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SOLVING THE INVERSE PROBLEM OF ESTIMATING FUZZY VISCOELASTIC CONSTITUTIVE PARAMETERS

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Abstract. *This paper presents a numerical approach to solve the inverse problem of estimating fuzzy viscoelastic constitutive parameters. The direct fuzzy viscoelastic problem is solved by utilizing Scale Boundary Finite Element Method(SBFEM), a temporally adaptive algorithms and the General Transformation Method (GTM). On the basis of the solution of direct problem, the fuzzy estimation can be realized using an inverse fuzzy arithmetic and an evolutionary algorithm of Differential Evolution(DE). Two numerical examples are provide to verify the proposed approach.*

1 INTRODUCTION

The experiment based estimation of viscoelastic constitutive parameters is fundamental for the viscoelastic analysis, and is an important issue related with many engineering aspects. In the estimation procedure, there are various uncertainties caused by measurements, observation, and modeling etc.. Such uncertainties may result in fault results, and necessitate to take into account.

As a matter of fact, there is a big pool containing a number of literatures related with analytical and numerical analysis for the estimation of viscoelastic constitutive parameters, However, to the best of authors' knowledge, nearly all these literatures are based on the deterministic assumption.

There are mainly three mathematical means to describe the uncertainty, including probabilistic method, interval analysis and fuzzy theory that can account epistemic uncertainties caused by a lack of knowledge or imperfections in the modeling procedure[1-3],and is becoming increasingly popular for the analysis of numerical models that incorporate uncertainty in their description [4-5].

In the context of fuzzy analysis, many well documented experimental, numerical and analytical results exist in literature concerned with various aspects, but there seems no any report directly related with the inverse fuzzy viscoelastic problems of parameters estimation.

This paper aims at developing a numerical approach to solve inverse problems of estimating unknown fuzzy constitutive parameters.

2 VISCOELASTIC CONSTITUTIVE RELATIONSHIP

In this paper, the viscoelastic constitutive relationships is specified by a three-parameter solid viscoelasticity model (see Figure 1) in a differential form [6]

$$\alpha_1 \mathbf{D}\boldsymbol{\varepsilon}(t) + p_1 \mathbf{D}\frac{\mathrm{d}\boldsymbol{\varepsilon}(t)}{\mathrm{d}t} = (\alpha_1 + \alpha_2)\boldsymbol{\sigma}(t) + p_1 \frac{\mathrm{d}\boldsymbol{\sigma}(t)}{\mathrm{d}t} \qquad (t > 0) \tag{1}$$

$$\boldsymbol{\sigma}(t) = \mathbf{D}\boldsymbol{\varepsilon}(t) \qquad (t = 0) \tag{2}$$

where

$$\mathbf{D} = \frac{E_2}{1-v^2}\begin{pmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{pmatrix} \quad \text{for the plane stress problem} \tag{3}$$

$$p_1 = \frac{\eta_1}{E_1 + E_2} \qquad \alpha_1 = \frac{E_1}{E_1 + E_2} \qquad \alpha_2 = \frac{E_2}{E_1 + E_2} \tag{4}$$

where v, E_1, E_2 and η_1 are constitutive parameters.

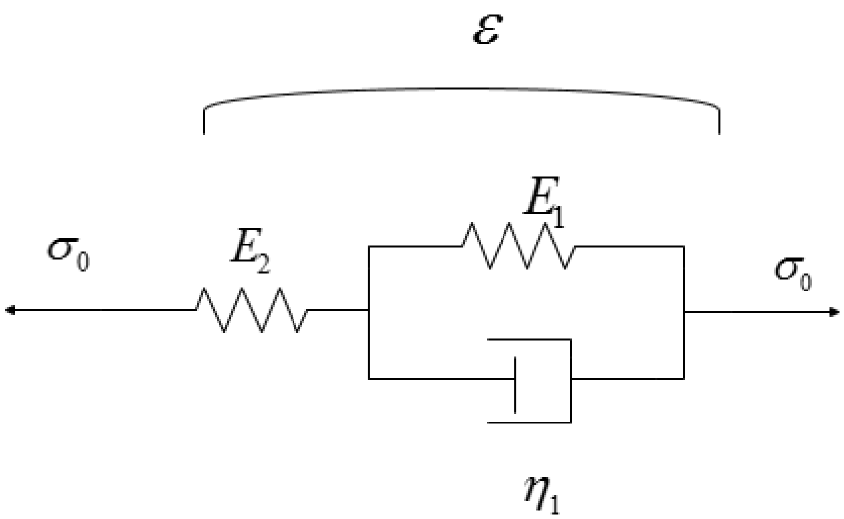


Figure 1: Three-parameter solid viscoelasticity model.

3 DESCRIPTION OF SOLVING DIRECT VISOELASTIC PROBLEMS WITH FUZZY UNCERTAINTY

In the direct visoelastic problem, E_1, E_2, η_1, and ν, are defined by fuzzy numbers $\tilde{p}_i(i = 1,2,3,4)$, the corresponding unknown fuzzy output of displacement ,$u(t)$, is defined by $\tilde{q}$.

The deterministic relationship between , E_1, E_2, η_1, ν and $u_s(t,\vec{x})$ can be obtained in the form [7]

$$u_s(t,\vec{x}) = u(t, \vec{x}, E_1, E_2, \eta_1, \nu..) \quad ,s = 1,2,3 \tag{5}$$

where t stands for time, and $\vec{x}$ refers to a vector of coordinates.
The indeterministic relationship between E_1, E_2, η_1, ν and $u_s(t,\vec{x})$ is defined by

$$\tilde{q}_r = \tilde{q}_r(\tilde{p}_1, \tilde{p}_2 \tilde{p}_3 . \tag{6}$$

where $\tilde{p}_i = \left\{(x_i, \mu_{\tilde{p}_i}(x_i)) \mid \mu_{\tilde{p}_i}(x_i) \in [0,1]\right\}$, x_i is a value of $\tilde{p}_i$, $i = 1,2,...,4$

$\tilde{q}_r, r = 1,2,...,N$ is a set of N fuzzy-valued outputs at *r-th* degree of freedom (DOF) in SBFEM.

If $\mu_{\tilde{p}_i}(x_i) = 1$, x_i is a member of the fuzzy set $\tilde{p}_i$.

If $\mu_{\tilde{p}_i}(x_i) = 0$, x_i is not a member of the fuzzy set $\tilde{p}_i$.

If $0 < \mu_{\tilde{p}_i}(x_i) < 1$, the membership of x_i is uncertain.

Using Eq.(5) and the General transformation method[8], $\tilde{q}_r$ can be obtained. The major steps to determine the membership function of $\tilde{q}_r$ using GTM include

(1) Subdividing $\mu_{\tilde{p}_i}$ into a number of mm segments, equally spaced by

$$\Delta\mu_{\tilde{p}_i} = \frac{1}{mm} \quad ,i = 1,2,\ldots,4 \tag{7}$$

where mm is the number of discrete segments.

(2) Decompose $\tilde{p}_i$ into a number of intervals

$X_i^{(j)} = [a_i^{(j)}, \mathrm{b}_i^{(j)}], a_i^{(j)} \le \mathrm{b}_i^{(j)}, i = 1,2,\ldots,4, j = 0,1,\ldots,mm$ by the α-cut at the *j-th* level ,and set $c_{l,i}^{(j)}$ within $X_i^{(j)}, l = 2,3,\ldots,mm-j, j = 0,1,\ldots,mm-2$ by

$$c_{l,i}^{(j)} = \begin{cases} a_i^{(j)} & l = 1, j = 0,1,\ldots,mm \\ \dfrac{(\mathrm{c}_{l-1,i}^{(j+1)} + \mathrm{c}_{l,i}^{(j+1)})}{2} & l = 2,3,\ldots,mm-j, j = 0,1,\ldots,mm-2 \\ b_i^{(j)} & l = mm-j+1, j = 0,1,\ldots,mm \end{cases} \tag{8}$$

$a_i^{(j)}, b_i^{(j)}$ denote the lower and upper bounds of the interval at the *j-th* membership level for $\tilde{p}_i$.

The decomposition procedure is shown in Figure 2.

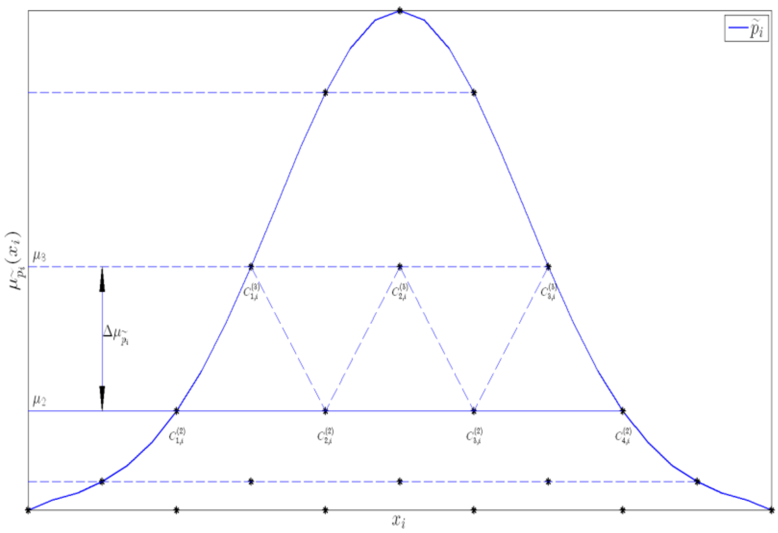


Figure 2:Decomposition of the i-*th* uncertain parameter $\tilde{p}_i$

(3) Construct a matrix $X^{(j)}$ at the j *-th* level

$$X^{(j)} = \begin{bmatrix} X_1^{(j)} \\ X_2^{(j)} \\ \cdots \\ X_4^{(j)} \end{bmatrix} \tag{9}$$

where

$$X_i^{(j)} = (\underbrace{(\gamma_{1,i}^{(j)}, \gamma_{2,i}^{(j)}, \ldots, \gamma_{mm+1-j,i}^{(j)}), \ldots, (\gamma_{1,i}^{(j)}, \gamma_{2,i}^{(j)}, \ldots, \gamma_{mm+1-j,i}^{(j)})}_{(mm+1-j)^{i-1} \; (mm+1-j) \; tuples}) \tag{10}$$

$$\gamma_{l,i}^{(j)} = \underbrace{(c_{l,i}^{(j)}, \ldots, c_{l,i}^{(j)})}_{(mm+1-j)^{4-i} \; elements} \tag{11}$$

$$i = 1,2,\ldots,4,\ j = 0,1,2,\ldots,mm,\ l = 1,2,3,\ldots,mm+1-j \tag{12}$$

(4) Determine ${}^k\hat{q}_r^{(j)}$ at *j-th* level

$${}^k\hat{q}_r^{(j)} = \tilde{q}_r({}^kX_1^{(j)}, {}^kX_2^{(j)}, \ldots, {}^kX_4^{(j)}, t),\ j = 0,1,2,\ldots,mm,\ k = 1,2,\ldots,(mm+1-j)^4 \tag{13}$$

where ${}^k\hat{q}_r^{(j)}$ denotes the k *-th* element of $\hat{q}_r^{(j)}$ and ${}^kX_i^{(j)}, i = 1,2,\ldots,4$ denotes the k *-th* column of the matrix $X^{(j)}$.

(5) Evaluate $\tilde{q}_r$ in a decomposed form

$$\tilde{q}_r^{(j)} = [a^{(j)}, b^{(j)}],\ a^{(j)} \le b^{(j)},\ j = 0,1,\ldots,mm \tag{14}$$

where

$$\begin{aligned} a^{(j)} &= \min_k(a^{(j+1)}, {}^k\hat{q}_r^{(j)}) \\ b^{(j)} &= \max_k(b^{(j+1)}, {}^k\hat{q}_r^{(j)}) \end{aligned},\ j = 0,1,\ldots,mm-1 \tag{15}$$

$$a^{(mm)} = \min_k({}^k\hat{q}_r^{(mm)}) = \max_k({}^k\hat{q}_r^{(mm)}) = b^{(mm)} \tag{16}$$

$\tilde{q}_r^{(j)}$ is a decomposition interval for $\tilde{q}_r$ at the *j-th* level μ_j.

(6) Evaluate $\tilde{q}_r$ in an ensemble form

By recomposing $\tilde{q}_r^{(j)}$ according to their levels of membership μ_j, an approximation for the ensemble $\tilde{q}_r$ can be achieved.

4 DESCRIPTION OF SOLVING INVERSE VISCOELASTIC PROBLEMS WITH FUZZY UNCERTAINTY

The target of solving inverse fuzzy viscoelastic problems is to seek membership functions of unknown constitutive parameter $E_1, E_2, \eta_1, \ \nu$, it can be achieved by an inverse fuzzy arithmetic proposed by Hanss [9] with Eq. (5) via some partially known membership functions of $u(t)$ at some discrete time points $t_0, t_1, t_2, ..., t_{nn}$.

The major procedure is described as following.

(1) Evaluation of peak values

Under the condition in which $\tilde{q}$ is strictly monotonic with respect to each of $\tilde{p}_i$, $\bar{x}_1$, $\bar{x}_2$,..., $\bar{x}_4$, the peak values of $\tilde{p}_i, i = 1, 2, ..., n$ can be evaluate via the inverse solutions of following equations.

$$u(t_{\mathrm{m}}) = u(t_m, \ \bar{x}_1, \bar{x}_2, .., \bar{x}_4) \ \ (m = t_0, t_1, t_2, ..., t_{nn}) \tag{17}$$

In this paper, the inverse solutions are given by DE [10].

(2) Computation of gain factors

Single-sided gain factors $\eta_{i+}^{(j)}, \eta_{i-}^{(j)}$,which describes the effect of the *i-th* uncertainty parameter $\tilde{p}_i$ on the uncertainty of $\tilde{q}_r$ when only positive deviations and negative deviations from the peak value i=1, 2, ..., 4 are considered, are given by

$$\eta_i^{(j)} = \frac{1}{2^3 (b_i^{(j)} - a_i^{(j)})} \times \sum_{k=1}^{2^{4-i}} \sum_{l=1}^{2^{i-1}} ({}^{k_2}\hat{q}_r^{(j)} - {}^{k_1}\hat{q}_r^{(j)}) \tag{18}$$

$$\begin{aligned} & i = 1, 2, ..., 4, \ j = 0, 1, ..., mm - 1 \\ & k_1 = k + (l-1) 2^{5-i}, \\ & k_2 = k + (2l - 1) 2^{4-i}. \end{aligned} \tag{19}$$

where $a_i^{(j)}$ and $b_i^{(j)}$ are guessed lower and upper bounds of $\tilde{p}_i$ at membership μ_j, ${}^{k_2}\hat{q}_r^{(j)}$ and ${}^{k_1}\hat{q}_r^{(j)}$ refer to Eq. (13).

(3) Assembly of $\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_4$

$\breve{a}_i^{(j)}$ and $\breve{b}_i^{(j)}$, which are the lower and upper bounds of the intervals of $\tilde{p}_i$ at membership μ_j can be obtained by

$$H^{(j)}\begin{bmatrix} \breve{a}_1^{(j)}-\bar{x}_1 \\ \breve{b}_1^{(j)}-\bar{x}_1 \\ \breve{a}_2^{(j)}-\bar{x}_2 \\ \breve{b}_2^{(j)}-\bar{x}_2 \\ \vdots \\ \breve{a}_4^{(j)}-\bar{x}_4 \\ \breve{b}_4^{(j)}-\bar{x}_4 \end{bmatrix}=\begin{bmatrix} c_1^{(j)}-\bar{q}_1 \\ d_1^{(j)}-\bar{q}_1 \\ c_2^{(j)}-\bar{q}_2 \\ d_2^{(j)}-\bar{q}_2 \\ \vdots \\ c_{NN}^{(j)}-\bar{q}_{NN} \\ d_{NN}^{(j)}-\bar{q}_{NN} \end{bmatrix} \tag{20}$$

where

$$H^{(j)}=\begin{bmatrix} H_{11}^{(j)} & H_{12}^{(j)} & \cdots & H_{14}^{(j)} \\ H_{21}^{(j)} & H_{22}^{(j)} & \cdots & H_{24}^{(j)} \\ \vdots & \vdots & \vdots & \vdots \\ H_{\mathrm{NN1}}^{(j)} & H_{NN2}^{(j)} & \cdots & H_{NN4}^{(j)} \end{bmatrix} \tag{21}$$

$$\begin{aligned} & H_{ri}^{(j)}=\frac{1}{2}\begin{bmatrix} \eta_{i-}^{(j)}(1+\mathrm{sgn}(\eta_{i-}^{(j)})) & \eta_{i+}^{(j)}(1-\mathrm{sgn}(\eta_{i+}^{(j)})) \\ \eta_{i-}^{(j)}(1-\mathrm{sgn}(\eta_{i-}^{(j)})) & \eta_{i+}^{(j)}(1+\mathrm{sgn}(\eta_{i+}^{(j)})) \end{bmatrix} \\ & i=1,2,\ldots 4.r=1,2,\ldots NN \end{aligned} \tag{22}$$

$c_i^{(j)}$ and $d_i^{(j)}$ are the known lower and upper bounds of the intervals of $\tilde{q}_r$ at membership μ_j.NN is the number of DOF with known output information.

The membership functions of the unknown $\tilde{p}_i$ can be described by $\breve{a}_i^{(j)}$,$\breve{b}_i^{(j)}$ and $\bar{x}_k$, $i,k=1,2,\ldots,4$.

5 NUMERICAL EXAMPLES

For the simplicity, all variables are assumed dimensionless. The DE parameters are set as following: maximum number of iterations is 1000, population size is 50, lower bound of scaling factor is 0.2, upper bound of scaling factor is 0.8, crossover probability is 0.2.

Example 1 Considers the three-parameter solid viscoelasticity model(see Figure 1) under a constant loading σ_0. In this case, the model material is rock, and the constitutive relationship is defined by

$$\varepsilon(t)=\frac{\sigma_0}{E_2}+\frac{\sigma_0}{E_1}(1-e^{-\frac{E_1}{\eta_1}t}) \tag{23}$$

where $\sigma_0=1$ $E_2=9800$. E_1,η_1 refers to $\tilde{p}_1,\tilde{p}_2$, and is defined by

$$\mu_{\tilde{p}_i}(x_i) = \begin{cases} \exp\left[-\dfrac{(x_i - \overline{x}_i)^2}{2d_i^2} \right], \; for \left| x_i - \overline{x}_i \right| < 3d_i \\ 0, \; for \left| x_i - \overline{x}_i \right| \geq 3d_i \end{cases}, i = 1,2 \tag{24}$$

where

$$\begin{array}{ll} \overline{x}_1 = 1960 & \overline{x}_2 = 52083 \\ d_1 = 98 & d_1 = 2604.15 \end{array} \tag{25}$$

The range of time is [0,10] and the time step is set to 0.1.

Figure 3 shows the membership function of $\tilde{p}_1, \tilde{p}_2$ where mm is set to 20.

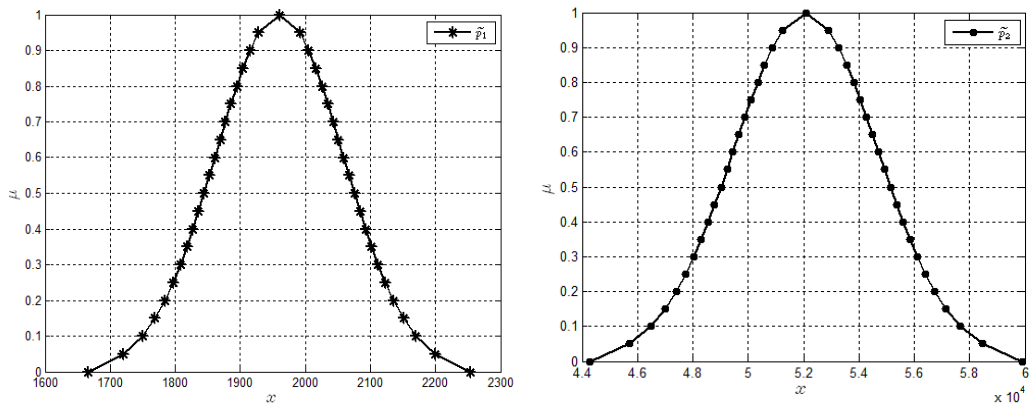
Figure 4 shows the fuzzy response of ε with respect to prescribed E_1, η_1.

The membership functions of ε at *t*=0,0.1,0.2,0.3..1.4 are employed as known information.

Figure 5 exhibits a comparison of $\tilde{p}_1$ and $\tilde{p}_2$ given by the proposed approach with prescribed ones, the maximum relative error between prescribed and estimated $\tilde{p}_1$ is 2.60%,and 1.04% between prescribed and estimated $\tilde{p}_2$.

Figure 6 shows the iteration history of DE at *0-th* membership level used in the step of inversion.

The interval boundary of prescribed and estimated $\tilde{p}_1, \tilde{p}_2$ at each membership is shown in Table 1 and Table 2 respectively where LB means left boundary and RB means right boundary.



(a) (b)

Figure 3:Symmetrical Gaussian fuzzy numbers $\tilde{p}_1$ and $\tilde{p}_2$.(a) $\tilde{p}_1$.(b) $\tilde{p}_2$

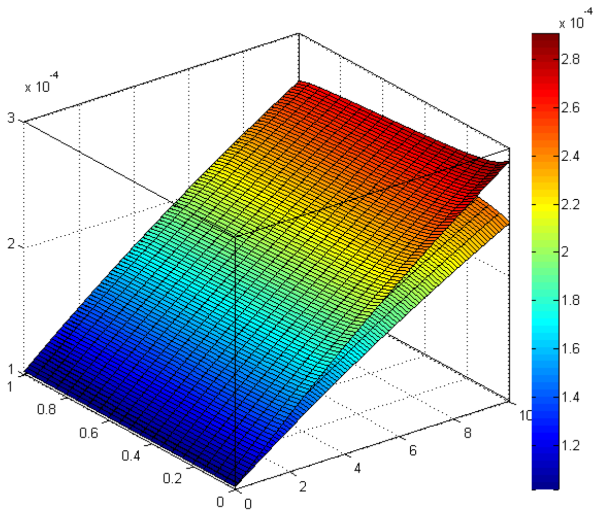


Figure 4: The fuzzy response of ε

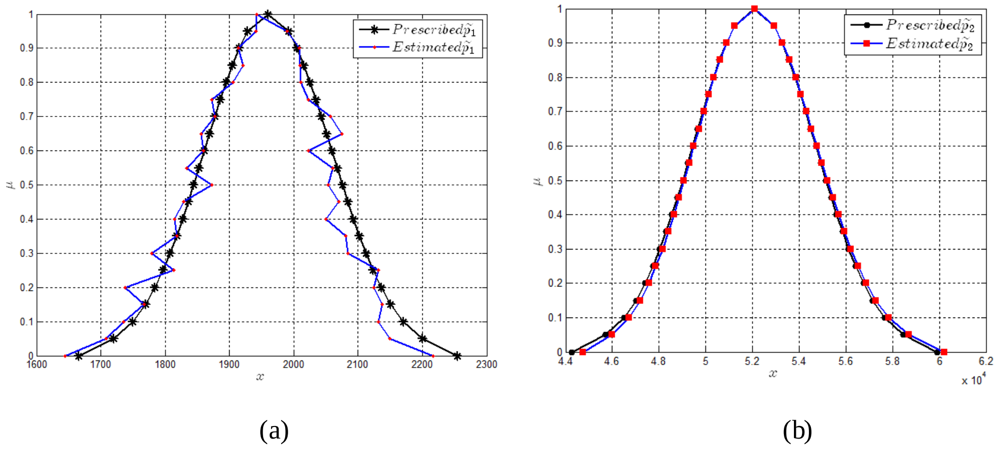


Figure 5 A comparison of prescribed and estimated $\tilde{p}_1$, $\tilde{p}_2$.(a).comparison of prescribed and estimated $\tilde{p}_1$.(b). comparison of prescribed and estimated $\tilde{p}_2$.

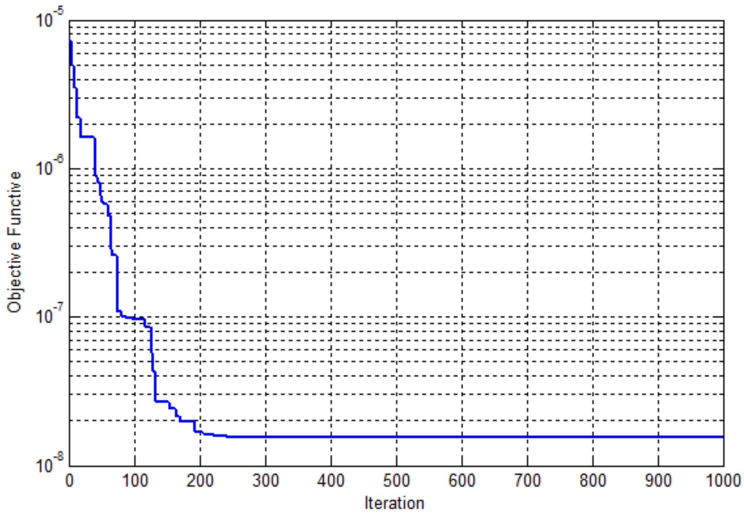


Figure 6: Iteration history of DE at *0-th* membership level.

Table 1:The interval boundary of prescribed $\tilde{p}_1, \tilde{p}_2$ at each membership.

Membership $\mu_{\tilde{p}_i}(x_i)$	LB of $\tilde{p}_1$	RB of $\tilde{p}_1$	LB of $\tilde{p}_2$	RB of $\tilde{p}_2$
1.000000	1960.000000	1960.000000	52083.000000	52083.000000
0.950000	1928.611441	1991.388558	51248.913118	52917.086881
0.900000	1915.013726	2004.986273	50887.581595	53278.418404
0.850000	1904.128150	2015.871849	50598.319616	53567.680383
0.800000	1894.531371	2025.468628	50343.304803	53822.695196
0.750000	1885.664293	2034.335706	50107.680307	54058.319692
0.700000	1877.229157	2042.770842	49883.533787	54282.466212
0.650000	1869.035840	2050.964159	49665.813111	54500.186888
0.600000	1860.944770	2059.055229	49450.809417	54715.190582
0.550000	1852.840058	2067.159941	49235.443242	54930.556757
0.500000	1844.613817	2075.386182	49016.847689	55149.152310
0.450000	1836.154387	2083.845612	48792.055583	55373.944416
0.400000	1827.334584	2092.665415	48557.687338	55608.312661
0.350000	1817.996537	2102.003462	48309.547786	55856.452213
0.300000	1807.927945	2112.072054	48041.995514	56124.004485
0.250000	1796.819296	2123.180703	47746.805818	56419.194181
0.200000	1784.175987	2135.824012	47410.835688	56755.164311
0.150000	1769.107672	2150.892327	47010.425974	57155.574025
0.100000	1749.695329	2170.304670	46494.582572	57671.417427
0.050000	1720.120810	2199.879189	45708.700090	58457.299909
0.000000	1666.000000	2254.000000	44270.550000	59895.450000

Table 2:The interval boundary of estimated $\tilde{p}_1, \tilde{p}_2$ at each membership.

Membership $\mu_{\tilde{p}_i}(x_i)$	LB of $\tilde{p}_1$	RB of $\tilde{p}_1$	LB of $\tilde{p}_2$	RB of $\tilde{p}_2$
1.000000	1942.091888	1942.091888	52092.133589	52092.133589
0.950000	1941.704284	1989.675237	51241.514079	52924.630769
0.900000	1913.567391	2008.507551	50895.673356	53284.662664
0.850000	1920.806585	2009.527200	50600.240660	53582.141626
0.800000	1905.677396	2011.281705	50356.896392	53849.245699
0.750000	1872.697636	2022.596843	50135.697758	54084.679594
0.700000	1877.325231	2057.531698	49919.216384	54302.629658
0.650000	1855.714355	2074.421790	49709.882968	54518.237541
0.600000	1860.045326	2022.486221	49492.167331	54777.565568
0.550000	1833.528683	2060.997639	49299.879755	54974.083302
0.500000	1872.131222	2053.463510	49067.589422	55208.184748
0.450000	1829.143780	2070.269237	48866.174422	55443.235282
0.400000	1814.798895	2049.936918	48646.071934	55699.262524
0.350000	1817.706941	2080.728220	48403.983274	55946.477782
0.300000	1779.252500	2083.824668	48167.880228	56224.833575
0.250000	1812.915434	2131.030043	47867.654497	56507.772061
0.200000	1737.820053	2124.651896	47582.681851	56879.581643
0.150000	1767.853034	2137.985729	47188.502901	57297.399235
0.100000	1735.500517	2131.909586	46718.589420	57848.333607
0.050000	1708.101554	2149.767512	46002.731009	58698.303257
0.000000	1644.807367	2216.345710	44730.752739	60209.475501

Example 2 Considers an estimation of E_1 and η_1 in Figure 1 for a square viscoelastic plate subjected to a uniform tension $p=1$ as shown in Figure 7 where $a=1, E_2=9800, \nu=0.3$.Figure 8 describes a node distribution of SBFEM with an adjacent node distance $ds=0.1$.

The membership functions of E_1 and η_1, refer to $\tilde{p}_1$ and $\tilde{p}_2$ defined by Eqs.(24-25), are prescribed in Figure 9 where mm is set to 10.

The range of time is [0,10] and the time step is set to 0.1.

The fuzzy response of displacements are given using Eq. (5), and the membership functions of displacement of nodes 12,13,14,..20 along x direction at t=1,2,3..10 are employed as known information.

Figure 10 shows the fuzzy response of u_x at node 16 with respect to prescribed E_1, η_1 where u_x is the displacement along x direction.

Figure 11 exhibits a comparison of $\tilde{p}_1$ and $\tilde{p}_2$ given by the proposed approach with prescribed ones, the maximum relative error between prescribed and estimated $\tilde{p}_1$ is 0.74%,and 0.59% between prescribed and estimated $\tilde{p}_2$.

Figure 12 shows the iteration history of DE at 0-th membership level used in the step of inversion.

The interval boundary of prescribed and estimated $\tilde{p}_1, \tilde{p}_2$ at each membership is shown in Table 3 and Table 4 respectively.

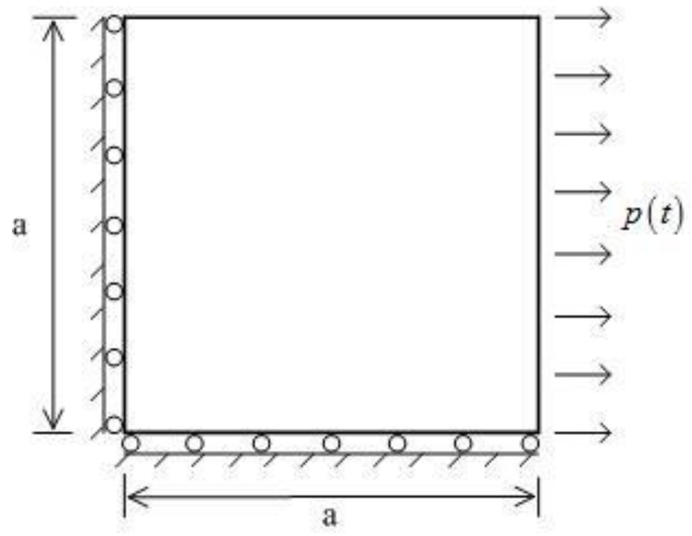


Figure 7: A square viscoelastic plate subjected to a uniform tension

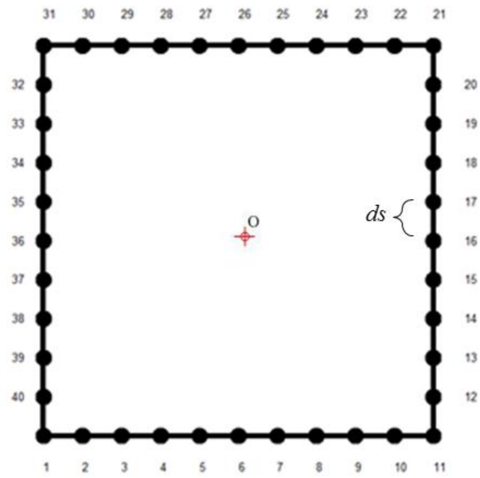
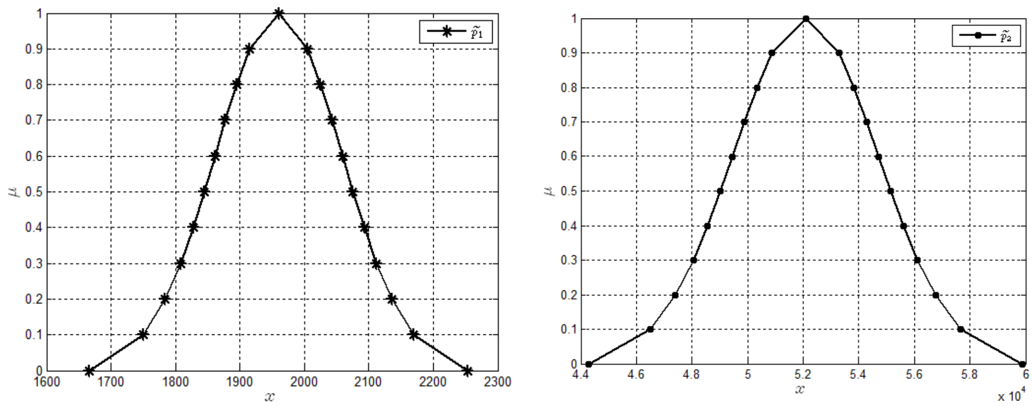


Figure 8: Node distribution of SBFEM



(a) (b)

Figure 9: Symmetrical Gaussian fuzzy numbers $\tilde{p}_1$ and $\tilde{p}_2$.(a) $\tilde{p}_1$.(b) $\tilde{p}_2$

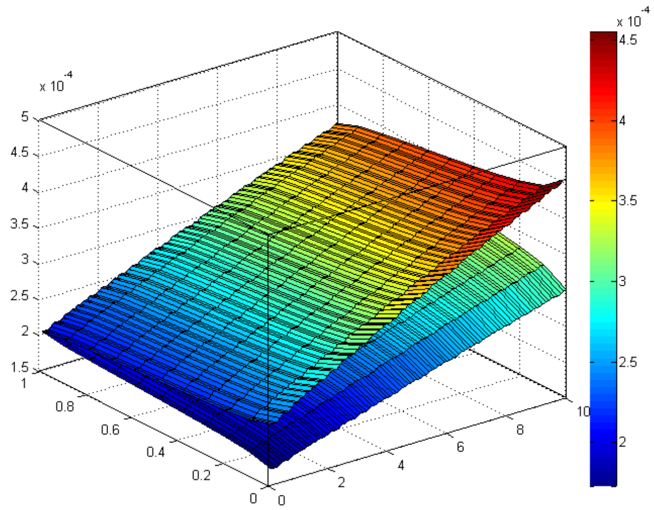
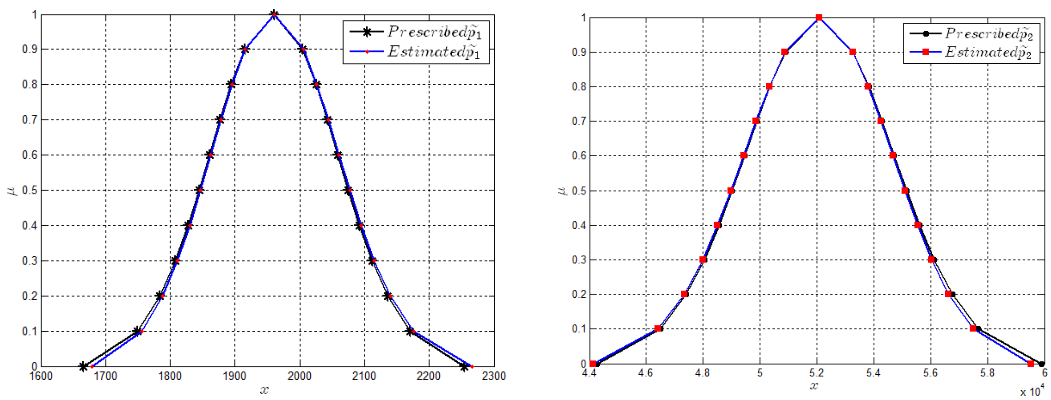


Figure 10: The fuzzy response of u_x at node 16.



(a) (b)

Figure 11: A comparison of prescribed and estimated $\tilde{p}_1$, $\tilde{p}_2$.(a).comparison of prescribed and estimated $\tilde{p}_1$.(b). comparison of prescribed and estimated $\tilde{p}_2$.

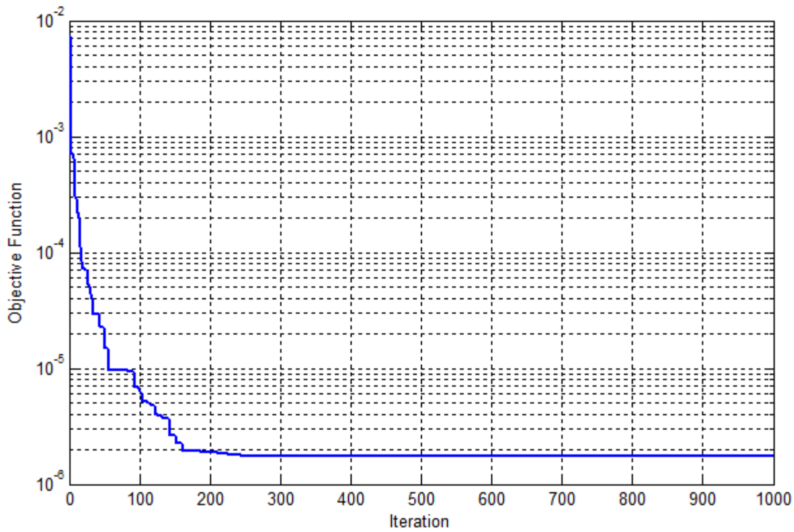


Figure 12: Iteration history of DE at 0-th membership level.

Table 3:The interval boundary of prescribed $\tilde{p}_1, \tilde{p}_2$ at each membership.

Membership $\mu_{\tilde{p}_i}(x_i)$	LB of $\tilde{p}_1$	RB of $\tilde{p}_1$	LB of $\tilde{p}_2$	RB of $\tilde{p}_2$
1.000000	1960.000000	1960.000000	52083.000000	52083.000000
0.900000	1915.013726	2004.986273	50887.581595	53278.418404
0.800000	1894.531371	2025.468628	50343.304803	53822.695196
0.700000	1877.229157	2042.770842	49883.533787	54282.466212
0.600000	1860.944770	2059.055229	49450.809417	54715.190582
0.500000	1844.613817	2075.386182	49016.847689	55149.152310
0.400000	1827.334584	2092.665415	48557.687338	55608.312661
0.300000	1807.927945	2112.072054	48041.995514	56124.004485
0.200000	1784.175987	2135.824012	47410.835688	56755.164311
0.100000	1749.695329	2170.304670	46494.582572	57671.417427

Table 4:The interval boundary of estimated $\tilde{p}_1, \tilde{p}_2$ at each membership.

Membership $\mu_{\tilde{p}_i}(x_i)$	LB of $\tilde{p}_1$	RB of $\tilde{p}_1$	LB of $\tilde{p}_2$	RB of $\tilde{p}_2$
1.000000	1960.054091	1960.054091	52084.882501	52084.882501
0.900000	1915.336738	2005.351962	50885.112497	53272.798515
0.800000	1895.154611	2026.093082	50332.132062	53809.456212
0.700000	1878.243287	2043.728483	49865.878574	54261.986362
0.600000	1862.373259	2060.422459	49426.076379	54684.788371
0.500000	1846.496256	2077.209759	48983.346190	55104.098930
0.400000	1829.802602	2095.060415	48514.290231	55546.667206
0.300000	1811.175145	2115.208505	47987.524316	56041.476602
0.200000	1788.518746	2139.975804	47339.448358	56641.253246
0.100000	1755.898031	2176.257461	46401.577000	57501.404072

6 CONCLUSIONS

The major merits of this paper include

1. The fuzzy uncertainty is taken into account for the inverse viscoelastic problem. Although there are a number of reports concerned with fuzzy uncertainty analysis in various aspects, it seems no any directly related with the inverse viscoelastic problem.

2. By integrating SBFEM, GTM, and DE, a numerical approach is presented to solve inverse problems of estimating fuzzy viscoelastic parametes.

3. Numerical verification is provided via comparisons of membership functions given by the proposed approach with prescribed ones.

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