ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

BEAM DYNAMIC STRESSES INCREMENTS AFTER PARTIAL DECONSTRUCTION OF FOUNDATION

Vladimir A. Gordon, Olga V. Pilipenko, Timur F. Gasimov

Oryol State University
address
e-mail: gordon1312@mail.ru

Keywords: Beam on elastic foundation, partial damage to the foundation, sudden structural changes, transient dynamic processes, the dynamic increments of strain and stress.

Abstract. *One of the major problems of modern structural mechanics is sensitivity analysis of actual and projected designs to various effects not provided for under normal operating conditions, arising in particular in emergency situations, as a result of wear and damage accumulation, etc. The paper presents a physical and mathematical model of the transient dynamical processes initiated in a beam on an elastic foundation with a sudden partial or fully destruction of the foundation. Mathematical models are the differential equations of the static bending of initial and damaged beam, equations of the free and forced vibrations of sections beams formed after damage foundation, with the initial and boundary conditions, conjugations conditions of the beam's parts, assumptions and limitations. Exact solution of the problem of forced vibrations loaded beam on an elastic foundation (partially or fully destroyed) was obtained, using the Fourier method of separation of variables, the expansion of dynamic movements on the modes of free vibrations of the damaged beams and Duhamel integral.*

1 INTRODUCTION

An important problem in structural mechanics is a sensitivity analysis of load-bearing systems in structural rearrangements under load such as a sudden turn off the links, cracks, partial damage, etc. Obtaining this information for real structures requires the development of special methods, because the problem can not be solved by universal methods. From the viewpoint of structural mechanics in these problems there is a need for calculating such systems as a constructively nonlinear with dynamic increments of strain and stress caused by sudden impacts beyond design basis. And if the design accidental situations are analyzed and regulated by the relevant documents, the situation is not beyond design basis and classified as responses to specific impacts on structural elements are insufficiently investigated. Engineering design and calculation methods that take into account the sudden restructuring and sudden damage to the structural systems, are few and far from being perfect. Absence and lack of knowledge about deformation and stress state of structural elements during dynamic processes initiated by a sudden damage, constrain the development of the theory and design techniques that take into account the possibility and potential consequences of beyond design impacts and ensure a high level of safety of their maintenance. As an example of work performed in the discussed direction, we note a number of publications [1-4] containing the results of the modeling of transient dynamic processes that occur in stressed beams at the sudden breakage of the support links, formation of transverse and longitudinal cracks, delamination and exfoliation, change conditions interfaces parts construction and others. All these works are made in relation to the free beams, that are not supported by solid foundations. It is of theoretical interest and practical importance of the spread of similar approaches to the beams on an elastic foundation.

Beams on elastic foundation, columns, piles, supported by length – widely used building blocks are long and detailed study of objects in many fields of mechanics and construction. However, despite the abundance of books, articles and reports related to the dynamic problems of structural elements resting on all sorts of grounds, in the scientific literature insufficiently investigated cases where structure simply supported partly initially or lost some (or all) of the foundation during exploitation. The problem of vibration of a homogeneous beam, partially supported by the Winkler foundation, first staged by Doyle and Pavlovic in [5], which investigated with two cases of symmetrical bearing beam ends: hinges and free ends. The authors identified three cases occurring in different ranges of values of the natural vibration frequencies, associated with the mechanical characteristics of the beam and foundation. Solutions are obtained analytically. Numerical results are presented for the first five modes of natural vibrations. Further development of the problem was in Eisenberger’s at al work [6], in relation to arbitrary boundary conditions. The paper deduced the precise mass and stiffness matrix, developed by special finite elements, the displacement functions which are the solutions of the equations for the static beam on an elastic foundation. Dynamic solution produced a finite element procedure. These results take the form of generalized frequency dependency of rigidity generalized system of "beam-foundation" for the first four forms of eigenfunction. Later, the results of [5] extended to partially embedded piles [7]. The paper Kukla [8] investigated the free vibrations of a homogeneous beam, supported by the Winkler foundation with areas of piecewise constant stiffness which is given by the Heaviside function. The frequency equation was obtained by standard procedure satisfying the boundary conditions and the conditions of conjugation parts. As a special case, one of the numerical examples describes the vibration of a cantilever beam, consisting of two sections, one of which is not supported by the elastic foundation. It is shown an effect of the partial support on the first four natural frequencies. The paper Motaghian et al [9] provides a method for the analysis of natural vibrations of the beam, in part supported by the Winkler foundation. The approach is to provide a foundation reactions as a some "external load", distributed along the length and depending on the desired deflection. Next eigenfunctions and "load" are expanded in Fourier series. The coefficients of the series are calculated by successive approximations by definite integrals, which bounds are the coordinates of the beginning and the end of the foundation part. The method is illustrated by calculations of natural frequencies of the beams with different boundary conditions, location and length of the supported part. In Cazzani work [10] studied in detail the natural vibrations of the beams, partially supported by an elastic foundation. Presents a number of exact solutions of the problem on the eigenvalues, in particular, the transcendental equations for the eigenfrequencies and their solutions. Analyzed three types of vibration modes, depending on the combination of the stiffness of the beam and the foundation. It is shown possibility of transfer of vibrations from one type to another with increasing number of forms of natural vibrations.

Thus, the vast majority of work on the dynamics of interaction between the beam and the grounds devoted to the analysis of natural vibrations. Forced vibrations caused by various external forces are considered in cases where the calculated scheme of the system "beam-foundation" in the process of loading is not changed.

In this paper, it is assumed that static load applied to the beam, fully supported by the elastic foundation, and only at some point part (or all) of the foundation suddenly collapses and the beam is set in motion, in which getting additional dynamic strain and stress. The aim of this study is to construct a mathematical model of transient dynamic processes in a loaded beam on an elastic foundation with the sudden occurrence of the defect foundation – total or partial destruction of it.

The structure of the paper is as follows: in the first part is given an analysis of the stress-strain state of the loaded beam with quasi-static fracture supporting its elastic foundation. Decisions of static problems are used in the second part to form the initial conditions of dynamic tasks and for comparison with the stress-strain state of the beam during the dynamic process initiated by the sudden damage to the foundation.

2 STATIC STRESS – STRAIN STATE OF THE BEAM SUPPORTED ON THE ELASTIC FOUNDATION

2.1 Beam fully supported on the elastic foundation

Consider the bending of the beam with stiffness *EI*, resting on the Winkler foundation with stiffness k, rigid clamped at both ends, under the action of uniformly distributed load of intensity q (Figure 1 *a*). The solution of the problem is carried out in Cartesian coordinates related to the beam axis. All movements and linear dimensions attributed to the length of the beam *L*. The deflection of the beam of the beam in the dimensionless variables and parameters described by the equation

$$\frac{d^4 w_{cm}}{d\xi^4} + 4\alpha^4 w_{cm} = \bar{q}, \tag{2.1}$$

where $\xi = \frac{x}{L}$, $w_{cm} = \frac{w}{L}$, $\alpha = \sqrt[4]{\frac{kL^4}{4EI}}$, $\bar{q} = \frac{qL^3}{EI}$.

The general solution of the inhomogeneous equation (2.1) in the case of clamped of the ends

$$w_{cm}(\xi)=\bar{q}\left\{\frac{1-R_4(\alpha\xi)}{4\alpha^4}+C_1(\alpha)R_2(\alpha\xi)+C_2(\alpha)R_1(\alpha\xi)\right\}, \qquad (2.2)$$

where

$$C_1(\alpha)=\frac{(R_4(\alpha)-1)R_2(\alpha)+4\alpha^4R_1^2(\alpha)}{R_5(\alpha)},\quad C_2(\alpha)=\frac{(R_4(\alpha)-1)R_3(\alpha)+4\alpha^4R_1(\alpha)R_2(\alpha)}{R_5(\alpha)};$$

$$R_1(\alpha\xi)=\frac{\sin\alpha\xi ch\alpha\xi-\cos\alpha\xi sh\alpha\xi}{4\alpha^3},\quad R_2(\alpha\xi)=\frac{\sin\alpha\xi sh\alpha\xi}{2\alpha^2},$$

$$R_3(\alpha\xi)=\frac{\sin\alpha\xi ch\alpha\xi+\cos\alpha\xi sh\alpha\xi}{2\alpha},\quad R_4(\alpha\xi)=\cos\alpha\xi ch\alpha\xi,\quad R_5(\alpha\xi)=\frac{sh^2(\alpha\xi)-\sin^2(\alpha\xi)}{2}.$$

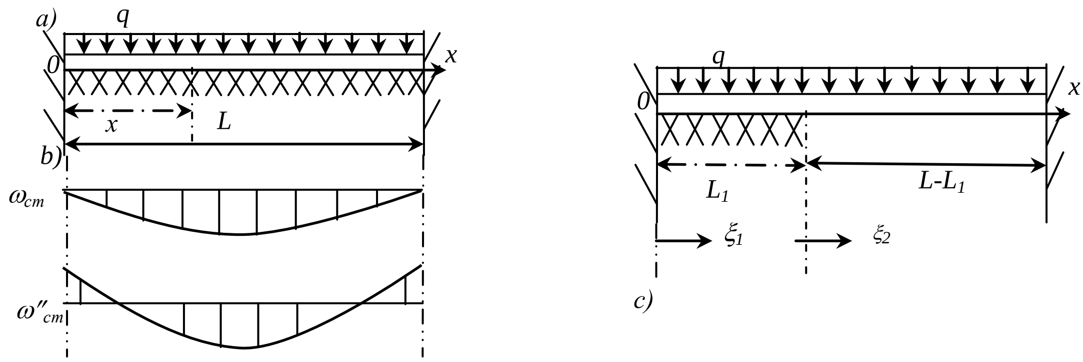


Figure 1: Beam on elastic foundation: *a)* completely simply supported on a foundation, *b)* diagrams of deflections and bending moments, *c)* simply supported on the part of foundation.

The function (2.2) is then used as the initial condition of the dynamic process resulting from the total or partial destruction of the foundation.

2.2 Beam partly supported on an elastic base

It is assumed that the formation of a defect – the removal a part of foundation of the loaded beams – was slowly (quasi-statically), i.e. without inertial forces. In this case, there is the problem of calculating the beam consisting of two coupled segments one of which is supported by an elastic foundation, the other – free (Figure 1 *c*).

Local coordinates are entered for each segment

$$\xi_1=\xi \quad \text{and} \quad \xi_2=\xi-\nu, \quad \nu=\frac{L_1}{L}.$$

Let us consider each segment bend sequence.

1) 1st segment bend described by the equation

$$\frac{d^4w_1}{d\xi_1^{\ 4}}+4\alpha^4w_1=\bar{q}, \quad w_1=\frac{v_1}{L}, \qquad (2.3)$$

the general solution (2.3) has the form

$$w_1(\xi_1)=\frac{\bar{q}}{4\alpha^4}(1-R_4(\alpha\xi_1))+w_{10}R_4(\alpha\xi_1)+w'_{10}R_3(\alpha\xi_1)+w''_{10}R_2(\alpha\xi_1)+w'''_{10}R_1(\alpha\xi_1), \qquad (2.4)$$

where $w_{10}=w_1(0),\ w'_{10}=w'_1(0),\ w''_{10}=w''_1(0),\ w'''_{10}=w'''_1(0)$ – the initial parameters.

We introduce:

– the vector of state of the 1st segment $\overline{W}_1(\xi_1)$

$$\overline{W_1}(\xi_1) = \{w_1(\xi_1) \quad w_1'(\xi_1) \quad w_1''(\xi_1) \quad w_1'''(\xi_1)\};$$

– the vector of initial parameters $\overline{W}_{10}$

$$\overline{W}_{10} = \{w_{10} \; w_{10}' \; w_{10}'' \; w_{10}'''\};$$

– functional matrix $V_1(\xi_1)$

$$\underset{4\times4}{V_1}(\xi_1) = \begin{pmatrix} R_4(\alpha\xi_1) & R_3(\alpha\xi_1) & R_2(\alpha\xi_1) & R_1(\alpha\xi_1) \\ -4\alpha^4 R_1(\alpha\xi_1) & R_4(\alpha\xi_1) & R_3(\alpha\xi_1) & R_2(\alpha\xi_1) \\ -4\alpha^4 R_2(\alpha\xi_1) & -4\alpha^4 R_1(\alpha\xi_1) & R_4(\alpha\xi_1) & R_3(\alpha\xi_1) \\ -4\alpha^4 R_3(\alpha\xi_1) & -4\alpha^4 R_2(\alpha\xi_1) & -4\alpha^4 R_1(\alpha\xi_1) & R_4(\alpha\xi_1) \end{pmatrix}$$

– load vector $\overline{q}\overline{V}_{1q}(\xi_1)$

$$\overline{q}\overline{V}_{1q}(\xi_1) = \overline{q}\left\{\frac{1 - R_4(\alpha\xi_1)}{4\alpha^4} \; R_1(\alpha\xi_1) \; R_2(\alpha\xi_1) \; R_3(\alpha\xi_1)\right\}.$$

Then the state in an arbitrary section of the 1st segment describes the matrix equation

$$\overline{W}_1(\xi_1) = V_1(\xi_1)\overline{W}_{10} + \overline{q}\overline{V}_{1q}(\xi_1). \tag{2.5}$$

The state vector of the end 1st segment, with $\xi_1 = v$

$$\overline{W}_1(v) = V_1(v)\overline{W}_{10} + \overline{q}\overline{V}_{1q}(v). \tag{2.6}$$

2) 2nd segment bend described by the equation

$$\frac{d^4 w_2}{d\xi_2^4} = \overline{q}, \quad w_2 = \frac{v_2}{L}. \tag{2.7}$$

The general solution of equation (1.7) has the form

$$w_2(\xi_2) = w_{20}T_4(\xi_2) + w_{20}'T_3(\xi_2) + w_{20}''T_2(\xi_2) + w_{20}'''T_1(\xi_2) + \overline{q}_0 T_0(\xi_2), \tag{2.8}$$

where $T_0(\xi_2) = \frac{\xi_2^4}{24}$, $T_1(\xi_2) = \frac{\xi_2^3}{6}$, $T_2(\xi_2) = \frac{\xi_2^2}{2}$, $T_3(\xi_2) = \xi_2$, $T_4 = 1$; $w_{20} = w_1(v)$, $w_{20}' = w_1'(v)$, $w_{20}'' = w_1''(v)$, $w_{20}''' = w_1'''(v)$ – the initial parameters of the 2nd segment. These equations are also conditions of conjugation segments.

We introduce:

– the vector of state of the 2nd segment $\overline{W}_2(\xi_2)$

$$\overline{W_2}(\xi_2) = \{w_2(\xi_2) \quad w_2'(\xi_2) \quad w_2''(\xi_2) \quad w_2'''(\xi_2)\};$$

– the vector of initial parameters, the components of which are expressed through the components of the vector of initial parameters of the previous one, the 1st segment $\overline{W}_1(v)$

$$\overline{W}_{20} = \overline{W}_1(v) = V_1(v)\overline{W}_{10} + \overline{q}\overline{V}_{1q}(v). \tag{2.9}$$

– functional matrix $V_2(\xi_2)$

$$\underset{4\times 4}{V_2}(\xi_2)=\begin{pmatrix}1 & T_3(\xi_2) & T_2(\xi_2) & T_1(\xi_2)\\ 0 & 1 & T_3(\xi_2) & T_2(\xi_2)\\ 0 & 0 & 1 & T_3(\xi_2)\\ 0 & 0 & 0 & 1\end{pmatrix};$$

– load vector $\bar{q}\bar{V}_{2q}(\xi_2)$

$$\bar{q}\bar{V}_{2q}(\xi_2)=\bar{q}\{T_0(\xi_2)\ T_1(\xi_2)\ T_2(\xi_2)\ T_3(\xi_2)\}.$$

Then the state of the 2nd segment described by the matrix equation

$$\bar{W}_2(\xi_2)=V_{21}(\xi_2,v)\bar{W}_{10}+\bar{q}(\bar{V}_{21}(\xi_2,v)+\bar{V}_{2q}(\xi_2)), \qquad (2.10)$$

where the matrix $\underset{4\times 4}{V_{21}}(\xi_2,v)=\underset{4\times 4}{V_2}(\xi_2)\underset{4\times 4}{V_1}(v)$ and vector $\underset{4\times 1}{\bar{V}_{21}}(\xi_2,v)=\underset{4\times 4}{V_2}(\xi_2)\underset{4\times 1}{\bar{V}_{1q}}(v)$.

Thus, the state of the beam at both segment is expressed through the initial parameters of the 1st segment at $\xi_1=0$. In this case, two parameters known

$$w_{10}=w'_{10}=0.$$

The remaining two parameters w''_{10} and w'''_{10} are determined from boundary conditions at the end of 2nd segment with $\xi_2=1-v$. Denoted the elements of matrix $V_{21}(\xi_2,v)$ and vector $\bar{q}(\bar{V}_{21}(\xi_2,v)+\bar{V}_{2q}(\xi_2))$ as $a_{ij}(\xi_2,v)$ and $b_i(\xi_2,v)$ $(i,j=1\div 4)$, respectively, and write the equation (2.10) in expanded form for $\xi_2=1-v$ taking into account the known boundary conditions

$$w_{10}=w'_{10}=w_2(1-v)=w'_2(1-v)=0$$

$$\begin{pmatrix}0\\ 0\\ w''_2(1-v)\\ w'''_2(1-v)\end{pmatrix}=\begin{pmatrix}a_{11}(1-v) & a_{12}(1-v) & a_{13}(1-v) & a_{14}(1-v)\\ a_{21}(1-v) & a_{22}(1-v) & a_{23}(1-v) & a_{24}(1-v)\\ a_{31}(1-v) & a_{32}(1-v) & a_{33}(1-v) & a_{34}(1-v)\\ a_{41}(1-v) & a_{42}(1-v) & a_{43}(1-v) & a_{44}(1-v)\end{pmatrix}\begin{pmatrix}0\\ 0\\ w''_{10}\\ w'''_{10}\end{pmatrix}+\bar{q}\begin{pmatrix}b_1(1-v)\\ b_2(1-v)\\ b_3(1-v)\\ b_4(1-v)\end{pmatrix} \qquad (2.11)$$

where, in particular

$$a_{13}(\xi_2,v)=R_2(\alpha v)+R_3(\alpha v)\xi_2+R_4(\alpha v)\frac{\xi_2^2}{2}-4\alpha^4R_1(\alpha v)\frac{\xi_2^3}{6};$$

$$a_{14}(\xi_2,v)=R_1(\alpha v)+R_2(\alpha v)\xi_2+R_3(\alpha v)\frac{\xi_2^2}{2}+R_4(\alpha v)\frac{\xi_2^3}{6};$$

$$a_{23}(\xi_2,v)=R_3(\alpha v)+R_4(\alpha v)\xi_2+4\alpha^4R_1(\alpha v)\frac{\xi_2^2}{2};$$

$$a_{24}(\xi_2,v)=R_2(\alpha v)+R_3(\alpha v)\xi_2+R_4(\alpha v)\frac{\xi_2^2}{2};$$

$$b_1(\xi_2,v)=\frac{1-R_4(\alpha v)}{4\alpha^4}+R_1(\alpha v)\xi_2+R_2(\alpha v)\frac{\xi_2^2}{2}+R_3(\alpha v)\frac{\xi_2^3}{6}+\frac{\xi_2^4}{24};$$

$$b_2(\xi_2,v)=R_1(\alpha v)+R_2(\alpha v)\xi_2+R_3(\alpha v)\frac{\xi_2^2}{2}+\frac{\xi_2^3}{6}.$$

From the matrix equation (2.11) we obtain a system of two algebraic equations for the unknown w''_{10} and w'''_{10}

$$\begin{pmatrix} a_{13}(1-v) & a_{14}(1-v) \\ a_{23}(1-v) & a_{24}(1-v) \end{pmatrix} \begin{pmatrix} w''_{10} \\ w'''_{10} \end{pmatrix} + \bar{q} \begin{pmatrix} b_1(1-v) \\ b_2(1-v) \end{pmatrix} = 0,$$

where by Cramer method obtain

$$w''_{10} = \frac{\Delta_1}{\Delta}, \quad w'''_{10} = \frac{\Delta_2}{\Delta}, \tag{2.12}$$

where determinants Δ, Δ_1 and Δ_2 have the form

$$\Delta = \begin{vmatrix} a_{13}(1-v) & a_{14}(1-v) \\ a_{23}(1-v) & a_{24}(1-v) \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} -\bar{q}b_1(1-v) & a_{14}(1-v) \\ -\bar{q}b_2(1-v) & a_{24}(1-v) \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} a_{13}(1-v) & -\bar{q}b_1(1-v) \\ a_{23}(1-v) & -qb_2(1-v) \end{vmatrix}.$$

Now dimensionless deflections and bending moments on the areas determined by dependencies

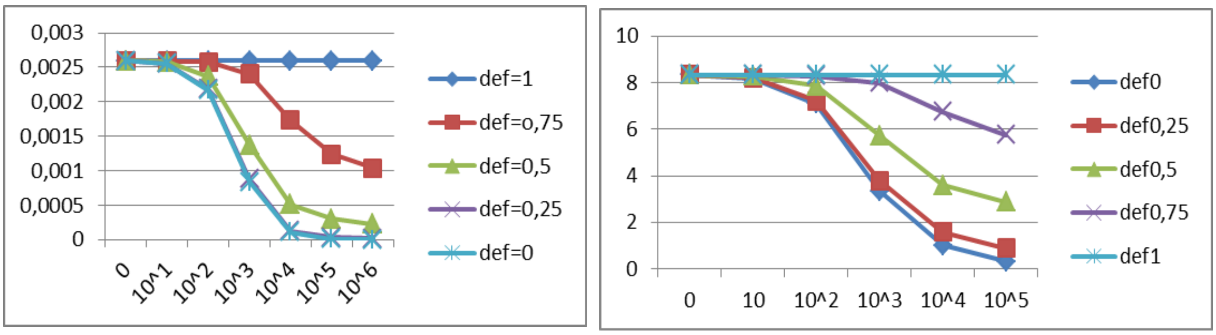
$$w_1(\xi_1) = w''_{01}R_2(\alpha\xi_1) + w'''_{10}R_1(\alpha\xi_1) + \frac{\bar{q}}{4\alpha^4}(1 - R_4(\alpha\xi_1)), \quad 0 \le \xi_1 \le v$$

$$w''_1(\xi_1) = w''_{01}R_4(\alpha\xi_1) + w'''_{10}R_3(\alpha\xi_1) + \bar{q}R_2(\alpha\xi_1), \tag{2.13}$$

$$w_2(\xi_2) = w''_{10}a_{13}(v,\xi_2) + w'''_{10}a_{14}(v,\xi_2) + \bar{q}b_1(v,\xi_2), \quad 0 \le \xi_2 \le 1-v$$

$$w''_2(\xi_2) = w''_{10}a_{33}(v,\xi_2) + w'''_{10}a_{34}(v,\xi_2) + \bar{q}b_3(v,\xi_2).$$

Figure 2 shows the relationship (2.13), respectively, the largest deflection $w_{\max}$ and bending moments $w''_{\max}$ with the generalized parameter of rigidity of the system "beam-foundation" $\bar{\lambda} = 4\alpha^4 = \frac{KL^4}{EI}$ for a number of values of the parameter $v = 0; 0{,}25; 0{,}5; 0{,}75; 1,$ characterizing the degree of damage to the foundation: $v = 1$ – fully justified, $v = 0$ – absence of a foundation. The calculations correspond to a single dimensionless load



a) *b)*

Figure 2: The dependence:
a) the largest deflections; *b)* the largest bending moment in the right clamp – from length damage area.

Figure 3 shows the reduction (as a percentage of the free beam) the largest bending moment in the beam at the reinforcement of its foundation of varying length and relative stiffness.

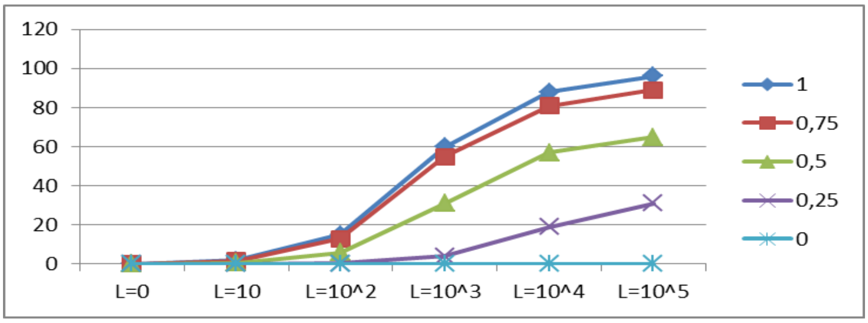


Figure 3: Effect of the length and rigidity to the foundation on value of the largest bending moment in the right clamp of beam.

3 FREE VIBRATION BEAM SUPPORTED ON AN ELASTIC FOUNDATION

To date, there are a number of works devoted to the study of dynamic processes caused by the sudden formation of defects in beams and plates. In particular, in papers [2-4] considers the transition dynamic process caused by the sudden formation of transverse cracks in a loaded beam. Beam modeled conjugation of two segments connected by a torsion spring whose stiffness is determined by the depth of cracks. One of the initial conditions is the static deflection of the intact beam. In [5-8] studied the dynamic process in the loaded component beams arising during sudden longitudinal stratification, caused break shift links in the seam. In [9-11] considered vibrations plates with a sudden change of support conditions, caused by partial separation of support links. In [12-14] solved the problem of the vibrations of the beam when it is sudden partial destruction. In [4, 6, 11, 16] studied transients dynamical process caused by sudden transformation of the internal structure of rod systems: coupling and bearing conditions their elements. In [17] is modeled by a sudden breaking of the reinforcing bar and the longitudinal vibrations caused by this defect. Note ,that in all these studies considered forced movement of the beam (plates) under the load ,produced by expansion into an infinite series of the initial state and the external load on the damaged natural vibration modes of the beam (the plate), and using the Duhamel integral.

It is practically important to develop methods of analysis of transient dynamic processes in the beams on an elastic foundation, initiated by the sudden operational damage of foundation: in the simplest case of total or partial destruction of it. Manifestations of structural nonlinearity, i.e. changes in the calculation scheme loaded beam on an elastic foundation, and the consequences of them – are not described in the prior literature.

In this part, an algorithm for determining the forms and frequencies of free vibrations of bending beams, in whole or in part simply supported on an elastic Winkler foundation. In its construction were used the approaches, the effectiveness of which is shown in the above-cited work: method of initial parameters, the vectors of state and the initial parameters, the matrix of influence of initial parameters and other. Eigenfunctions and eigenfrequencies of free vibrations are important dynamic characteristics of the "beam – foundation" system and used to solve inhomogeneous differential equations, describing forced vibrations.

3.1 Free vibration beam partially supported on the elastic foundation

Consider free transverse vibrations of a continuous beam (Figure 1 *c*) Deflections of the beams in the cross – sections denoted x_1 and x_2, respectively, v_1 and v_2. Without reducing the generality of the constructions it is supposed to be specific rigid clamping beam ends. Differential equations of natural vibrations of the beam segments are integrated separately, and integration constants are determined from the boundary conditions and the conditions of conjugation of segments.

3.1.1. Free transverse vibrations of the 1st segment

Natural vibrations of the beam in the area $0 \le \xi_1 \le v$ described by differential equations of the form [10]

$$\frac{\partial^4 w_1}{\partial \xi_1^4} + 4\alpha^4 w_1 + \frac{\partial^2 w_1}{\partial \tau^2} = 0, \tag{3.1}$$

where we have introduced the dimensionless variables and parameters

$$\xi_i = \frac{x_1}{L}; w_1 = \frac{v_1}{L}; \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}}; A = bh; \alpha = \sqrt[4]{\frac{kL^4}{4EI}}; v = \frac{v_1}{L}.$$

Putting free harmonic vibrations, we divide the variables in the equation (3.1) the representation

$$w_1(\xi_1, \tau) = W_1(\xi_1)\sin \bar{\omega}\tau, \tag{3.2}$$

where $\bar{\omega} = \omega L^2 \sqrt{\frac{\rho A}{EI}} -$ the dimensionless natural frequency of transverse vibration (natural frequency parameter); $\omega = \frac{\bar{\omega}}{L^2}\sqrt{\frac{EI}{\rho A}}$ – dimensional frequency.

Substituting (3.2) into (3.1) gives the equation forms its free vibrations of segment

$$W_1^{IV} + 4\alpha^4 \left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right) W_1 = 0, \tag{3.3}$$

where $\omega_0 = \sqrt{\frac{k}{\rho A}}$ – parameter having a dimension of frequency [1/*c*], and called "conventional", which compares the vibration frequency ω investigated beams.

We are looking for the solution of equation (3.3) Euler substitution

$$W_1 = Ce^{n\xi_1}, \tag{3.4}$$

where *C* and *n* – constants to be determined.

Substituting (3.4) into (3.3), we obtain the characteristic equation for the differential equation (3.3)

$$n^4 + 4\alpha^4 \left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right) = 0. \tag{3.5}$$

Multiplier in brackets depending on the value $\frac{\omega}{\omega_0}$ can be positive ($\frac{\omega}{\omega_0} < 1$), negative ($\frac{\omega}{\omega_0} > 1$) and zero ($\frac{\omega}{\omega_0} = 1$).

Let $\frac{\omega}{\omega_0} < 1$. Then the roots of the equation (3.5) can be represented in the form

$$n_j = (\pm i \pm 1)\beta_1, \quad (j=1,2,3,4),$$

where $\beta_1 = \alpha \sqrt[4]{1 - \left(\frac{\omega}{\omega_0}\right)^2}$.

In this case, the natural vibration shape function defined by [10]

$$W_1(\xi_1) = w_{10}S_4(\xi_1) + w'_{10}S_3(\xi_1) + w''S_2(\xi_1) + w'''S_1(\xi_1), \tag{3.6}$$

where $w_{10} = w_1(0), w'_{10} = w'_1(0), w''_{10} = w''_1(0), w'''_{10} = w'''_1(0)$ – the initial parameters of the 1st segment.

$$S_1(\xi_1) = \frac{ch\beta_1\xi_1 \sin\beta_1\xi_1 - sh\beta_1\xi_1 \cos\beta_1\xi_1}{4\beta_1^3}; S_2(\xi_1) = \frac{sh\beta_1\xi_1 \sin\beta_1\xi_1}{2\beta_1^2};$$

$$S_3(\xi_1) = \frac{ch\beta_1\xi_1 \sin\beta_1\xi_1 + sh\beta_1\xi_1 \cos\beta_1\xi_1}{2\beta_1}; S_4(\xi_1) = ch\beta_1\xi_1 \cos\beta_1\xi_1 \text{– Krylov function.}$$

Note, that the $S'_1 = S_2, S'_2 = S_3, S'_3 = S_4, S'_4 = -4\beta_1^4 S_1$ и

$$S_1(0) = S_2(0) = S_3(0) = 0, S_4(0) = 1.$$

Let ω $\frac{\omega}{\omega_0}$ >1. Then the equation (3.5) in the form

$$W_1^{IV} - 4\alpha^4\left(\left(\frac{\omega}{\omega_0}\right)^2 - 1\right)W_1 = 0. \tag{3.7}$$

In this case, the roots of the characteristic equation

$$n^4 - 4\alpha^4\left(\left(\frac{\omega}{\omega_0}\right)^2 - 1\right) = 0, \tag{3.8}$$

take the form

$$n_{1,2} = \pm\lambda_1;\ n_{3,4} = \pm i\lambda_1;\ \lambda_1 = \lambda_1 = \sqrt{2}\alpha\sqrt[4]{\left(\frac{\omega}{\omega_0}\right)^2 - 1} \tag{3.9}$$

and forms of natural vibrations determined by the function

$$W_1\xi_1 = w_{10}K_4(\lambda_1\xi_1) + w'_{10}K_3(\lambda_1\xi_1) + w''_{10}K_2(\lambda_1\xi_1) + w'''_{10}K_1(\lambda_1\xi_1), \tag{3.10}$$

where

$$K_1(\lambda_1\xi_1) = \frac{sh\lambda_1\xi_1 - \sin\lambda_1\xi_1}{2\lambda_1^3}; \qquad K_2(\lambda_1\xi_1) = \frac{ch\lambda_1\xi_1 - \cos\lambda_1\xi_1}{2\lambda_1^2}; \qquad K_3(\lambda_1\xi_1) = \frac{sh\lambda_1\xi_1 + \sin\lambda_1\xi_1}{2\lambda_1};$$

$K_4(\lambda_1\xi_1) = \frac{ch\lambda_1\xi_1 + cos\lambda_1\xi_1}{2}$ – another type of Krylov's functions.

Their properties

$$K_1' = K_2,\ K_2' = K_3,\ K_3' = K_4,\ K_4' = \lambda_1^4 K_1 \ \ \textit{and} \ \ K_1(0) = K_2(0) = K_3(0) = 0,\ K_4(0) = 1.$$

Finally let $\frac{\omega}{\omega_0} = 1$. Then the equation (3.3) takes the form

$$W_1^{1V} = 0 \tag{3.11}$$

and deflection function $W_1(\xi_1)$ obtain the successive integration of the equation (3.11) using the initial parameters

$$W_1(\xi_1) = w_{10} + w_{10}'\xi_1 + w_{10}''\frac{\xi_1^2}{2} + w_{10}'''\frac{\xi_1^3}{6} \tag{3.12}$$

Further action on the first section we consider the case $\frac{\omega}{\omega_0} > 1$.

We introduce:

– the vector of state of the first segment: $\bar{W}_1(\xi_1)$:

$$\overline{W_1}(\xi_1) = \left\{ W_1(\xi_1)\ \ W_1'(\xi_1)\ \ W_1''(\xi_1)\ \ W_1'''(\xi_1) \right\},$$

where $W_1(\xi_1)$ – dimensionless deflection of the beam; W_1', W_1'', W_1''' – dimensionless cross-sectional angle of rotation, bending moment and shear force, respectively;

– the vector of initial parameters: $\overline{W_{10}}$

$$\overline{W}_{10} = \left\{ w_{10}\ \ w_{10}'\ \ w_{10}''\ \ w_{10}''' \right\}$$

– Influence function matrix: $V(\xi_1)$

$$\underset{4\times 4}{V_1(\xi_1)} = \begin{pmatrix} K_4(\lambda_1\xi_1) & K_3(\lambda_1\xi_1) & K_2(\lambda_1\xi_1) & K_1(\lambda_1\xi_1) \\ \lambda_1^4 K_1(\lambda_1\xi_1) & K_4(\lambda_1\xi_1) & K_3(\lambda_1\xi_1) & K_2(\lambda_1\xi_1) \\ \lambda_1^4 K_2(\lambda_1\xi_1) & \lambda_1^4 K_1(\lambda_1\xi_1) & K_4(\lambda_1\xi_1) & K_3(\lambda_1\xi_1) \\ \lambda_1^4 K_3(\lambda_1\xi_1) & K_2(\lambda_1\xi_1) & \lambda_1^4 K_1(\lambda_1\xi_1) & K_4(\lambda_1\xi_1) \end{pmatrix}.$$

Then the state of the first segment describes how the matrix equation

$$\bar{W}_1(\xi_1) = V(\xi_1)\bar{W}_{10}. \tag{3.13}$$

The vector of state $\bar{W}_1(\xi_1)$ at the end of the first segment

$$\bar{W}_1(\nu) = V(\nu)\bar{W}_{10}. \tag{3.14}$$

3.1.2 Free transverse vibrations of the 2nd segment.

Free transverse vibrations of the segment described by the equation [10]

$$\frac{\partial^4 w_2}{\partial \xi_2^4} + \frac{\partial^2 w_2}{\partial \tau^2} = 0. \tag{3.15}$$

Separating the variables, we obtain

$$w_2(\xi_2, \tau) = W_2(\xi_2)\sin\omega\tau, \tag{3.16}$$

$$W_2^{IV}(\xi_2)-\bar{\omega}^2 W_2(\xi_2)=0, \quad W_2=Ae^{r_2\xi_2}, \tag{3.17}$$

where $r_2^4=\overline{\omega}^2$; $r_{21}=\lambda_2$; $r_{22}=-\lambda_2$; $r_{23}=i\lambda_2$; $r_{24}=-\lambda_2$; $\lambda_2=\sqrt{\bar{\omega}}$.
The general solution of equation (2.17) has the form

$$W_2(\xi_2)=w_{20}K_4(\lambda_2\xi_2)+w'_{20}K_3(\lambda_2\xi_2)+w''_{20}K_2(\lambda_2\xi_2)+w'''_{20}K_1(\lambda_2\xi_2),$$

where $w_{20}, w_{20}', w_{20}'', w_{20}'''$ – the initial parameters of the second segment.

We introduce:

– the vector of state of the second section: $\overline{W}_2(\xi_2)$

$$\overline{W}_2(\xi_2)=\{W_2(\xi_2)\ \ W_2'(\xi_2)\ \ W_2''(\xi_2)\ \ W_2'''(\xi_2)\};$$

– the vector of initial parameters: $\overline{W}_{20}$

$$\overline{W}_{20}=\{w_{20}\ \ w'_{20}\ \ w''_{20}\ \ w'''_{20}\};$$

– influence function matrix: $V_2(\xi_2)$

$$\underset{4\times4}{V_2}(\xi_2)=\begin{pmatrix} K_4(\lambda_2\xi_2) & K_3(\lambda_2\xi_2) & K_2(\lambda_2\xi_2) & K_1(\lambda_2\xi_2)\\ \lambda_2^4K_1(\lambda_2\xi_2) & K_4(\lambda_2\xi_2) & K_3(\lambda_2\xi_2) & K_2(\lambda_2\xi_2)\\ \lambda_2^4K_2(\lambda_2\xi_2) & \lambda_2^4K_1(\lambda_2\xi_2) & K_4(\lambda_2\xi_2) & K_3(\lambda_2\xi_2)\\ \lambda_2^4K_3(\lambda_2\xi_2) & K_2(\lambda_2\xi_2) & \lambda_2^4K_1(\lambda_2\xi_2) & K_4(\lambda_2\xi_2) \end{pmatrix}.$$

Then the state of the second segment describes the matrix equation

$$\bar{W}_2(\xi_2)=V_2(\xi_2)\bar{W}_{20}. \tag{3.18}$$

From conditions of conjugation of segments areas has

$$\bar{W}_1(\nu)=\bar{W}_2(0) \ \text{ or } \ V_1(\nu)\bar{W}_{10}=V_2(0)\bar{W}_{20}.$$

Given that $V_2(0)$ – the identity matrix, we obtain $\bar{W}_{20}=V_1(\nu)\bar{W}_{10}$ and, consequently, from (3.18) we have $\bar{W}_2(\xi_2)=V_2(\xi_2)V_1(\nu)\bar{W}_{10}$.

Those: state at both segments is determined by the parameters of the first segments.

State at the end of the second segment when $\xi_2=1-\nu$ determined by the vector $\bar{W}_2(1-\nu)$

$$\bar{W}_2(1-\nu)=V_2(1-\nu)V_1(\nu)\bar{W}_{10}=V_{21}(1-\nu,\nu)\bar{W}_{10}. \tag{3.19}$$

Here the matrix of influence $V_{21}(1-\nu,\nu)=V_2(1-\nu)V_1(\nu)$.

Two initial parameter known $w_{10}=w'_{10}=0$.

There are also known two components of the vector of state at the end of the second segment

$$W(1-\nu)=W_2'(1-\nu)=0.$$

Substituting these values in the matrix equation (3.19), written in block form, we obtain

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} W_2''(1-\nu) \\ W_2'''(1-\nu) \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} & \begin{pmatrix} c_{13} & c_{14} \\ c_{23} & c_{24} \end{pmatrix} \\ \begin{pmatrix} c_{31} & c_{32} \\ c_{41} & c_{42} \end{pmatrix} & \begin{pmatrix} c_{33} & c_{34} \\ c_{42} & c_{44} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} w_{10}'' \\ w_{10}''' \end{pmatrix} \end{pmatrix}.$$

Whence

$$\begin{pmatrix} c_{13} & c_{14} \\ c_{23} & c_{24} \end{pmatrix} \begin{pmatrix} w_{10}'' \\ w_{10}''' \end{pmatrix} = 0. \tag{3.20}$$

The frequency equation is obtained by equating to zero the determinant

$$\begin{vmatrix} c_{13} & c_{14} \\ c_{23} & c_{24} \end{vmatrix} = 0, \tag{3.21}$$

where

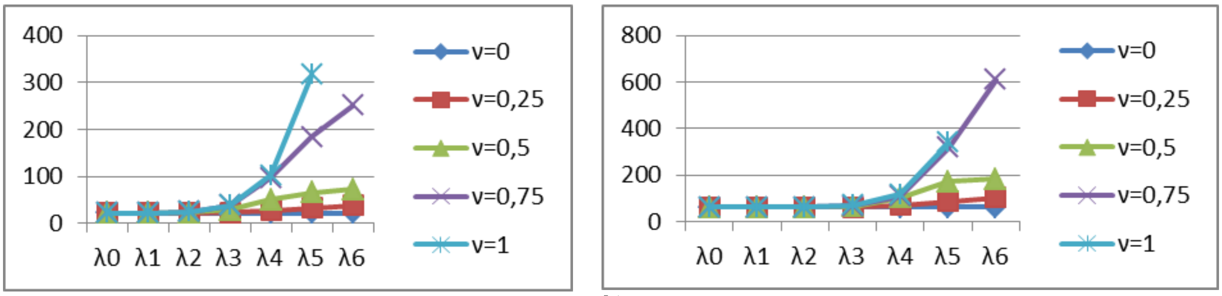
$$C_{13} = K_4(\lambda_2(1-\nu))K_2(\lambda_1\nu) + K_3(\lambda_2(1-\nu))K_3(\lambda_1\nu) + K_2(\lambda_2(1-\nu))K_4(\lambda_1\nu) + \lambda_1^4 K_1(\lambda_2(1-\nu))K_1(\lambda_1\nu).$$

$$C_{14} = K_4(\lambda_2(1-\nu))K_1(\lambda_1\nu) + K_3(\lambda_2(1-\nu))K_2(\lambda_1\nu) + K_2(\lambda_2(1-\nu))K_3(\lambda_1\nu) + K_1(\lambda_2(1-\nu))K_4(\lambda_1\nu).$$

$$C_{23} = \lambda_2^4 K_1(\lambda_2(1-\nu))K_2(\lambda_1\nu) + K_4(\lambda_2(1-\nu))K_3(\lambda_1\nu) + K_3(\lambda_2(1-\nu))K_4(\lambda_1\nu) + \lambda_1^4 K_2(\lambda_2(1-\nu))K_1(\lambda_1\nu).$$

$$C_{24} = \lambda_2^4 K_1(\lambda_2(1-\nu))K_1(\lambda_1\nu) + K_4(\lambda_2(1-\nu))K_2(\lambda_1\nu) + K_3(\lambda_2(1-\nu))K_3(\lambda_1\nu) + K_2(\lambda_2(1-\nu))K_4(\lambda_1\nu).$$

Figure 4 shows the dependence of the two frequencies of the free vibrations of the beam from relative stiffness of grounds for different sizes of the damaged area of the foundation



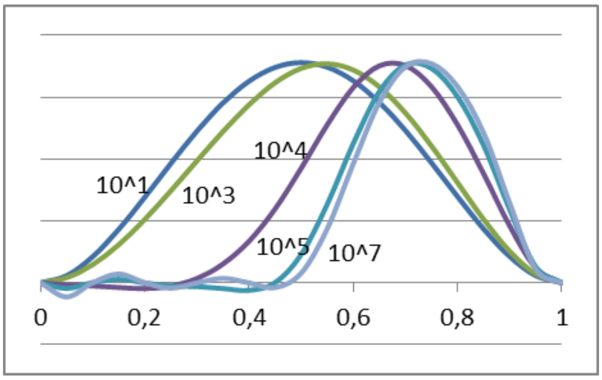
a) *b)*

Figure 4: The natural frequencies of flexural vibrations damaged beam: *a)* – first, *b)* – second.

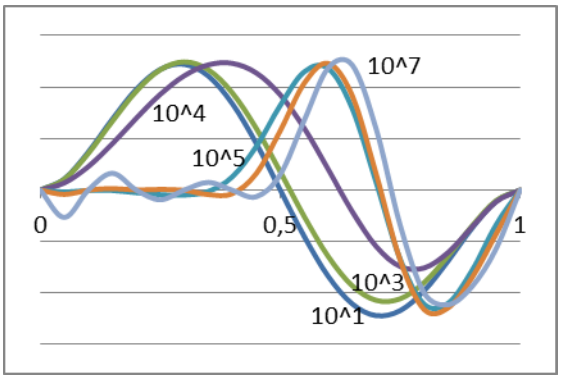
4 FORCED VIBRATIONS BEAM PARTIALLY SUPPORTED ON THE ELASTIC FOUNDATION

Forced vibrations of the beam consisting of two segments, formed after the partial destruction of the foundation, described by corresponding non-homogeneous equation

$$\frac{\partial^4 w_1}{\partial \xi_1^{\,4}} + 4\alpha^4 w_1 + \frac{\partial^2 w_1}{\partial \tau^2} = q' \quad \text{and} \quad \frac{\partial^4 w_2}{\partial \xi_2^{\,4}} + \frac{\partial^2 w_2}{\partial \tau^2} = \bar{q} \tag{4.1}$$



a)



b)

Figure 5: Forms of free vibration of the beam: *a*) – first , *b*) – second.

Expending movement $w_i(\xi_i,\tau)$ (i = 1, 2) in the forms of natural vibrations (3.13) and (3.18), respectively, with coefficients in the form of well-known functions of time using orthogonal forms of natural vibrations, satisfying the initial conditions

$$w_i(\xi_i,0)=w_{cm}(\xi_i),\quad \left.\frac{\partial w_i}{\partial\tau}\right|_{\xi_i,0}=0 \tag{4.2}$$

obtain function of deflection on segments [11]

$$w_i(\xi_i,\tau)=\sum_{n=1}^{\infty}\left[\frac{\int_0^{\nu_i}w_{cm}(\xi)W_{in}(\xi)d\xi}{\int_0^{\nu_i}W_{in}^2(\xi)d\xi}\cos\bar{\omega}_n\tau+\frac{2\bar{q}\int_0^{\nu_i}(\xi)W_{in}(\xi)d\xi}{\bar{\omega}_n^2\int_0^{\nu_i}W_{in}^2(\xi)d\xi}\sin^2\frac{\bar{\omega}_n\tau}{2}\right]W_{in}(\xi), \tag{4.3}$$

where $\nu_1=\nu,\quad \nu_2=1-\nu.$

5 A NUMERICAL EXAMPLE

Reinforced concrete beam length L = 6,7 *m*, rectangular cross-section with sides of width b = 0,25 *m*, height h = 0,18 *m*, moment of inertia of the cross-section of the $I=1{,}215\cdot10^{-4}\,m^4$, Young's modulus of the beam material $E=3{,}05\cdot10^{10}\,\frac{N}{m^2}$. Material of foundation – a ballast layer of crushed stone with module $K_1=75\frac{MPa}{m}$. Coefficient of stiffness of foundation $K=K_ib=18{,}75\cdot10^6\,Pa$. Parameter $\bar{\lambda}=\frac{KL^4}{EI}\approx10^4$. Setting ends beams are clamped. The beam is loaded with a uniformly distributed load of intensity $q=13{,}5\frac{kN}{m}$ (brick wall height of 3 *m*, a length of 6.7 *m*, a thickness of 0.25 *m*, the proportion of the material $18\frac{kN}{m^3}$).

In the initial (unstrained) state the maximum bending moment $\bar{M}_{\max 0}=0{,}099\bar{q}$, $\bar{M}=\frac{ML}{EI}$, $\bar{q}=\frac{qL^3}{EI}$. Normal stress $\sigma=39\frac{kg}{cm^2}$.

Table 1 contains the ratios $K_{quasi} = \frac{M_{quasi}}{M_{st.}}$ and $K_{dinam} = \frac{M_{dinam}}{M_{st.}}$ increasing the maximum bending moment in the beam at different degrees, respectively, quasi-static and sudden damage to the base

The degree of damage, $1-\nu$	1	0,75	0,5	0,25	0
Quasi-static, K_{quasi}	1	1,41	3,53	6,56	8,33
Sudden, K_{dinam}	1	1,57	4,52	11,29	14,53

Table 1: Splash of max bending moment at the foundation damage.

6 CONCLUSIONS

A mathematical model of free transverse vibrations of simply supported beam partially on an elastic Winkler foundation. The beam appears conjugation of two segments, one of which is fully supported by the foundation, the other is free. The spectrum of the natural frequencies of the beam on the foundation of partial damage frequency is determined by the equation obtained procedure similar to that used in the finite element method (FEM) in the construction of finite element stiffness matrix. We introduce four component vectors of the state – the dimensionless deflection, turn the cross-section, the bending moment and shear force. In matrix form obtained according to the state vectors in an arbitrary section of the vector of initial parameters. Vectors of state presented a set of blocks, including kinematic and power parameters. This matrix is the influence of the initial section of the cell becomes final, always 4×4. The frequency equation is obtained by equating the determinant of this matrix is zero. The use vectors of state, and the initial parameters of the procedure reduces the influence of the matrix to the fourth, with any number of mating parts of the beam, which significantly reduces the complexity of the calculations.

As a result, we obtained the relationship between the natural frequencies, the mechanical characteristics of the beam and the foundation and damage parameters: the length of the damaged portion of the foundation and its localization in the overall length of the beam.

REFERENCES

[1] V.A. Gordon, V.I. Brusova, E.A. Kravtsova, T.V. Poturaeva, Mathematical modeling of dynamic processes in the core system to the sudden changes in their structure, *Fundamental and applied problems of engineering and technology*, **5(283)**, 3-9, 2010.

[2] V.A. Gordon, V.I. Kolchunov, Y.S. Stepanov, Dynamical processes analysis in the load beams during partial destruction, *Proceedings of XVII International congress on sound and vibration*, Cairo, Egypt, July 18-22, 2010.

[3] V.A. Gordon, V.I. Kolchunov, Y.S. Stepanov, Dynamic loading of composite beam with the sudden longitudinal stratification, *Proceedings of XIX International congress on sound and vibration*, Vilnius, Lithuania, July 8-12, 2012.

[4] P.E. Doyle, M. Pavlovic, Vibration of beams on partial elastic foundation, *Earthquake Engineering and Structural Dynamics*, **10**, 663-674, 1982.

[5] M. Eisenberger, D.Z. Yankelensky, M.A. Adin, Vibration of beams fully or partially supported on elastic foundation, *Earthquake Engineering and Structural Dynamics*, **13**, 651-660, 1985.

[6] A. Valsangkar, R.Pradhanang, Vibrations of beam-columns on two-parameter elastic foundations *Earthquake Engineering and Structural Dynamics*, **16**, 217-225, 1988.

[7] S. Kukla, Free vibration of a beam supported on a stepped elastic foundation, *Journal of Sound and Vibration*, **149(2)**, 259-265, 1991.

[8] S.E. Motaghian, M. Mofid, P. Alanjari, Exact solution to free vibration of beams partially supported by an elastic foundation, *Scientia Iranica A*, **18(4)**, 861-866, 2011.

[9] A. Cazzani, On the dynamics of a beam partially supported by an elastic foundation: an exact solution-set, *International Journal of Structural Stability and Dynamics,***13**, 8, 1350045 (30 pages), 2013.

[10] V.A. Gordon, O.V. Pilipenko, Vibration of loaded beam initiated by fully or partially destruction of the elastic foundation, *Proceedings XVII International congress on sound and vibration*, Florence, Italy, July 12-16, 2015.

[11] V.A. Gordon, T.V. Poturaeva, G.A. Semenova, Stress-strain state of the beam, in part simply supported on an elastic foundation, *Fundamental and applied problems of engineering and technology,* **5-2**, 57-68, 2015.

[12] V.A . Gordon, T.V. Poturaeva, Frequencies free flexural vibrations of a freely supported beam with a crack, *Building mechanics and calculation of constructions,* **3**, 19-23, 2009.

[13] T.V. Poturaeva, V.A. Gordon, Dynamic loading freely supported beams initiated the formation of a crack , *Construction and reconstruction*, **4(24)**, 28-33, 2009.

[14] V.A. Gordon, E.A. Kravtsova, Effect of longitudinal stratification of the composite rod in the natural frequency of the flexural vibrations, *Building mechanics and calculation of constructions*, **1**, 19-24, 2011.

[15] V.A .Gordon, E.A .Kravtsova, Dynamic loading in the beam with multilevel longitudinal stratification, *Construction and reconstruction*, **6(44)**, 17-27, 2012.

[16] V.A. Gordon, E.A. Kravtsova, Methods of determining the spectrum of frequencies of flexural vibrations of the beam with multilevel longitudinal stratification, *Fundamental and applied problems of engineering and technology*, **2-5(292)**, 87-95, 2012.

[17] V.A. Gordon, T.A. Pavlova, Dynamic processes in the composite plate with a sudden longitudinal stratification, *Proceedings of the TulaSU. Series: Building materials, design and construction*, 40-49, 2006.

[18] V.A. Gordon, Y.S. Stepanov, P.N. Anokhin, Transitional processes in the constructions with the sudden structural reconstructions, *Proceedings XV International congress on sound and vibration*, Daejeon, Korea, July 6-10, 2008.

[19] V.A. Gordon, N.V. Klyueva, V.I. Brusova, Axisymmetric vibrations of circular plates with a sudden change of support conditions, *Building mechanics and calculation of constructions*, **1**, 41-43, 2009.

[20] V. A. Gordon, Y.S. Stepanov, V.S. Shorkin, Concepts for estimating of structural safety of bar system, *Proceedings XII International congress on sound and vibration*, Lisbon, Portugal, July 11-14, 2005.

[21] N.V. Klyueva, V.A. Gordon, Calculation of dynamic spatial loading in core system suddenly shuts down elements, *Building mechanics of engineering structures*, **6**, 72-79, 2008.

[22] V.A .Gordon, V.I. Brusova ,The transition process in the load beam at a detachment of the longitudinal layer ,*Construction and reconstruction*, **4(24)**, 8-10, 2009.

[23] T.V. Poturaeva, Transient dynamic process in the core design with a sudden change of support conditions, *Fundamental and applied problems of engineering and technology*, **4(272)**, 27-31, 2008.

[24] V.A. Gordon, T.V. Poturaeva, Evaluation of the dynamic effect of a sudden restructuring of the construction, *Fundamental and applied problems of engineering and technology*, **1**, 3-8, 2008.

[25] V.A. Gordon, A.G. Tamrazyan, T.V. Savostikova, Dynamic stress reinforcing bar with a sudden crack formation, *Herald SIC "Construction*", **2**, 167-176, 2010.