwell and Burgers models, is that unlike the MM's model which estimates well the creep of masonry at short to long terms, the relevance of this version of the Burgers model is limited to mortars with too high Maxwell's relaxation times ($\tau_M \geq 10^7(s)$) otherwise it predicts vanishing stiffness at short and medium terms and exaggerated local strains. In addition, the effects of the damage parameter - crack density d_c - and the ratio between the mortar's and bricks stiffness at short and long terms have been assessed [18].

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