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# ELECTRICAL FIELDS SIMULATION IN HETEROGENOUS DOMAINS USING THE PROPER GENERALIZED DECOMPOSITION

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**Abstract.**

*Recently microwave heating is replacing classical methods in composite manufacturing processes. In fact one of the advantages of microwave heating is achieving a volumetric heating, which can improve the final material properties. However electrical and magnetic fields propagation is complicated to simulate and understand inside a heterogeneous domain. In fact, carbon fibers are highly conductor with respect to the composite matrix. Such behavior would alter the wave propagation inside the part and reduces the desired volumetric heating effects. In this work, we simulate the propagation of electric fields inside a heterogeneous part by using the Proper Generalized Decomposition (known as PGD). In fact this method helps us achieving full 3D simulations by the complexity of few 1D simulations.*

## 1 INTRODUCTION

Composite parts are getting popular thanks to their good mechanical properties and low weight. Aeronautic and transport industries are turning to composite materials to reduce the energy use and therefore the transportation costs.

However composite materials manufacturing processes are not fulfilling industrial needs yet. In fact, classical manufacturing techniques can't follow up the increasing industrial demand of faster and reliable production. Therefore, researchers are turning their attention to innovative production techniques such as the microwave curing of composite parts. Moreover, microwave curing can lead to better parts with mechanical properties similar to the autoclave-made parts [1], thanks to the volumetric heating achieved by this method [2]. On the other hand, volumetric curing can also shorten the total manufacturing process time [3]. Such advantages are leading the industrials to microwave heating of thermoplastic as well as thermoset polymers.

On the other hand, microwave heating relies on the propagation of an electromagnetic field inside composite materials. However, such materials are heterogeneous, and usually with a degenerated thickness dimension. Moreover, we note that carbon fibers are electrical conductors, while, in general, the composite matrix is highly insulating. Thus, the electrical field behavior in such conditions is still not mastered yet to the knowledge of the authors. Therefore a full 3D simulation is required to analyze the wave propagation inside the composite material and understand the microwave heating process. Despite the large progress in computing power during the last decade, such simulations are still difficult to perform with the required resolution using classical calculation techniques. In fact the simulation of the wave propagation in the composite material is highly dependent on the degenerated thickness direction.

Thus, model order reduction techniques appeared recently as a replacement of classical numerical methods, which allow faster simulation with acceptable accuracy [4]. The Proper Generalized Decomposition or PGD is one of these techniques which allows overcoming the curse of dimensionality by using separated representation of the domain [4, 5]. In fact this technique allows the simulation of a full 3D models with a complexity of few 2D or 1D simulations [6, 7].

The objective of this work is to simulate the electrical field propagation using a full 3D model in an heterogeneous domain, containing carbon fibers inside a thermoplastic matrix. Using the PGD, the simulation will be separated in a sequence of three 1D simulations which will lead to the full 3D solution [8]. We will be showing the results inside two different domains. In the first one, the domain dimensions are large with respect to the wavelength, while in the second one, the domain dimensions are small with respect to the wavelength.

## 2 SIMULATED MODEL

In this section we describe the model used in the simulation. As illustrated in figure 1, we have a matrix domain in which a conductor which has a rectangular cross section is placed. The conductor is at $\frac{3}{5}b$ from the bottom of the domain and centered in the $(x, z)$ plane. The cross section of the conductor has the dimensions $\frac{1}{5}b \times 0.05c$.

The physical model consists of the Maxwell equations in the phasor domain, that after some

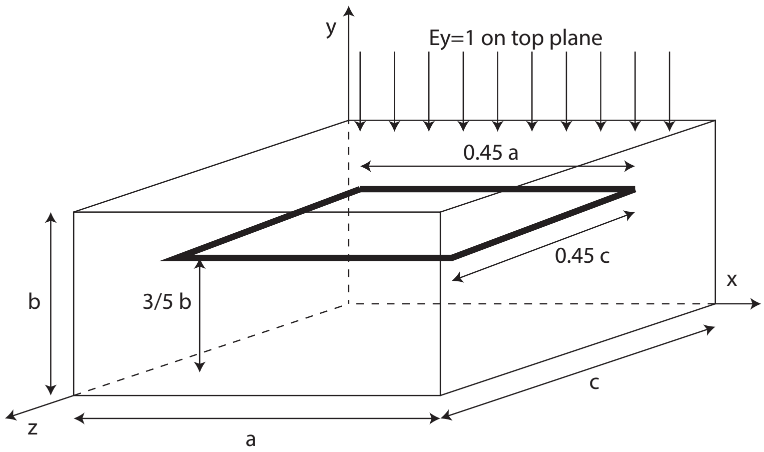


Figure 1: The model used in the simulation

manipulations and assuming an harmonic electric field, is reduced to:

$$\nabla^2 \mathbf{E} - \gamma^2 \mathbf{E} = 0 \tag{1}$$

where $\mathbf{E}$ is the electric field in the phasor domain having 3 components denoted as $\mathbf{E} = (E_x; E_y; E_z)$. While $\gamma$ is defined by:

$$\gamma^2 = i \cdot w \cdot \mu \left(\sigma + i \cdot w \cdot \epsilon\right) \tag{2}$$

where $i$ being the complex number, $\sigma$ is the electrical conductivity, $\mu$ is the electrical permeability, $\epsilon$ is the permittivity and $\omega$ is the angular frequency of the wave. In our simulations we use a frequency of 2.45 GHz, which has a wavelength of about 12 cm. The boundary conditions of the simulated domain can be written by:

$$\begin{array}{l}
E_y = 0 \text{ at } y = 0; \\
E_y = 1 \text{ at } y = b; \\
E_x(x=0) = E_x(x=a); \\
E_x(z=0) = E_x(z=c); \\
\frac{\partial E_x}{\partial x}(x=0) = \frac{\partial E_x}{\partial x}(x=a); \\
\frac{\partial E_x}{\partial z}(z=0) = \frac{\partial E_x}{\partial z}(z=c); \\
E_z(x=0) = E_z(x=a); \\
E_z(z=0) = E_z(z=c); \\
\frac{\partial E_z}{\partial x}(x=0) = \frac{\partial E_z}{\partial x}(x=a); \\
\frac{\partial E_z}{\partial z}(z=0) = \frac{\partial E_z}{\partial z}(z=c); \\
E_y(x=0) = E_y(x=a); \\
E_y(z=0) = E_y(z=c); \\
\frac{\partial E_y}{\partial x}(x=0) = \frac{\partial E_y}{\partial x}(x=a); \\
\frac{\partial E_y}{\partial z}(z=0) = \frac{\partial E_y}{\partial z}(z=c);
\end{array} \tag{3}$$

The boundary conditions depicted in equation 3, can be resumed by an imposed electrical field on the $y$ direction and symmetrical boundary conditions on the $x$ and $z$ directions.

In table 1 we indicate the considered parameters for both the matrix and the fibers (conductor) in the domain.

| Material | conductivity | permeability | permittivity |
|---|---|---|---|
| Conductor | inf | $\mu_0$ | $\epsilon_0$ |
| Matrix | 0 | $\mu_0$ | $\epsilon_0$ |

Table 1: Parameters used in the simulation of the proposed model

## 3 USING THE PROPER GENERALIZED DECOMPOSITION

Using the Proper Generalized Decomposition, we will separate the domain into $3 \times 1D$ domains by assuming that $E_x$, $E_y$ and $E_z$ can be written as:

$$\begin{aligned} E_x &= \sum_{i=0}^{i=N} X_x^i(x) \cdot Y_x^i(y) \cdot Z_x^i(z) \\ E_y &= \sum_{i=0}^{i=N} X_y^i(x) \cdot Y_y^i(y) \cdot Z_y^i(z) \\ E_z &= \sum_{i=0}^{i=N} X_z^i(x) \cdot Y_z^i(y) \cdot Z_z^i(z) \end{aligned} \tag{4}$$

Such representation is appealing for solving 3D problems efficiently, especially for problems with rich physics in the in-plane and out-of-plane directions, while ensuring a computational complexity of few 1D resolutions [6]. Such representation allows the use of a fine mesh in the degenerated thickness direction if needed. However, classical techniques should employ a 3D extremely fine mesh in the entire domain to capture the discontinuity of the electrical field around the conductors, which involves a prohibitive number of degrees of freedom. Moreover, for the resolution of the separated 1D problems, one could use any suitable numerical technique (for example finite elements for one problem and finite differences for the second one).

The resolution procedure starts by replacing equations (4) into the weak form of the Maxwell equation (1). Afterwards, a fixed-point algorithm is used to compute the unknown functions, one at a time. All the details of the resolution algorithm can be found in [9].

## 4 RESULTS

### 4.1 Large domain

As a first step, we propose to simulate the problem detailed in section 2 on a large domain such that the wavelength is small compared to $a$, $b$ or $c$ values. Therefore we start by showing the results on a domain having the dimensions $a = 1m$; $b = 0.4m$ and $c = 1m$. We use 110 nodes along each of the $x$ and $z$ directions, while we use 500 nodes along the $y$ direction. All the following figures are showing the amplitude of the electrical fields components. Figure 2 illustrated the $E_y$ component in the domain, while figure 3 shows $E_y$ inside the domain using 2 cuts normal to the $x$ and $z$ axis. Figure 4 illustrates $E_y$ in the domain with a horizontal cut, normal to the y-axis, and passing by the conductor. On the other hand, figures 5, 7 and 6 show respectively $E_x$, $E_y$ and $E_z$ on a cut passing by $z = 0.5m$.

### 4.2 Small domain

In this section, we solve the same model on a domain that is relatively small compared to the wavelength, therefore we consider $a = 1mm$; $b = 0.4mm$ and $c = 1mm$. The result for $E_y$ in a

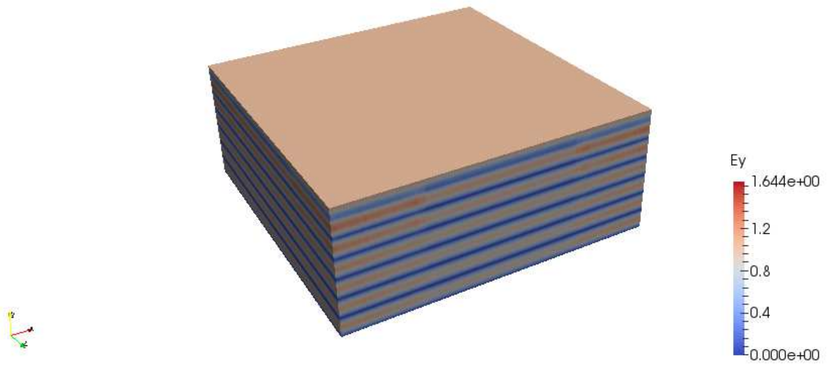


Figure 2: The electrical fields on the domain $a = 1m$; $b = 0.4m$ and $c = 1m$

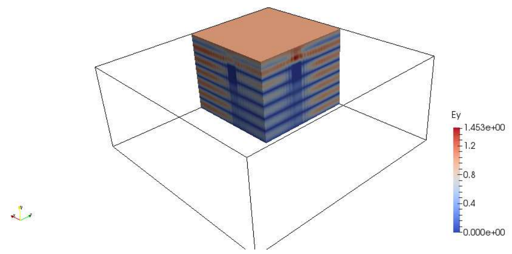

Figure 3: $E_y$ shown inside the domain $a = 1m$; $b = 0.4m$ and $c = 1m$

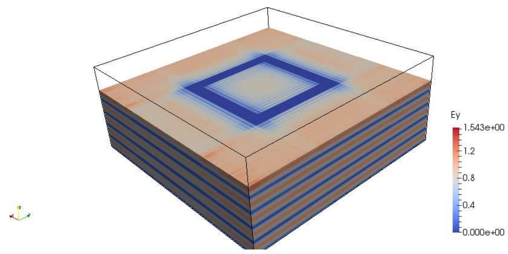

Figure 4: $E_y$ on an horizontal cut passing by the conductor

vertical section passing by the fibers is shown in figure 8. Moreover, figures 10, 9 and 11 show respectively $E_x$, $E_y$ and $E_z$ in a section normal to the z-axis and passing by $z = 0.5 \cdot 10^{-3}m$.

## 5 CONCLUSION

In this work, we solved the Maxwell equations in a heterogeneous domain to simulate the electrical fields. The resolution is performed on two different domains, the first one large with respect to the wavelength, while the second one is small with respect to it.

The shown results are different in each case. In the large domain, the wavelength is small with respect to the domain dimensions therefore the electric field can oscillate inside the domain. However, in the second case, the electric field can't oscillate due to the large wavelength with respect to the dimensions of the domain.

For the first time to the knowledge of the authors we are able to simulate in full 3D the electric fields with a resolution up to more than 6 000 000 nodes (18 000 000 degrees of freedom) with about an hour on a standard laptop, core i7. Such simulation wasn't possible without the use of the PGD which allows separating the 3D domain into $3 \times 1D$ domains. Therefore the computation time is reduced several orders of magnitude, from few days to few minutes. Moreover, the used resolution allows capturing the 3D effects generated by the presence of the

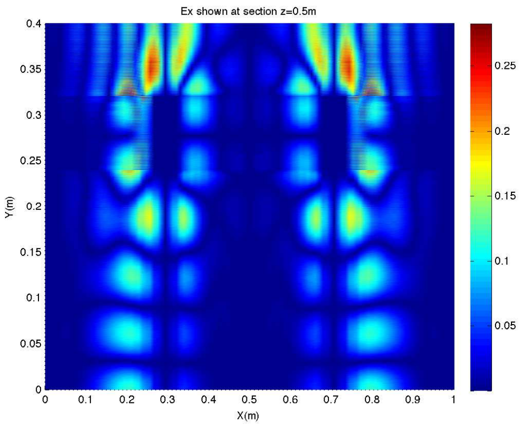


Figure 5: $E_x$ on a section passing by $z = 0.5$ m

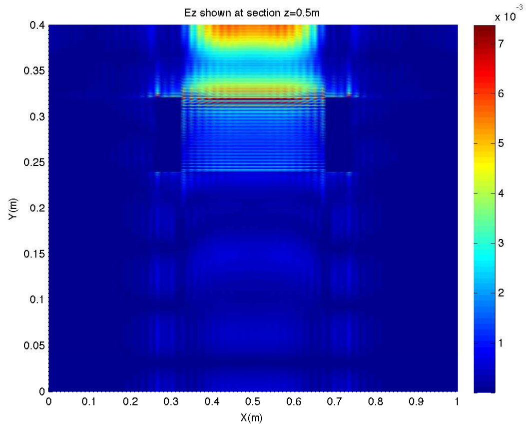


Figure 6: $E_z$ on a section passing by $z = 0.5$ m

conductor in the domain, which changes the electrical fields around the conductor.

This present work will be generalized to a large number of fibers inside the domain, leading to effective simulation of composite materials and therefore a better understanding of the microwave heating process in both thermoplastics and thermosets.

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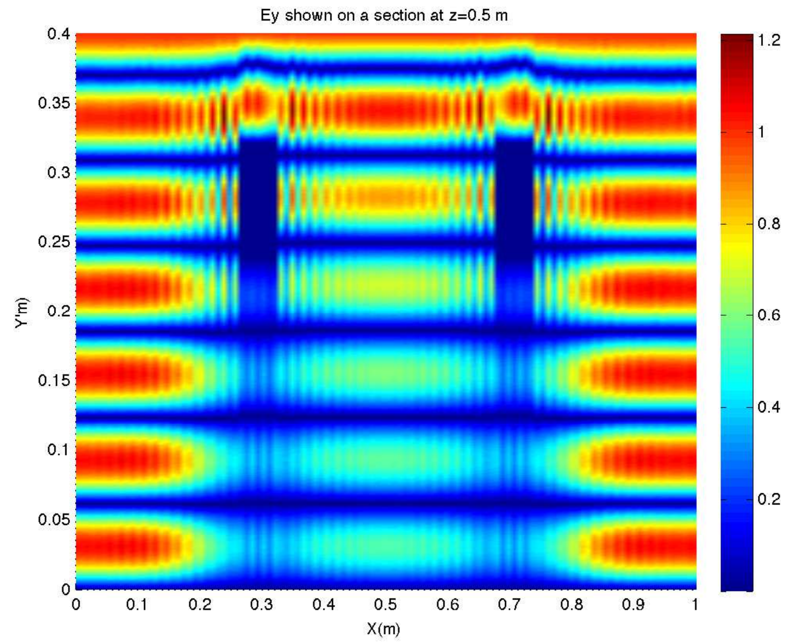


Figure 7: $E_y$ on a section passing by $z = 0.5$ m

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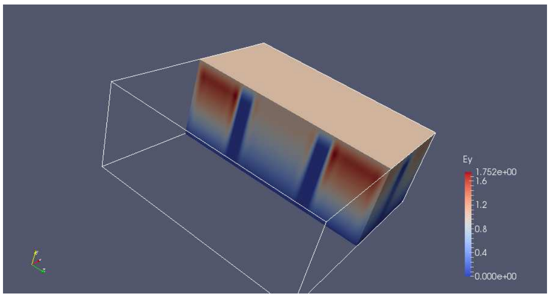


Figure 8: The electrical field $E_y$ in the domain $a = 1 \times 10^{-3}m$; $b = 0.4 \times 10^{-3}m$ and $c = 1 \times 10^{-3}m$, shown on a section normal to $z$

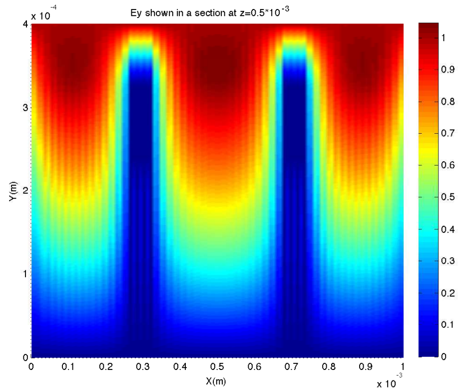


Figure 9: $E_y$ on a section passing by $z = 0.5 \times 10^{-3}$ m

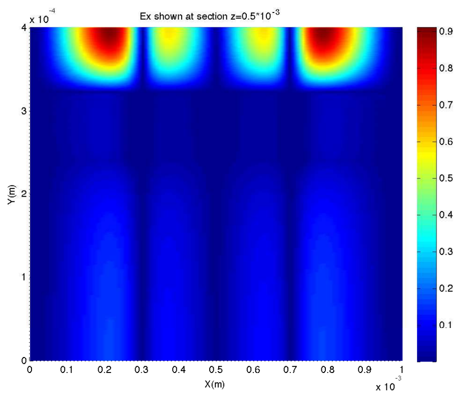


Figure 10: $E_x$ on a section passing by $z = 0.5 \times 10^{-3}$ m

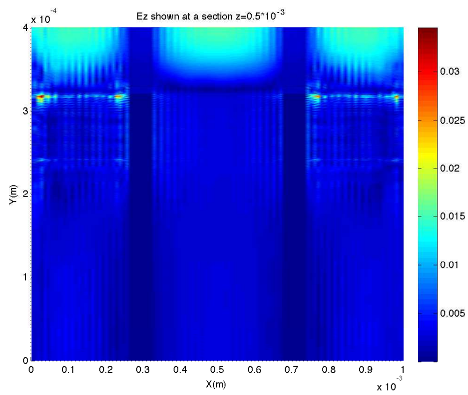


Figure 11: $E_z$ on a section passing by $z = 0.5 \times 10^{-3}$ m