ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

TO ESTIMATE NON-STATIONARY STOCHASTIC DISTRIBUTED LOAD ON A BEAM STRUCTURE FROM RESPONE SAMPLES

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Keywords: Spatially distributed load identification, Dynamic, Stochastic, non-stationary, Mode shape function.

Abstract. *A stochastic load identification approach is proposed based on a beam structure. The dynamic force is assumed as a non-stationary Gaussian random field varying with both time and space. The mode shape functions are selected as orthogonal basis for the projection of the distribution function of the stochastic dynamic force which enables the excitation force can be estimate from limit measurement response data. The statistics of the stochastic distributed force are estimated by the Mode Superposition Method (MSM) combined with the Monte Carlo Approach (MCA). A numerical example is studied. A trapezium-shaped function is assumed for the distribution of the stochastic dynamic force. The stochastic time history of the excitation forces at each location on beam is assumed to be Gaussian and have a time-varying mean value and power density function. Numerical simulations are conducted to verify the load identification algorithm. Results show that the proposed method is effective to identify the stochastic distributed load on the structure.*

1 INTRODUCTION

Dynamic load is one of the key factors in the design and condition assessment of various engineering structures. In the cases when the load information is difficult to be obtained by direct measurement, indirect approaches which noted as force identification techniques, are often adopted.

Numerous force identification approaches have been proposed in the past decades which can successfully estimate the dynamic excitation forces on engineering structure from measurement dynamic response. These methods can mainly be classified into two categories: frequency-domain method [1, 2] and time-domain method [3, 4], etc. Just as their name imply, the relationship between the measurement response and the dynamic force to be estimated are formulated in frequency domain and time domain respectively in the two kinds of method. Most of the aforementioned approaches are deterministic and focusing on the identification of concentrated force.

It is noted that various kinds of the dynamic load are spatially distributed along the structure. Two approaches namely the finite difference method and the modal method were proposed by Pezerat and Guyader [5] to estimate the spatially distributed dynamic force on structure and the regularization approach was adopted to solve the ill-posed problem [6]. In the modal method, when the force to be identified only spans over the contact region, a prohibitively large number of basis functions is often needed. An improved method was proposed by Liu and Shepard [7] in which new basis functions are introduced to overcome the drawback. A mode-selection method for distributed dynamic force was proposed by Jiang and Hu [8] and the optimal range of frequency and spatial modes was studied; to improve the accuracy on the identified force at fix boundary, the Legendre polynomials are used as the consistent orthogonal base functions to describe the distributed dynamic loads instead of the mode shape function, similar work was done by Li et al. [9] with the Chebyshev orthogonal polynomials. The temporal-dependent external forces for the cutting tools were estimated by Huang et al. [10] based on the conjugate gradient method. The acoustic holography and the force analysis technique were adopted by Pezerat et al. [11] to identify vibration sources from radiated noise measurements. The Virtual Fields Method was adopted by Berry et al. [12] to identify both mechanical point load excitation and distributed acoustic excitation of a bending panel.

Since randomness often exists in excitation forces due to the random nature, e.g. wind load, earthquake, excitation due to the road surface roughness, excitation from the ocean waves, etc. Stochastic force identification techniques have been developed. To identify the concentrated stochastic force on structure, the system uncertainty is not included at first, which can be divided into two kinds: frequency domain method [13] and time domain method [14, 15]; later, both the uncertainty in system parameters and excitation forces have been included [16, 17, 18]. These research work provide statistical estimations on the stochastic excitation forces on structure, however, stochastic excitation forces such as the wind load, load due to ocean waves, etc. are spatially distributed, new techniques should be developed to deal with the distributed stochastic dynamic force identification. A frequency domain method was proposed by Granger and Perotin [19] to identify the distributed random excitation acting on a vibrating structure and this method was further be adopted to a fluid-induced vibration problem [20]. The distributed random excitation is assumed as a stationary Gaussian random field and the mode method was adopted to identify the distribution and power spectrum density (PSD) function of the excitation force from the PSD of the measurement random response.

In this paper, a time domain approach is proposed in which a non-stationary stochastic distributed dynamical force is identified from the response sample sets from limit number of sensors under a beam structure. The mode shape functions are selected as orthogonal basis for the projection of the stochastic distributed force. The statistics of the stochastic distributed force are estimated by the mode superposition technique combined with the MCA.

2 STOCHASTIC RESPONSE ANALYSIS OF A DYNAMICAL SYSTEM

2.1 Description of the dynamical system

A uniform simply supported Euler-Bernoulli beam subjected to a stochastic distributed dynamic force is shown in Figure 1 and this dynamic system will be adopted to demonstrate the new identification algorithm in this paper. It is noted that the proposed method may also be effective on other kinds of structure with different boundary conditions as long as the vibration modes are available.

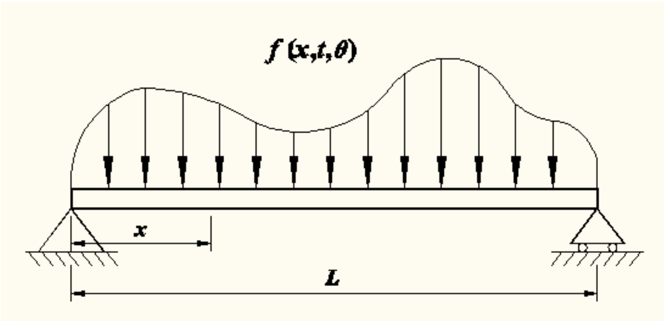

Figure 1: A simply supported beam subjected a stochastic distributed force

The equation of motion of the dynamical system is

$$\rho A\frac{\partial^2 w(x,t,\theta)}{\partial t^2}+c\frac{\partial w(x,t,\theta)}{\partial t}+EI\frac{\partial^4 w(x,t,\theta)}{\partial x^4}=f(x,t,\theta) \tag{1}$$

where ρ is the mass density and A is the cross-sectional area, therefore, ρA represents the mass per unit length. Symbols c and EI are the damping and the flexural rigidity of the beam, respectively. All the system parameters are assumed to be deterministic. $f(x,t,\theta)$ is a non-stationary Gaussian random field represents the stochastic distributed dynamic force; $w(x,t,\theta)$ is the stochastic displacement under the beam at location x and time t, and θ represents the random dimension.

2.2 Equation of motion in modal space

Though correlations may exist among the time, space and random dimension in stochastic excitations and random responses, in this research work, the distributed dynamic force is assumed as a non-stationary Gaussian random field and the distribution of the excitation is assumed to be independent of the other two dimensions and. With the ith mode shape function $\varphi_i(x)$, the stochastic distributed excitation $f(x,t,\theta)$ and the stochastic displacement under the beam $w(x,t,\theta)$ can respectively be expressed as

$$f\left(x,t,\theta\right)=\sum_{i=1}^{m}\varphi_i\left(x\right)F_i\left(t,\theta\right) \tag{2}$$

$$w\left(x,t,\theta\right)=\sum_{i=1}^{m}\varphi_i\left(x\right)q_i\left(t,\theta\right) \tag{3}$$

where the random processes $F_i(t,\theta)$ and $q_i(t,\theta)$ are the stochastic modal coordinates of the excitation and the displacement, respectively and m represents the number of modes adopted after truncation. When a simply supported Euler-Bernoulli beam is considered, the ith natural circular frequency ω_i and the mode shape $\varphi_i(x)$ have the following expressions respectively,

$$\begin{aligned}\omega_i &= \left(\frac{i\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \quad i=1,2,\cdots,m \\ \varphi_i(x) &= \sin\frac{i\pi x}{L}\end{aligned} \tag{4}$$

where L is the length of the beam.

Substituting Eqs.(2) and (3) into Eq.(1), by employing the orthogonal property between the mode shape functions,

$$\int_0^L \rho A \varphi_i(x)\varphi_j(x)\,\mathrm{d}x = m_i\delta_{ij} \tag{5}$$

where δ_{ij} is the Kronecker delta and m_i represents the ith modal mass. In this case $m_i=\rho AL/2$. Then Eq.(1) can be rewritten as

$$\ddot{q}_i(t,\theta)+2\zeta_i\omega_i\dot{q}_i(t,\theta)+\omega_i^2 q_i(t,\theta)=\frac{F_{mi}(t,\theta)}{m_i}, \quad i=1,2,\cdots,m \tag{6}$$

where the over-dot represents the differentiation respect to time t. The symbol ζ_i denotes the damping ratio of the ith mode; $F_{mi}(t,\theta)$ is the ith modal force and

$$F_{mi}(t,\theta)=\int_0^L f(x,t,\theta)\varphi_i(x)\,\mathrm{d}x \tag{7}$$

The stochastic modal coordinates of excitation $F_i(t,\theta)$ can be calculated by

$$F_i(t,\theta)=\frac{F_{mi}(t,\theta)}{\int_0^L \varphi_i^2(x)\,\mathrm{d}x} \tag{8}$$

Eq.(6) is a set of stochastic ordinary differential equations which can be solved by numerical methods combined with statistical analysis tools such as the Monte Carlo simulation, the KL method, etc. and then the statistics of the random dynamical response under the beam can be obtained.

3 IDENTIFICATION OF THE DISTRIBUTED STOCHASTIC DYNAMIC FORCE

3.1 Projection scheme of the distributed excitation

When the distribution of the excitation is assumed to be independent of the other two dimensions, the stochastic distributed force $f(x,t,\theta)$ can be expressed in the follow form:

$$f(x,t,\theta)=T(x)\cdot P(t,\theta) \tag{9}$$

where $T(x)$ is a function represents the distribution pattern of the force and $P(t,\theta)$ is a random process represents the stochastic time history at each location x.

It is more difficult to estimate a distributed dynamic force $f(x,t)$ than a concentrate one since the unknown time histories of the force at each location x should be identified from limited measurement response data. The number of the unknowns tends to be infinite. In order to reduce the unknowns in the identification process, a projection scheme is often adopted, e.g. to represent the distribution function $T(x)$ by a set of orthogonal bases $T_i(x)$,

$$f(x,t) = \sum_i T_i(x) d_i(t) \tag{10}$$

According to Eq.(10), once the coefficients $d_i(t)$ are estimated from limited measurement response data, the distributed dynamic force can be identified, which will significantly simplified the identification. With the projection scheme in Eq.(10), the ith modal force of a distributed dynamic force $f(x,t)$ gives,

$$F_{mi}(t) = \int_0^L f(x,t)\varphi_i(x)\mathrm{d}x = d_i(t)\int_0^L \sum_{ij} T_j(x)\varphi_i(x)\mathrm{d}x \tag{11}$$

When orthogonal property exists between $T_j(x)$ and $\varphi_i(x)$, the integration in Eq.(11) can be significantly simplified. Therefore, the mode shape function $\varphi_j(x)$ are adopted to replace $T_j(x)$ in Eq.(11) in this research work, according to Eq.(5), the expression of the ith modal force can be written as

$$F_{mi}(t) = d_i(t)\int_0^L \sum_{ij} \varphi_i(x)\varphi_j(x)\mathrm{d}x = \frac{m_i}{\rho A} d_i(t) \tag{12}$$

where

$$d_i(t) = \frac{\rho A F_{mi}(t)}{m_i} \tag{13}$$

Substituting Eq.(13) in to Eq.(10), according to Eqs.(5) and (8), the distributed dynamic force $f(x,t)$ can be represented by the mode shape functions as follows:

$$f(x,t) = \sum_i \varphi_i(x) d_i(t) = \sum_{i=1}^{m} \varphi_i(x) F_i(t) \tag{14}$$

It is noted that Eq.(14) will share the same expression with Eq.(2) when the random dimension θ is considered. To consider the random dimension θ in Eq.(12) and by substituting Eq.(12) into Eq.(6), we have

$$\ddot{q}_i(t,\theta) + 2\zeta_i\omega_i\dot{q}_i(t,\theta) + \omega_i^2 q_i(t,\theta) = \frac{1}{\rho A} d_i(t,\theta), \quad i=1,2,\cdots,m \tag{15}$$

Eq.(15) shows the relationship between the stochastic modal coordinates of the structural response and the stochastic coefficients of the distributed dynamic excitation force. The stochastic force identification algorithm can be formulated based on Eq.(15) with Monte Carlo approach.

3.2 Statistical force identification with MCA in modal space

Uncertainty in structural responses may originate from uncertainty in both the system parameters and excitation forces, in this research work, the system uncertainty is not addressed. The displacement measured at each location under the beam structure form a set of response sample and these sample sets belong to the population of the random displacement field $w(x,t,\theta)$. When the displacements at location $(x_1,x_2,\ldots x_n)$ are obtained from the rth measurement, these displacement data will form a vector as

$$\mathbf{W}_r = \{ w_r(x_1,t) \quad w_r(x_2,t) \quad \cdots \quad w_r(x_n,t) \}^{\mathrm{T}}, \; r = 1,\cdots,N_r \tag{16}$$

where N_r is the total number of the sample sets.

With the mode shape functions, the modal coordinates of each set of the dynamic response sample can be obtained from the following Equation，

$$\begin{Bmatrix} w_r(x_1,t) \\ w_r(x_2,t) \\ \vdots \\ w_r(x_n,t) \end{Bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_m(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_m(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_2) & \cdots & \varphi_m(x_n) \end{bmatrix} \begin{Bmatrix} q_{1,r}(t) \\ q_{2,r}(t) \\ \vdots \\ q_{m,r}(t) \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{1,r}(t) \\ \varepsilon_{2,r}(t) \\ \vdots \\ \varepsilon_{n,r}(t) \end{Bmatrix} \tag{17}$$

where $q_{i,r}(t)$ (i=1,$\cdots$,m) is the modal coordinate of the ith mode and rth measurement and $\varepsilon_{j,r}(t)$ (j=1,$\cdots$,n) is the error term of the jth location and rth measurement. The error term may include the measurement noise, model error, truncation of the higher order of modes and so on. Generally, the number of the modes m after truncation should be smaller than the number of the measurement locations n to avoid solving a set of underdetermined system of equations as shown in Eq.(17). When the rth sample of the coordinate of displacement $q_{i,r}(t)$ corresponding to the ith mode is obtained, the first and second derivatives of $q_{i,r}(t)$, which represent the coordinate of the velocity $\dot{q}_{i,r}(t)$ and acceleration $\ddot{q}_{i,r}(t)$, respectively, can be derived with differentiation. Therefore, the samples of the coefficient $d_{i,r}(t)$ for the distributed force can be calculated as

$$d_{i,r}(t) = \rho A\left[\ddot{q}_{i,r}(t) + 2\zeta_i\omega_i\dot{q}_{i,r}(t) + \omega_i^2 q_{i,r}(t)\right], \quad r=1,2,\cdots,N_r \tag{18}$$

Finally, the statistics of the identified stochastic distributed force, i.e. the mean value $\mu_f(x,t)$ and the variance $Var_f(x,t)$ can respectively be derived as

$$\mu_f(x,t) = \frac{1}{N_r}\sum_{r=1}^{N_r}\sum_{i=1}^{m}\varphi_i(x)\, d_{i.r}(t) \tag{19}$$

$$Var_f(x,t) = \frac{1}{N_r - 1}\sum_{r=1}^{N_r}\left(\sum_{i=1}^{m}\varphi_i(x)\, d_{i.r}(t) - \mu_f(x,t)\right)^2 \tag{20}$$

4 NUMERICAL SIMULATIONS

To verify the effectiveness of the proposed method and investigate different factors which may influence the accuracy of this stochastic distributed dynamic force identification algorithm, numerical simulations are conducted. The following parameters of the beam structure are adopted.

Length of the beam L=40 m; Cross-sectional area A=4.8 m^2; Second moment of inertia of cross-section I=2.5498 m^4; Rayleigh damping is assumed for the structure with damping ratio ξ=0.02 for all vibration modes; Young's modulus E and mass density ρ of material are 5×10^{10} N/m^2 and 2.5×10^3 kg/m^3 respectively. The first five natural frequencies of the beam are 3.9, 15.6, 35.1, 62.5 and 97.6 Hz, respectively.

A trapezium-shaped distribution function of the stochastic dynamic force is assumed in this study which has the following expression,

$$T(x)=\begin{cases}4x/L & 0\le x<L/4\\ 1 & L/4\le x\le 3L/4\\ -4x/L+4 & 3L/4<x\le L\end{cases} \tag{21}$$

It should be noted that the selection of distribution function is arbitrary unless it is periodic. Theoretically, any periodical distribution functions $T(x)$ can be represented by the orthogonal modal shape function bases; therefore, it will not affect the investigation on the proposed method.

The stochastic time history of the excitation forces at each location x denoted by $P(t,\theta)$ is assumed with the following mean values,

$$P_{\mathrm{d}}(t)=20000\left(1+0.1\sin(2\pi t)\right)\ \mathrm{N} \tag{22}$$

where the subscript 'd' denotes the deterministic part. The random part of $P(t,\theta)$ is assumed as a zero-mean non-stationary Gaussian random process with a time-varying PSD function $S(\omega,t)$ as

$$S\left(\omega,t\right)=C_f P_{\mathrm{d}}(t)\Phi\left(\omega\right) \tag{23}$$

where C_f is a coefficient represents the level of randomness and $\Phi(\omega)$ is a two-sided PSD function of a zero-mean stationary Gaussian random process,

$$\Phi\left(\omega\right)=\frac{1}{2\pi}\frac{2}{\omega^2+1} \tag{24}$$

According to the spectral representation[21] for a Gaussian process, the rth sample of the stochastic distributed force $f(x,t,\theta)$ at location x can be simulated by the following series,

$$f_{\mathrm{r}}(x,t)=T\left(x\right)\left[P_{\mathrm{d}}(t)+C_f P_{\mathrm{d}}(t)\sum_{k=1}^{N_k}\sqrt{4\Delta\omega\Phi\left(\omega_k\right)}\cos(\omega_k t+\varphi_k)\right] \tag{25}$$

where

$$\omega_k=\omega_{\min}+\Delta\omega\left(k-1\right) \tag{26}$$

$$N_k=\left(\omega_{\max}-\omega_{\min}\right)/\Delta\omega \tag{27}$$

It should be noted that Eq. (25) is not unique and other forms of expansion can be adopted. The variable φ_k is a random phase angle uniformly distributed in the interval [0, 2π]. The constant $\Delta\omega$ denotes the frequency increment and N_k is the total number of frequency divisions in the interval $[\omega_{\min}, \omega_{\max}]$. In this study, $\omega_{\min}$=0 Hz, $\omega_{\max}$=200π Hz, N_k=512, therefore, $\Delta\omega$=1.227 Hz and the sampling frequency in all the simulations is 200Hz, the total time is 1s.

The relative errors between the referenced and calculated results, *RE*, is defined as

$$RE=\frac{\left\|R_{calculated}-R_{referenced}\right\|_2}{\left\|R_{referenced}\right\|_2}\times 100\% \tag{28}$$

where $\left\|\bullet\right\|_2$ denotes the 2-norm.

4.1 Verification of the Forward Problem

To verify the forward problem of the proposed algorithm, the response samples of the beam structure under a stochastic distributed dynamic force are calculated with two methods: Modal superposition method with MCA (MMA) and FEM with MCA (FMA).

The samples of the stochastic distributed dynamic force are calculated from Eq. (25) in which the random phase angle φ_k is generated by "rand" in MATLAB software. C_f which represents the level of randomness is set to 0.2. When the number of response samples is sufficient, accuracy of the proposed algorithm will not be affected by the level of randomness due to the characteristic of MCA. In MMA, the samples of the *i*th modal force $F_{mi}(t,\theta)$ is calculated according to Eq. (7) in which the integration is conducted using "trapz" in MATLAB software with a step of 0.4m. 30 vibrational modes are adopted and the samples of the modal coordinates of the structural response are calculated by solving Eq. (6) using Newmark-β method. Response statistics of both 5000 samples and 10000 samples from MCS are obtained. In FMA, the samples of the nodal forces are calculated by integrations between the samples of the stochastic distributed dynamic force and the Hermitian cubic interpolation shape functions and. eight beam elements are adopted in the finite element model. The response samples of the beam structure obtained in FMA is similar to that in MMA and the response statistics of 5000 samples from MCS are obtained.

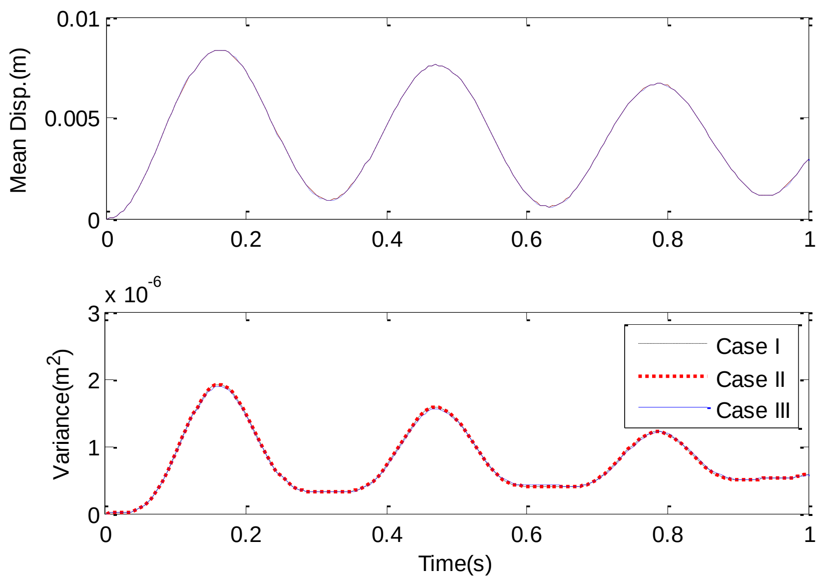

Figure 2. Response statistics of the mid-span displacement

Three sets of statistics of the mid-span displacement of the beam structure are demonstrated in Figure 2. They are from 5000 samples in MMA (Case I), 5000 samples in FMA (Case II) and 10000 samples in MMA (Case III), respectively. Results show that the time histories of mean value and covariance of the mid-span displacement from different simulations agree well with each other. If the response statistics of 10000 samples from MMA (Case III) is assumed as the "referenced" result, the relative errors of the mean value and variance between Case I and Case III according to Eq. (28) are 0.27% and 2.74%, respectively; the relative errors of the mean value and variance between Case II and Case III also equal to 2.85% and 5.82%, respectively. Results indicate that: (1) the time histories of response statistics in all Cases are agree well with each other; (2) 5000 runs in MCA are sufficient for obtaining accurate response statistics; (3) Response statistics from FMA in this case is sightly smaller than that from MMA.

4.2 Verifications of the Inverse Problem

To verify the inverse problem of the proposed algorithm, i.e. to identify the statistics of the non-stationary stochastic distributed dynamic force from measurement response samples, 5000 sets of 19 displacements evenly distributed underneath the beam structure are adopted, i.e. N_r=5000 and n=19, the distance between two sensors in this case is 2m. It is noted that strain signals can also be adopted in the force identification, since strains have the following relationship with displacement for a beam structure,

$$\varepsilon\left(x,t\right) = -z\frac{\partial^2 w\left(x,t\right)}{\partial x^2} \tag{29}$$

where z represents the distance from the neutral axis of the beam to the strain gauge.

Nineteen modes are adopted in the inverse problem, i.e. m=19. When the number of the measurement locations n is not less than the number of the modes m, the modal coordinates for each set of displacement sample are calculated according to Eq. (17). The cubic spline interpolation is applied on the modal coordinates for each set of displacement sample $q_{i,r}(t)$, then the first and second derivatives of $q_{i,r}(t)$ are calculated which represent the modal coordinate of the velocity and acceleration, respectively. Therefore, the samples of the coefficient $d_{i,r}(t)$ for the distributed force can be calculated according to Eq.(18). Finally, the statistics of the identified stochastic distributed dynamic force can respectively be obtained from Eqs. (19) and (20). The "referenced" mean value and variance of the stochastic distributed dynamic force can respectively be estimated from the following Equations,

$$\tilde{\mu}_f(x,t) = \frac{1}{N_r}\sum_{r=1}^{N_r} f_r\left(x,t\right) \tag{30}$$

$$\tilde{Var}_f(x,t) = \frac{1}{N_r-1}\sum_{r=1}^{N_r}\left(f_r\left(x,t\right) - \mu_f(x,t)\right)^2 \tag{31}$$

The identified and referenced statistics of the stochastic distributed dynamic force at the mid-span are shown in Figure 3. Results show that the time histories of two statistics agree well with each other, which indicates the statistics of the distributed non-stationary stochastic dynamic force can successfully be identified from response sample sets. The relative errors between the identified and the referenced mean value and variance are 4.55% and 7.08%, respectively.

Since the stochastic force is non-stationary, the distribution of the force will vary with respect to time. In order to demonstrate the accuracy on the identified distribution of the force, the identified distributions at different moments, e.g. at 0.25s, 0.5s and 0.75s, are normalized and compared with the referenced distribution function $T(x)$ in Eq. (21). Results are shown in Figure 4 in which the identified distributions of the mean force and the variance of the force at each moment are compared with $T(x)$ and $T(x)^2$, respectively. The distribution at each moment of the distributed non-stationary stochastic force can be successfully identified from limit number of measurement points. The relative errors between the identified and the referenced distribution for the mean value and variance of force at 0.5s are 0.57% and 0.92%, respectively. The relative errors among the identified distribution for the mean value and variance of force at different moments are smaller than 0.5%, i.e. the identification error is small and stable during the time history of the force.

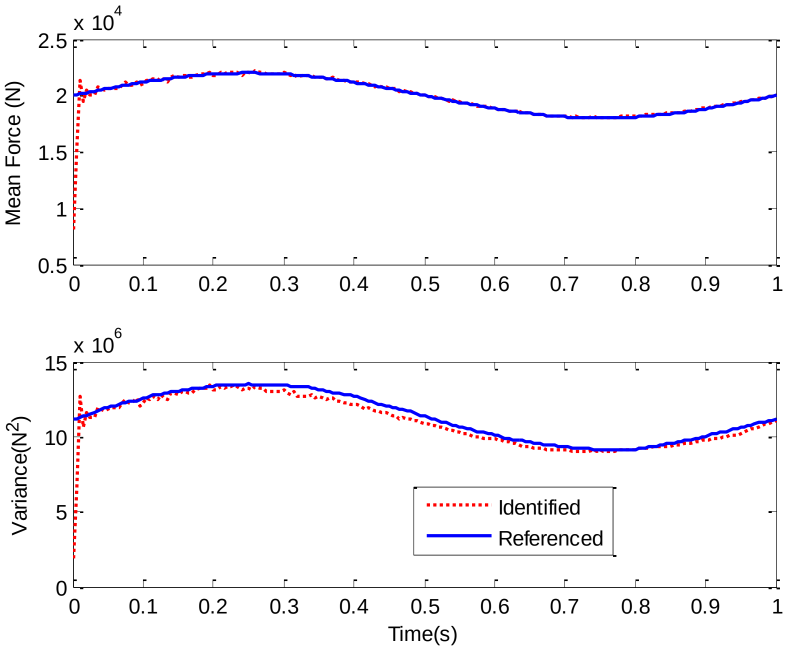

Figure 3. Comparison of the identified and referenced statistics of the distributed stochastic force at mid-span

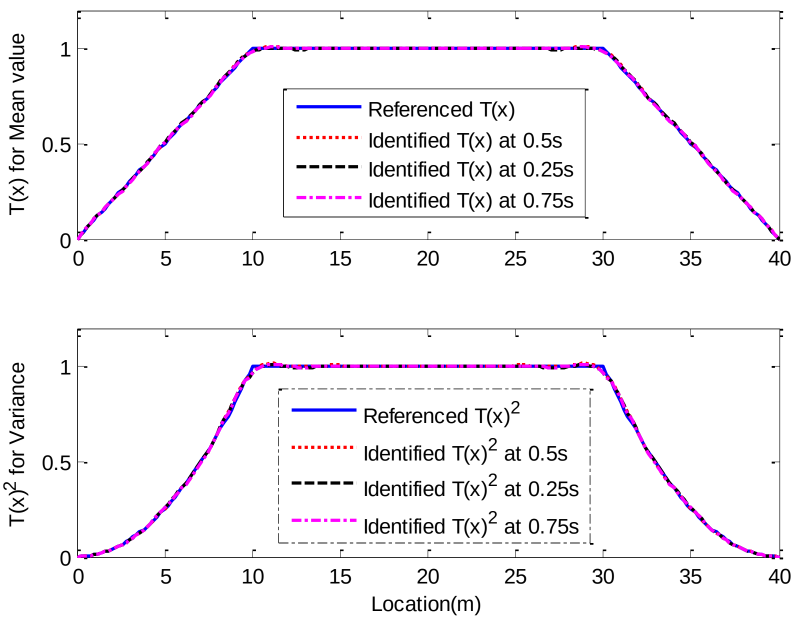

Figure 4. Comparison of the identified and referenced force distribution at different moments

5 CONCLUSIONS AND DISCUSSIONS

In order to estimate a distributed non-stationary random excitation on a vibrating structure, a time domain method is proposed and demonstrated on a beam structure. The dynamic force is assumed as a non-stationary Gaussian random field varying with both time and space. A trapezium-shaped distribution function is selected for the stochastic dynamic force in the numerical simulation. With a combination of MSM and MCA, both the distribution and the time-varying statistics of the random excitation can be estimated from response sample sets with good accuracy.

Different factors such as the number of vibration modes, the installation of sensors, the noise pollution in the response samples, etc. which may affect the accuracy of the proposed method are need to be further investigated and since the mode shape function is adopted as the orthogonal basis, when the random excitation only applied on part of the structure, numerous mode shape function should be included to achieve accurate results, which will further require more measurement sensors to be installed to acquire response samples. For this statistical identification approach in time domain, sufficient number of sample sets is assumed to avoid the unexpected errors in variance identification, when the number of samples sets reduce, the errors in the identified statistics of the random forces will increase and tend to be unstable. Investigation on aspects mentioned above will improve this primary work and will be conducted in the future work.

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