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A FAILURE CRITERIA FOR UNIDIRECTIONAL FIBER REINFORCED COMPOSITES BASED ON MICROMECHANICS BY ASYMPTOTIC HOMOGENIZATION

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Abstract. *This work presents a successful methodology for obtaining failure envelopes of unidirectional fiber reinforced composites based on micromechanical analysis by the asymptotic homogenization [1] method. Given a structure (in this case a composite material lamina) and external loads, plus having its material heterogeneity geometrically represented by a periodic unit cell of microstructure, the asymptotic homogenization method is able to predict its micromechanical stresses. Such stresses may be evaluated by failure criteria of the composite's constituents for several loading conditions, and this way it is possible to assess the composite's failure envelope. In the methodology developed, a periodic unit cell of the composite microstructure is isolated, consisting of a parallelepiped of polymeric matrix reinforced by cilindrical fibers oriented in one direction, and its behaviour is evaluated by an appropriate finite elements model. At first, the unit cell is tested in several directions to find strengths for the matrix and fiber and also matrix/fiber interface, thus evaluating failure characteristics of the composite constituents. The tests are carried out considering several possible orientations for the unit cell inside the related macroscopic media, which take into account the possible relative positions of the chosen unit cell inside the material heterogeneity of the composite. Then, the strengths of the constituents are used to predict the failure envelope for the fiber reinforced material, according to failure criteria devoted to the constituents. In this prediction, it is possible to say which is the constituent that fails first for each of the test loads. The results obtained are in good agreement with experimental data for carbon/epoxi and glass/epoxi composites. Moreover, the envelopes obtained are similar to the Puck & Schürmann [2] criterion, widely used to predict failure of such composites. This way, the present methodology renders good failure envelopes for fiber reinforced composites and gives information on the strengths of the constituents and material phase of failure, benefits from an incorporated micromechanical analysis.*

1 INTRODUCTION

Composite materials are made up of two or more materials (constituents) [3, 4] that have better properties than the constituents when alone [5]. The unidirectional fiber reinforced composite [3] is an example of such material that can achieve excellent mechanical properties having low mass. Because of that, they are heavily applied at the aeronautic industry [4]. The reinforcing phase are usually made of carbon or glass fibers and the base material is called matrix, usually made of epoxy [6].

It is very important in projects that use those materials to predict when failure is going to happen. A common procedure to investigate failure is to perform experiments. However, those are expensive, time consuming and it is not possible to investigate failure in all loading conditions. Then, with the increase of computer's capabilities, numerical methods arise as a possible manner to predict failure of composite materials.

Numerical methods can tell if failure is going to happen in a macromechanical or micromechanical analysis. The macromechanical analysis treats the composite structure as an orthotropic material and the failure criteria use the lamina strengths to determine failure [7]. The micromechanical analysis predict failure using the strengths of each constituent [8]. The micro analyses use stresses at each constituent to tell whether failure is going to happen and, because of that, different failure criteria can be considered for each constituent.

In this work, a micromechanical analysis of unidirectional fiber reinforced composites if performed using the asymptotic homogenization technique [1]. This method permits to calculate stresses at the micro level and then, failure can be predicted at each constituent. The methodology to determine failure presented here considers three regions: matrix, fiber and the interface between matrix and fiber.

2 ASYMPTOTIC HOMOGENIZATION

Asymptotic homogenization techniques allow to obtain micromechanics elastic properties of a composite considering its microstructure. In this technique, two levels are considered: the macro (**x**) and the micro (**y**). The micro level is considered to be periodic and it is represented by an unit cell formed by matrix and fiber. Its geometry can be changed in order to vary fiber volume fraction (V_f) and cross section shape. Inclusions and holes can also be inserted in it. In the present work, the hexagonal (Fig.1 a) and the square (Fig.1 b) unit cells are used to perform the homogenization.

2.1 Homogenized elastic properties

The homogenized elastic properties of the composite are obtained at micro scale by:

$$E^H_{ijkl} = \frac{1}{Y}\int_Y \left(E_{ijkl} - E_{ijkm}\frac{\partial \chi^{kl}_m}{\partial y_n} \right) dY. \tag{1}$$

In Eq.(1), tensorial notation is used, with the indexes vary from 1 to 3. Y is the volume of the unit cell, E_{ijkl} are the elastic properties of each point within the microstructure, y_n are the coordinates in the unit cell, χ^{kl}_m are the auxiliar displacement fields that are obtained from the following equations,

$$\int_Y E_{ijmn}\frac{\partial \chi^{kl}_m}{\partial y_n}\frac{\partial \delta u_i}{\partial y_j} dY = \int_Y E_{ijop}\epsilon^{kl}_{op}\frac{\partial \delta u_i}{\partial y_j} dY. \tag{2}$$

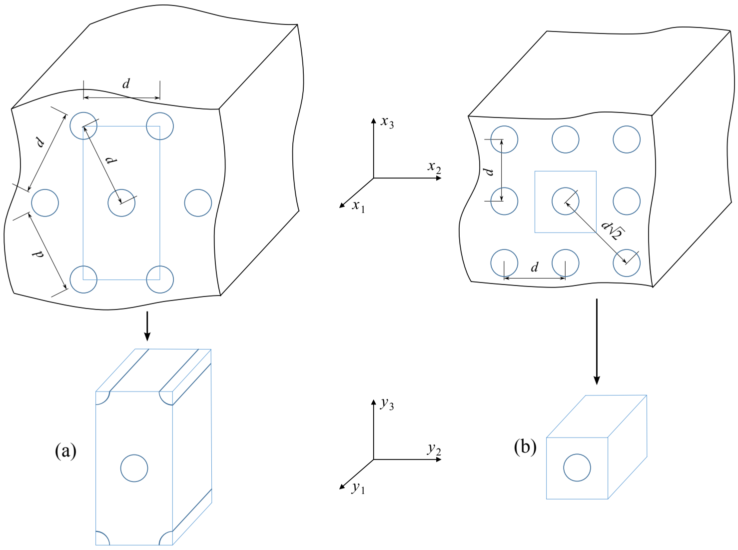

Figure 1: Periodical fiber distribution in the composite and the hexagonal unit cell.

In Eq.(2), ϵ_{op}^{kl} are unitary constant strain test fields and δu_i are the virtual displacements. Equation (2) is here solved in the software PREMAT [1] using finite element method [9].

It can be noted in Fig.1 that the distances between each fiber in the hexagonal geometry are the same, but this not happen in the square unit cell. Because of that, transversal isotropy is expected for the perfect hexagonal unit cell [10].

2.2 Micro stresses

After computing the elastic properties, stresses at micro level can be calculated with the aid of the χ_m^{kl} fields. Equation (3) allows the determination of the stress tensor σ_{ij} for each point of the unit cell:

$$\sigma_{ij} = \left(E_{ijkl} - E_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) \frac{\partial u_k^0}{\partial x_l} = \left(E_{ijkl} - E_{ijmn} \frac{\partial \chi_m^{kl}}{\partial y_n} \right) \epsilon_{kl}^0. \tag{3}$$

In Eq.(3), ϵ_{kl}^0 are the strains at macro level that are obtained from the average macroscopic displacement field $\mathbf{u}^0$ and x_l are the coordinates at macro level. In our case the stress calculations are performed using the software POSTMAT [1].

3 FAILURE ANALYSIS

In this section, a methodology to determine failure using the asymptotic homogenization technique is presented. The failure methodology presented here is based on the work presented by [11] in which the unidirectional strengths of the composite are used to calculate the strengths of the constituents. At the composite micro level, three regions are considered in the analysis: matrix, fiber and the interface between them, and each of them has its own failure criterion.

3.1 Matrix failure criterion

Epoxy matrices are considered isotropic materials with different tensile and compressive strengths. There are several experiments [11] showing that the matrix failure depends on the deviatoric stress invariant J_2 and on the volumetric stress invariant I_1. A failure criterion based on the relationship between those invariants is presented in [11]. It is known as the modified von Mises criterion for isotropic material that has different compressive and tensile strength, and is given as follows:

$$\left(\frac{\sigma_{VM}}{\sigma_{VM}^{cr}}\right)^{n_p} + \left(\frac{I_1}{I_1^{cr}}\right) = 1. \tag{4}$$

The mentioned invariants are calculated by:

$$I_1 = \sigma_{11}^m + \sigma_{22}^m + \sigma_{33}^m, \tag{5}$$

$$I_2 = -(\sigma_{11}^m\sigma_{22}^m + \sigma_{11}^m\sigma_{33}^m + \sigma_{33}^m\sigma_{22}^m) + (\sigma_{12}^m)^2 + (\sigma_{23}^m)^2 + (\sigma_{23}^m)^2, \tag{6}$$

$$J_2 = \frac{I_1^2}{3} + I_2. \tag{7}$$

In Eq.(5) and (6), σ_{ij}^m are the components of the stress tensor of the matrix and I_2 is the second volumetric stress invariant. In Eq.(4), n_p is a positive real number and the terms σ_{VM}, σ_{VM}^{cr} and I_1^{cr} are described below:

$$\sigma_{VM} = \sqrt{I_1^2 - 3I_2}, \tag{8}$$

$$\sigma_{VM}^{cr} = T_m\left(\frac{\alpha^{n_p}+\alpha}{\alpha+1}\right)^{1/n_p}, \tag{9}$$

$$I_1^{cr} = T_m\left(\frac{\alpha^{n_p}+\alpha}{\alpha-1}\right). \tag{10}$$

In Eq.(9) and Eq.(10) T_m is the tensile strength of the matrix and $\alpha = C_m/T_m$, where C_m is the compressive strength of the matrix.

3.2 Fiber failure criterion

Carbon and glass reinforcing fibers are considered to be transversally isotropic [11]. They have considerably higher elastic modulus and strength in the longitudinal direction than the matrix. In [11], the authors apply a quadratic failure criterion for fiber and show that terms regarding transverse strength can be eliminated from it. The result is a criterion that compares the normal stress in the fiber σ_{11}^f to fiber longitudinal strengths:

$$-C_f < \sigma_{11}^f < T_f. \tag{11}$$

Where C_f ($C_f > 0$) and T_f are the compressive and tensile strength of the fiber in the longitudinal direction, respectively.

3.3 Interface failure criterion

The interface failure can be caused by debonding or detachment between fiber and matrix [11]. In this work, it is assumed that normal and shear stresses at that region are responsible for interface failure. The failure criterion is also presented in [11]:

$$\left(\frac{\langle t_n\rangle}{Y_n}\right)^2 + \left(\frac{t_s}{Y_s}\right)^2 = 1. \tag{12}$$

In Eq.(12) Y_n and Y_s are the normal and shear strengths of the interface, respectively, and t_n and t_s are the normal and shear stresses, respectively, at the interface; and the symbol $\langle \cdot \rangle$ is an operator that returns 0 if the argument is negative. It means that only interface tractions cause failure.

3.4 Unit cell orientation

In this work the perfect hexagonal and square unit cells are used to sample the microstructure. It can be seen in Fig.1 that the fiber distribution using those cell within the composite has a constant pattern. However, in real unidirectional fiber reinforced composites the fiber distribution is rather random. Then, in order to account for this randomness, all possible unit cell orientations are considered as depicted in Fig.2.

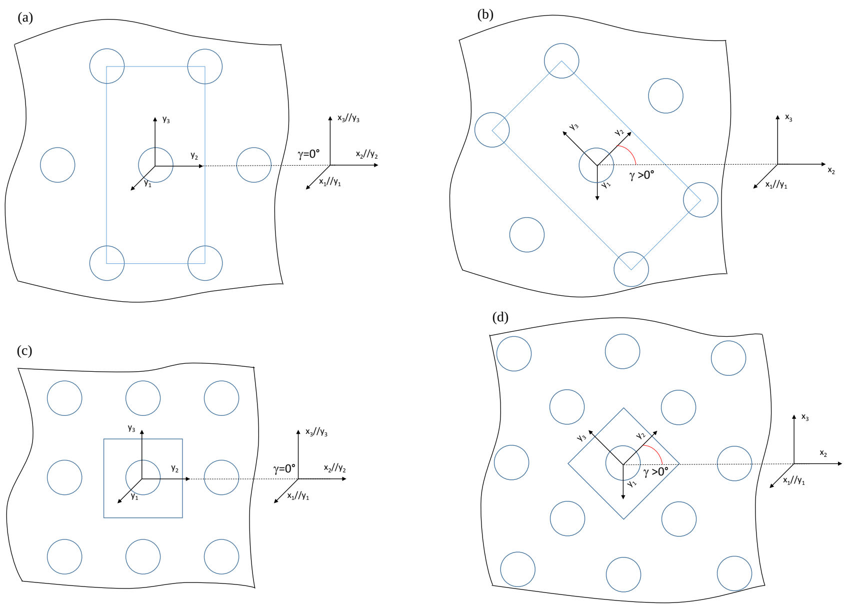

Figure 2: Possible orientations for the perfect hexagonal ((a) and (b)) and square unit cells ((c) and (d)).

Considering $\mathbf{x} = (x_1, x_2, x_3)$ the lamina coordinate system, the angle γ accounts for the rotation of the unit cell around the axis y_1. All possible fiber arrangements are within the domain $\gamma \in [0, \pi]$ rad.

3.5 Strengths of the constituents

The unidirectional strengths of the composite are used to calculate the strengths of the constituents. The five unidirectional strengths of the composite are: X_t and X_c are the tensile and compressive strengths at the longitudinal direction, Y_t and Y_c are the tensile and compressive strengths at the transversal direction and S_{12} is the in-plane shear strength. Those are considered as loads at the macro level and the micro stresses due to each of the macro loads are calculated for all possible unit cell orientations.

Three graphics can be plotted using the micro stresses, each of them representing a region of the microstructure: σ_{11}^f is plotted in the fiber graphic, σ_{VM} x I_1 in the matrix graphic and t_s x t_n in the interface graphic. Those are exemplified in Fig.3.

Those graphics combined with the failure criteria of Eq.(4), Eq.(11) and Eq.(12) are utilized to calculate the strengths of the constituents.

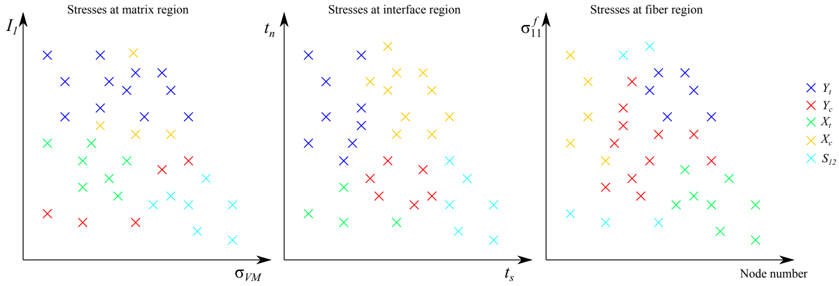

Figure 3: Stresses cloud of points of each region of the micro level.

For matrix, an optimization process is used to determine T_m, C_m and n_p, where the quadratic distance between the failure criterion and the micro stresses are minimized. For the fiber and interface, the strengths C_f, T_f, t_n and t_s are calculated in order to guarantee that the unidirectional strengths of the composite are exactly predicted.

Finally, with the strengths of the constituents, the failure envelope can be built.

4 Results

In this section, the failure methodology is employed to an AS4/3501-6 carbon epoxy composite and the numerical results are compared to experimental data [7].

The elastic properties of the constituents are shown in Tab.1.

Properties/Material	AS4 Carbon	3501-6 epoxy
E_1 (GPa)	225	4.2
$E_2 = E_3$ (GPa)	15	4.2
$G_{12} = G_{13}$ (GPa)	15	1.567
G_{23} (GPa)	7	1.567
$\nu_{12} = \nu_{13}$	0.2	0.34
ν_{23}	0.2	0.34

Table 1: Elastic properties of the constituents. Source:[12]

The elastic properties of the constituents are used as inputs to the homogenization problem. The homogenized elastic properties of the composite are calculated by Eq.(1). The fiber volume fraction used in the failure analysis was $V_f = 55\%$ due to this volume fraction yielded the least sum of error between experimental and numerical elastic properties when the perfect hexagonal unit cell was used, as can seen in Tab.2.

Now, the unidirectional strengths of the composite depicted in Tab.3 are applied as macro loads.

Applying those strengths as macro loads the micro stresses are calculated for each region of the microstructure and graphics can be built with those stresses. Those in conjunction with the failure criterion equation are used to determine the strengths of the constituents. The calculated strengths of the constituents are shown in Tab.4.

With the strengths of the constituents, the failure criterion equations of each region are defined. Figures 4 and 5 show the graphics for the perfect hexagonal and square unit cell plotted with the failure criterion equations.

	Experimental data	Asymptotic homogenization			
Elastic properties	$V_f = 60\%$	$V_f = 60\%$	Δ (%)	$V_f = 56\%$	Δ (%)
E_1 (GPa)	126.000	136.060	8.0	127.230	1.0
E_2(GPa)	11.000	8.856	19.5	8.437	23.3
E_3 (GPa)	-	8.841	-	8.425	-
G_{12} (GPa)	6.600	4.544	31.2	4.175	36.7
G_{23} (GPa)	-	3.386	-	3.186	-
G_{13} (GPa)	-	4.523	-	4.161	-
ν_{23}	0.400	0.361	9.8	0.374	6.5
ν_{13}	-	0.252	-	0.257	-
ν_{12}	0.280	0.251	10.4	0.257	8.2
			$\overline{\Delta} = 15.8\%$		$\overline{\Delta} = 15.1\%$

Table 2: Experimental elastic properties of the AS4/3501-6 carbon epoxy composite [12] and from asymptotic homogenization using perfect hexagonal unit cell. The difference between experimental data and homogenized is $\Delta = 100\frac{|Exp-AH|}{Exp}$. The average error is $\overline{\Delta}$. The sign "-" means that experimental data were not provided.

Strength properties	AS4/3501-6 carbon epoxy
Y_t (MPa)	60.2 (b)
Y_c (MPa)	273.3 (b)
X_t (MPa)	1950 (a)
X_c (MPa)	1480 (a)
S_{12} (MPa)	73.4 (b)

Table 3: Unidirectional strength properties of AS4/3501-6 carbon epoxy. Sources: (a) - [12]; (b) - [7].

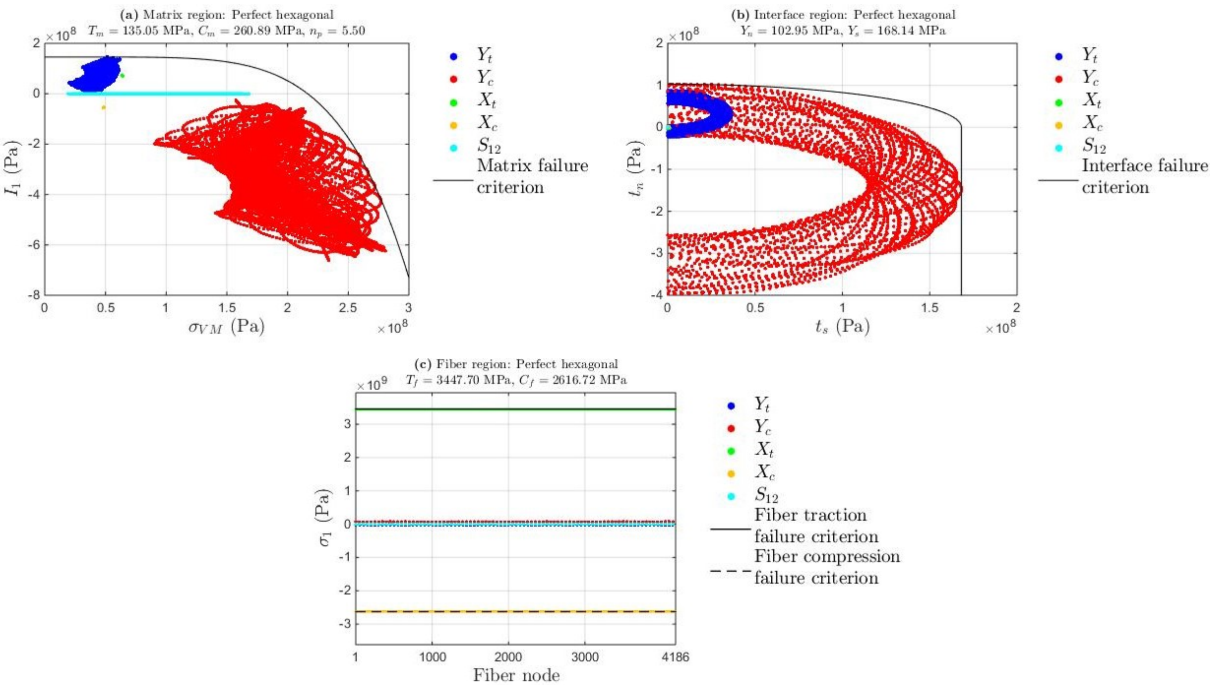

Figure 4: AS4/3501-6 carbon epoxy cloud of points and the failure criteria for each region using the perfect hexagonal unit cell.

Finally, the numerical failure envelopes can be constructed and be compared to experimental data. In addition to the asymptotic homogenization, the Puck & Schürmann [2] failure envelope is also plotted. The parameters that feed this criterion are $m_f = 1.1$ and $p_{\perp\|}^{(-)} = 0.3$. Figures 6

Material	Strength parameter	Perfect hexagonal	Square
Matrix	T_m (MPa)	135.05	130.84
	C_m (MPa)	260.89	270.33
	n_p	5.50	3.66
Interface	Y_n (MPa)	102.95	84.17
	Y_s (MPa)	168.14	168.53
Fiber	T_f (MPa)	3447.70	3434.07
	C_f (MPa)	2616.72	2606.37

Table 4: Strengths of the constituents and parameters for the perfect hexagonal and square unit cells of the AS4/3501-6 carbon epoxy.

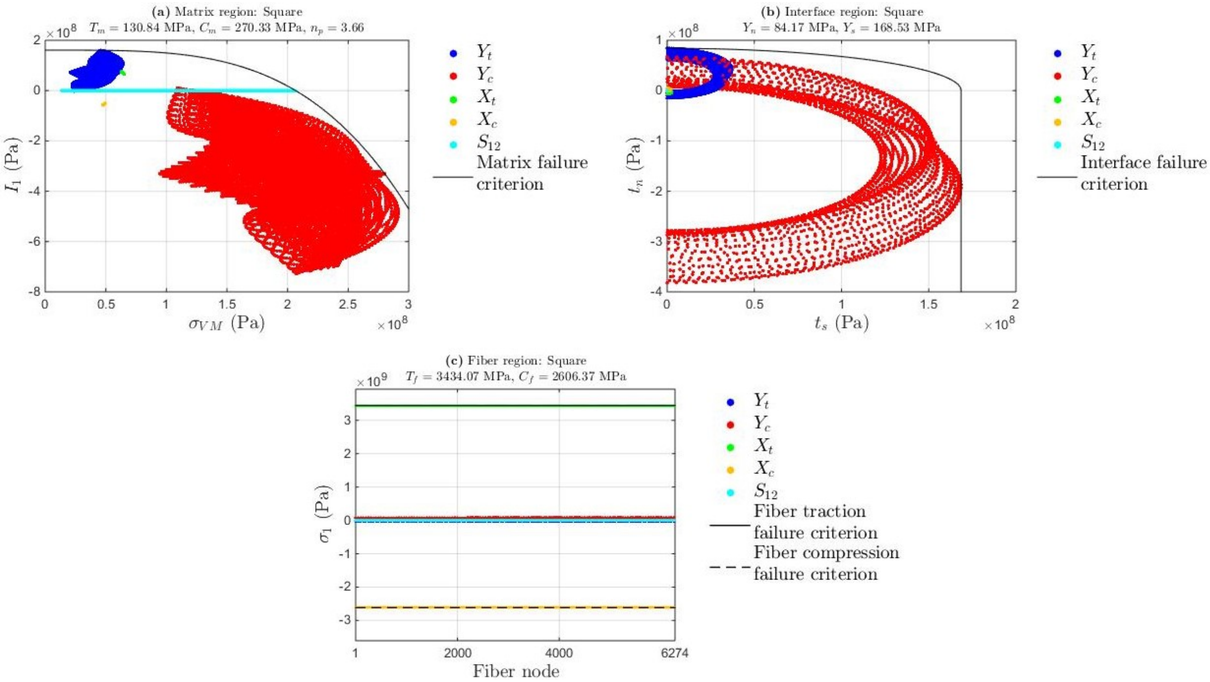

Figure 5: AS4/3501-6 carbon epoxy cloud of points and the failure criteria for each region using the square unit cell.

to 8 show the failure envelopes calculated.

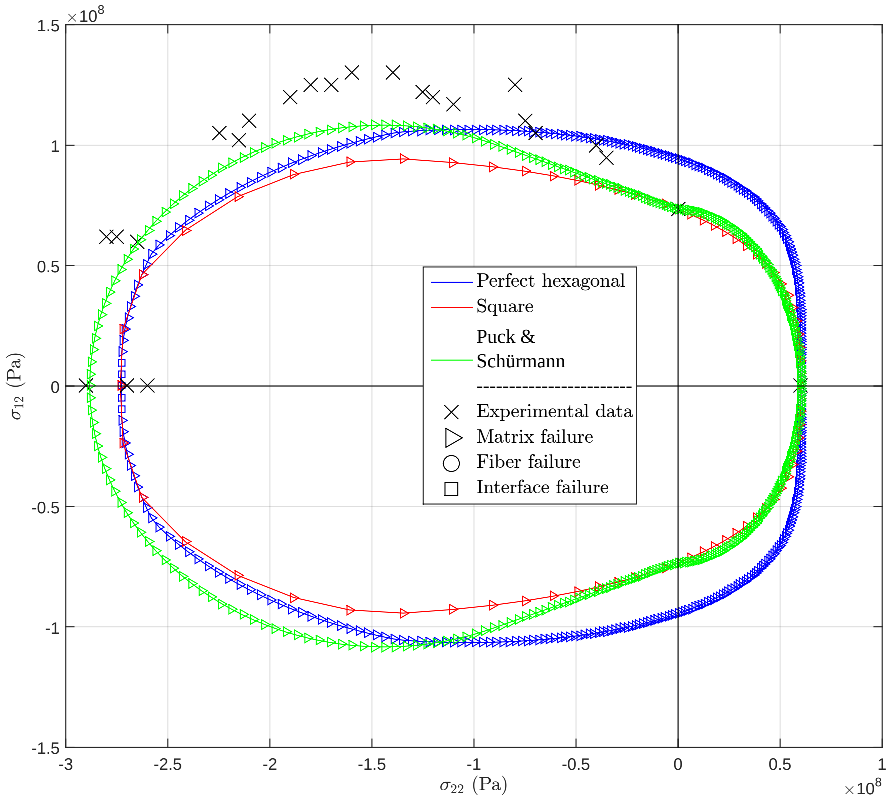

Figure 6: AS4/3501-6 carbon epoxy failure envelope at σ_{22} x σ_{12}.

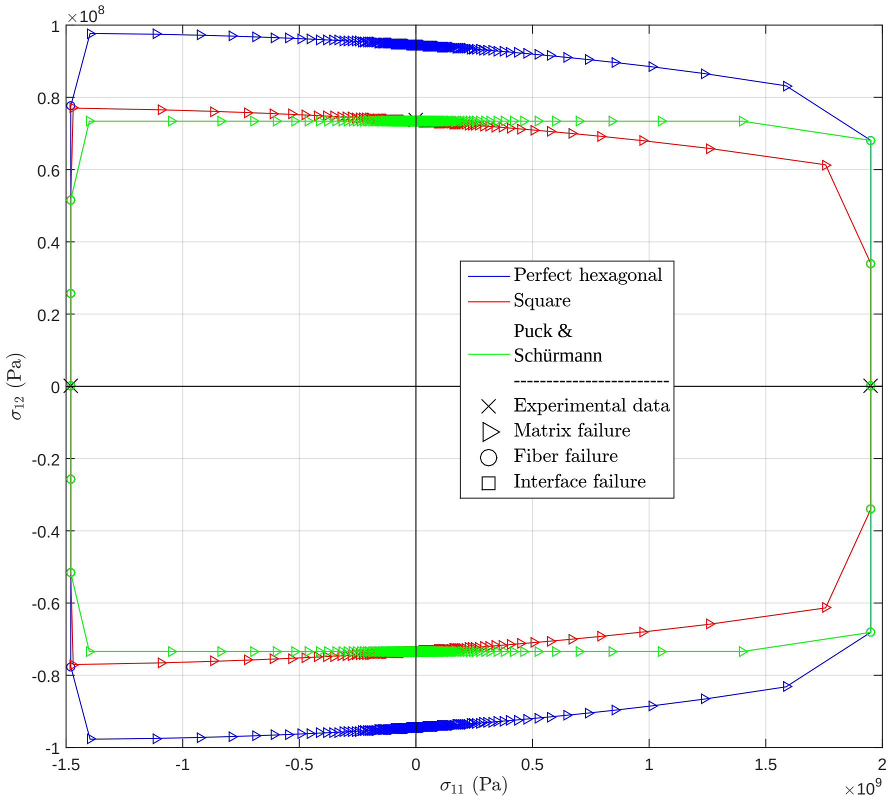

Figure 7: AS4/3501-6 carbon epoxy failure envelope at σ_{11} x σ_{12}.

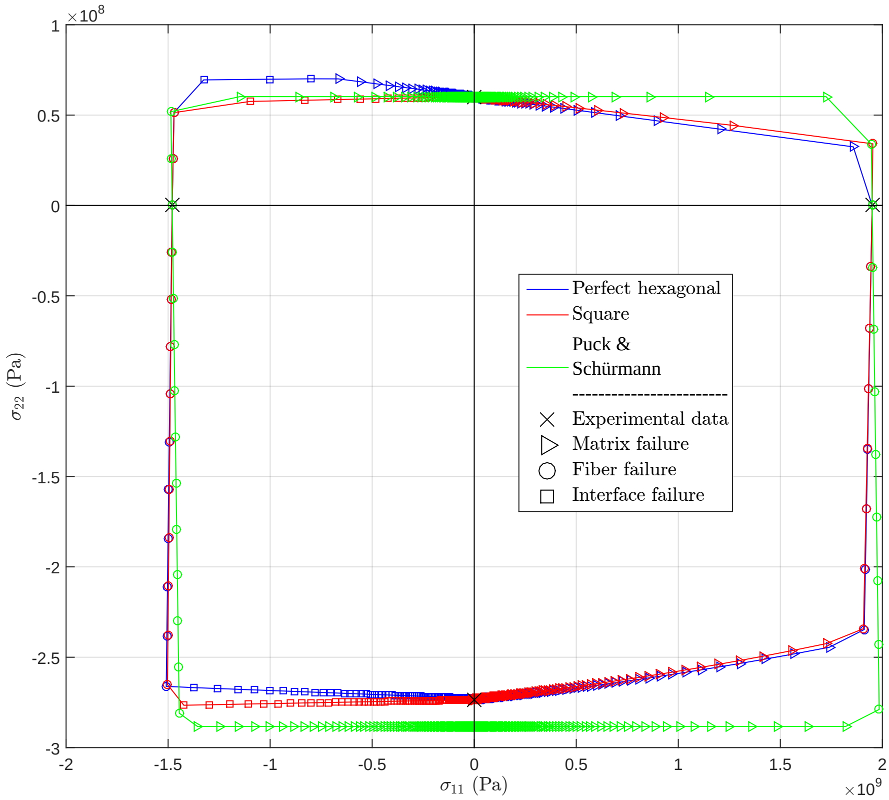

Figure 8: AS4/3501-6 carbon epoxy failure envelope at σ_{11} x σ_{22}.

5 CONCLUSIONS

This work proposed a methodology to predict failure of unidirectional fiber reinforced composite using the asymptotic homogenization technique. It can be seen that the failure envelopes calculated using the methodology predict well the experimental failure points. It can also be noted that the numerical failure envelopes calculated are very similar to the Puck & Schürmann failure envelope in some regions of the graphics. The proposed methodology revealed to be a good method to predict failure of unidirectional fiber reinforced composites using the unidirectional strengths of this material.

The differences between numerical and experimental failure envelopes are caused by simplifications on the mathematical model, which are:

- The interface between fiber and matrix is considered perfect, this is: the displacements of that region are the same for matrix and fiber. In a real composite, interfacial imperfections may exist, changing the composite behaviour;
- The fiber alignment is also perfect, all fibers are parallel due to the periodicity, which do not happen in the real material;
- There is no fiber waviness, a common characteristic in real composites;
- The fibers, because of the periodicity and the unit cell model, are perfectly distributed inside the macrostructure, this is: the fiber volume fraction is constant within the microstructure. Random distributions are seen in a manufactured composite.

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